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► **To cite this version:**

K. Y. Bliokh, Y. P. Bliokh, V. Freilikher, Azriel Genack, Patrick Sebbah. Coupling and Level Repulsion in the Localized Regime: From Isolated to Quasiextended Modes. *Physical Review Letters*, 2008, 101 (13), pp.133901. 10.1103/PhysRevLett.101.133901 . hal-00432798

HAL Id: hal-00432798

<https://hal.science/hal-00432798>

Submitted on 3 Dec 2009

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Coupling and Level Repulsion in the Localized Regime: from Isolated to Quasi-Extended Modes

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The interaction of localized states in an open 1D random system is studied experimentally and theoretically by manipulating their frequencies with changes in the internal structure of the sample. As the frequencies of two states come close, they are transformed into multiply-peaked quasi-extended modes. Level repulsion is observed experimentally and explained in terms of a model of coupled resonators. The spectral and spacial evolution of the coupled modes is described in terms of the coupling coefficient and Q-factors of resonators.

PACS numbers: 42.25.Dd, 78.70.Gq, 78.90.+t

Transport in open disordered media can be diffusive or localized, depending on the nature of the underlying quasimodes, which are, respectively, spread throughout the sample or exponentially peaked at random points, with a typical size given by the localization length [1–3]. The spatial overlap of localized modes which are close in frequency, couples these states and leads to the formation of a series of exponential peaks known as necklace states [4–8]. These states are short-lived with broadened spectral lines [7, 9] and contribute substantially to the overall transmission in samples much thicker than the localization length, $L \gg l_{loc}$ [6, 8]. Though such hybridized states are critically important in transport and may play an important role in the localization transition, their formation and the correlation between spatial and spectral properties has not been explored.

In this Letter, we study the transformation of localized states in a random sample as its configuration is altered and the coupling and hybridization of modes take place. Although level repulsion is ordinarily attributed to the diffusive regime [10, 11], the energy level correlation and repulsion in electron 1D localized systems has been found theoretically and numerically [12, 13]. Here we present the first experimental evidence of the level repulsion of the localized electromagnetic excitations. A simple theoretical model is introduced which explains the spectral and spatial characteristics of coupled modes in terms of the loss and the coupling strength.

The experiment involves a rectangular microwave waveguide opened at both ends, which supports only a single transverse mode [8]. The waveguide is filled with a sample comprised of five 4 mm-thick blocks each of low and high indices of refraction randomly mixed with 31 randomly oriented 8 mm-thick binary blocks with low and high index halves. The field inside the sample is

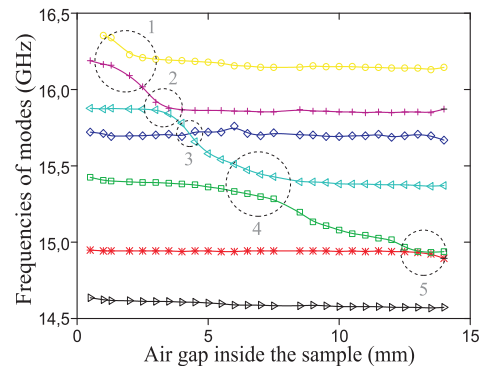


FIG. 1: (Color online.) Resonant frequencies of excited localized modes vs. the driving parameter - the air gap inside the sample. Five pair-interaction regions are circled.

weakly coupled to a cable which is translated along a 2-mm-wide slot cut along the waveguide in 1 mm steps. A sliding copper plate is pressed over the slot to eliminate leakage through the top of the waveguide. Field spectra are measured using a vector network analyzer.

Measurements are made in a sequence of configurations in which the spacing between two segments of the sample at a depth of 60 mm is increased in steps of 0.5 mm up to a maximum thickness of 14 mm. The spacing increment is sufficiently small to allow the identification of corresponding modes in configurations with different spacings. The position at which the air gap is introduced was chosen to correspond to the peak of a single localized mode. This allowed us to manipulate the frequency of the selected mode in the range from 14.5 to 16.5 GHz, which covers most of the band gap in the associated periodic sample. This is reminiscent of the tuning of a defect states through a band gap in a periodic structure as the

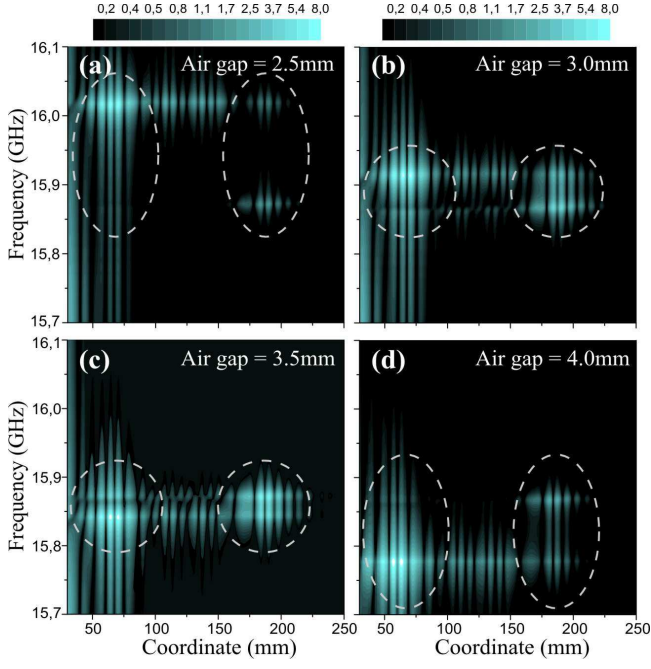


FIG. 2: (Color online.) Experimentally measured normalized wave intensity vs. frequency and coordinate inside the sample for two localized modes corresponding to region 2 in Fig. 1 at different values of the driving parameter. Excitations corresponding to the two effective resonators are circled.

defect width is increased up to a half-wavelength thickness. The mode frequency shifts across the frequencies of other localized states which makes it possible to study the coupling of two modes. We have also verified numerically that by changing the air spacing at other points in the sample at which other modes are peaked, it is possible to couple several localized modes, thereby creating necklace states extended throughout the sample.

The spectral positions of the localized states as functions of the air gap introduced into the sample are plotted in Fig. 1. The frequencies of modes may cross or exhibit an anti-crossing. In the latter case (regions 1,2,4,5 of Fig. 1), the coupling within the sample is accompanied by an exchange of shape, as is seen in Fig. 2. When the frequencies of the modes are closest, the two localized states couple into double peaked quasi-extended modes with the same spatial intensity distribution, Fig. 2b and c. In contrast, region 3 in Fig. 1 shows a mode crossing in which the shapes are not exchanged. This is seen in Fig. 3 which shows the driven mode passing through the broad mode closest to the input. The two modes remain practically independent of each other, except for the low-intensity zone (dark horizontal line in Fig. 3) at the input mode which traces the destructive interference with the driven state.

Resonant wave transmission through an isolated localized state in a random sample can be described by a simple model of a wave tunneling through a resonator

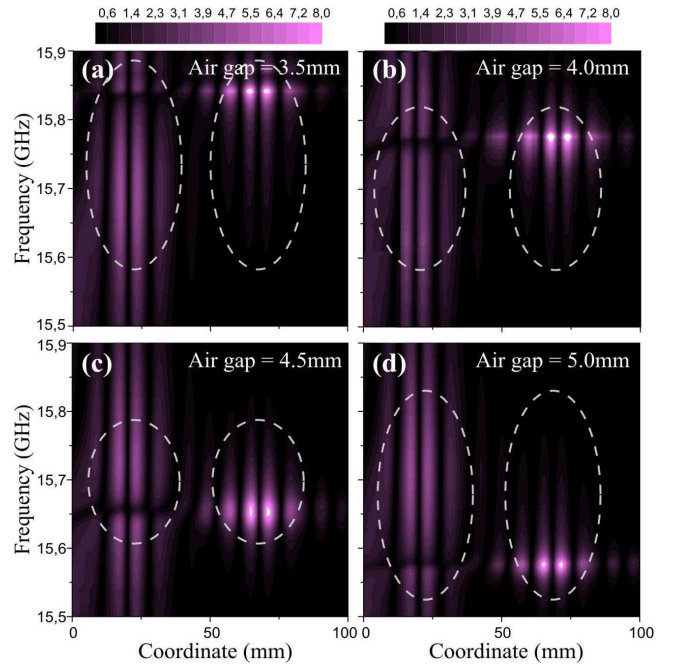


FIG. 3: (Color online.) The same as in Fig. 2 for interaction region 3 in Fig. 1.

with semitransparent walls [2, 14, 15]. Dynamics of the field in the resonator obeys the oscillator equation with an external force and damping, which accounts for the incident wave and the finite Q-factor of the resonator, respectively [15, 16]. Extending this model to the case of N localized states which are close in frequency, we arrive at a system of N coupled oscillators with the external force acting on the first of these:

$$\begin{cases} \psi_1'' + Q_1^{-1}\psi_1' + (1 - \Delta_1)^2 \psi_1 = q_{12}\psi_2 + f_0 e^{-i\nu\tau}, \\ \dots \\ \psi_l'' + Q_l^{-1}\psi_l' + (1 - \Delta_l)^2 \psi_l = q_{l+1}\psi_{l+1} + q_{l-1}\psi_{l-1}, \\ \dots \\ \psi_N'' + Q_N^{-1}\psi_N' + (1 - \Delta_N)^2 \psi_N = q_{NN-1}\psi_{N-1}. \end{cases} \quad (1)$$

Here $\psi_i(\tau)$ is the field in the i th resonator, $\tau = \omega_0 t$ is the dimensionless time (ω_0 is a characteristic central frequency of the problem), $1 - \Delta_i$ ($|\Delta_i| \ll 1$) is the dimensionless eigenfrequency of i th resonator, $Q_i \gg 1$ is the Q-factor describing the losses of the energy in the i th resonator, $q_{i+1} = q_{i+1} \ll 1$ is the coupling coefficient of i th and $(i+1)$ th resonators due to the spatial overlap of their modes; f_0 and ν , ($|\nu - 1| \ll 1$) are the amplitude and frequency of the external field exciting the first resonator. The Q-factors can be written as [16]:

$$Q_i^{-1} = \Gamma_i \quad (1 < i < N), \quad Q_{1,N}^{-1} = \Gamma_{1,N} + \frac{v_g T_{in,out}}{2l\omega_0}, \quad (2)$$

where Γ_i is the dissipation rate in the i th resonator, $T_{in,out}$ are the transmission coefficient of the input and output of the system, v_g is the wave group velocity inside

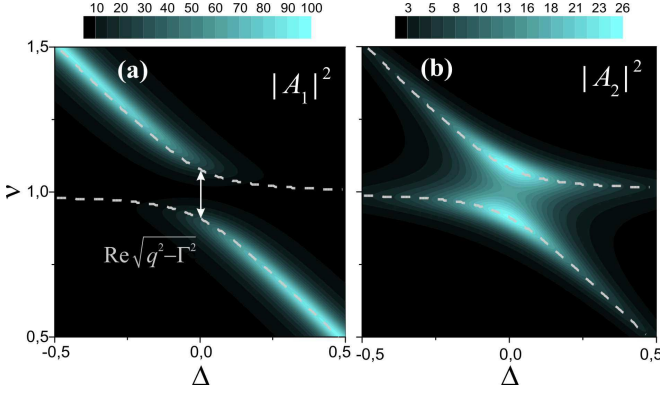


FIG. 4: (Color online.) Field amplitudes $|A_{1,2}|^2$ (6) in the two resonators as functions of the incident field frequency ν and detuning Δ between the resonators. The underlying frequencies $\delta\nu_{\text{res}}^{\pm}$ (7) are depicted by the dashed lines. Parameters are: $f_0 = 1$, $q = 0.2$, and $Q^{-1} = 0.1$ ($Q^{-1} < q$ regime).

the resonator cavity, and l is the cavity length. The last term in Eqs. (2) accounts for the energy leakage through the outermost resonators [15, 16].

To establish the correspondence between the model (1) and localized states in a random sample, we assume, following [14, 15], that ψ_i represents the peak field of i th localized state, $q_{i+1}\psi_i$ is the amplitude of the field penetrating into the adjacent cavity, and f_0 is the amplitude of the incident wave, ψ_0 , penetrating into the first localization cavity. Since the localization length is the only disorder-induced spatial scale in the problem, we have

$$\begin{aligned} q_{i+1} &\simeq \exp(-d_{i+1}/l_{\text{loc}}), \quad f_0 \simeq \psi_0 \exp(-d_{\text{in}}/l_{\text{loc}}), \\ T_{\text{in,out}} &\simeq \exp(-2d_{\text{in,out}}/l_{\text{loc}}), \quad l \sim l_{\text{loc}}. \end{aligned} \quad (3)$$

Here $d_{i+1} = |X_{i+1} - X_i|$ is the distance between adjacent states at coordinates X_{i+1} and X_i , whereas $d_{\text{in}} = X_1$ and $d_{\text{out}} = L - X_N$ are the distances from the edge localized states to the corresponding ends of the sample. Deterministic equations (1)–(3) provide an effective description of coupled modes in 1D random system.

Substituting $\psi_i = A_i \exp(-i\nu\tau)$, the set of equations (1) is reduced to an algebraic equation $\hat{H}\vec{\Psi} = \vec{F}$ with

$$\hat{H} = \begin{pmatrix} C_1 & -q_{1,2} & \dots & 0 & 0 \\ -q_{1,2} & \dots & -q_{l-1,l} & 0 & 0 \\ \dots & -q_{l-1,l} & C_l & -q_{l,l+1} & \dots \\ 0 & 0 & -q_{l,l+1} & \dots & -q_{N-1,N} \\ 0 & 0 & \dots & -q_{N-1,N} & C_N \end{pmatrix}, \quad (4)$$

$\vec{\Psi} = (A_1, \dots, A_N)^T$, and $\vec{F} = f_0(1, 0, \dots, 0)^T$, where $C_i = (1 - \Delta_i)^2 - \nu^2 - i\nu Q_i^{-1} \simeq 2(1 - \Delta_i - \nu) - iQ_i^{-1}$. The homogeneous equation $\hat{H}\vec{\Psi} = 0$ determines a set of independent eigenmodes of the system, with eigenfrequencies being the eigenvalues of the matrix (4).

For the sake of simplicity, we consider the case of two interacting modes, $N = 2$, and assume that $Q_1 = Q_2 \equiv$

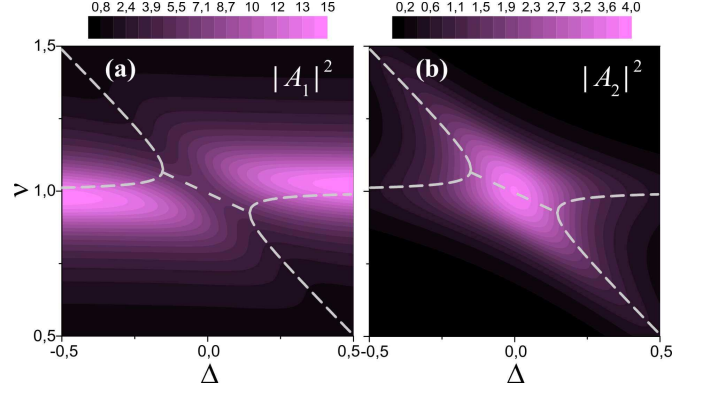


FIG. 5: (Color online.) The same as in Fig. 4 for the parameters $q = 0.2$ and $Q^{-1} = 0.25$ ($Q^{-1} > q$ regime).

Q and $q_{1,2} \equiv q$. Then, the complex eigenfrequencies $\nu^{\pm} = 1 + \delta\nu^{\pm}$ are given by

$$\delta\nu^{\pm} = -\frac{\Delta_1 + \Delta_2}{2} - i\frac{Q^{-1}}{2} \pm \frac{1}{2}\sqrt{(\Delta_1 - \Delta_2)^2 + q^2}, \quad (5)$$

This equation describes anti-crossing of levels (level repulsion) which occurs at finite $q \neq 0$, when the modes of isolated resonators couple into collective eigenmodes. The minimal frequency gap q is achieved at resonance, $\Delta_1 = \Delta_2$. Away from the resonance, $|\Delta_1 - \Delta_2| \gg q$, the eigenmodes tend to the modes of isolated resonators, exchanging when passing through the resonance, i.e., $+$ ($-$) eigenmode corresponds to the first (second) resonator at $\Delta_1 \ll \Delta_2$ and to the second (first) one when $\Delta_1 \gg \Delta_2$. It is important to note that the level repulsion of electromagnetic modes takes place in the regime of strong localization, cf. [13].

If the system is excited by an incident monochromatic wave with the real frequency $\nu = 1 + \delta\nu$, as it is in the experiment, the complex amplitudes $A_{1,2}$ in the two resonators can be obtained from $\hat{H}\vec{\Psi} = \vec{F}$, which yields

$$\begin{aligned} A_1 &= -\frac{[2(\Delta_2 + \delta\nu) + iQ_2^{-1}]f_0}{D}, \quad A_2 = \frac{qf_0}{D}, \\ D &= [2(\Delta_1 + \delta\nu) + iQ_1^{-1}][2(\Delta_2 + \delta\nu) + iQ_2^{-1}] - q^2 \end{aligned} \quad (6)$$

Behavior of $|A_{1,2}|^2$ is essentially determined by the denominator $|D|^2$, which is minimal at frequencies

$$\delta\nu_{\text{res}}^{\pm} = -\frac{(\Delta_1 + \Delta_2)}{2} \pm \frac{1}{2}\text{Re}\sqrt{(\Delta_1 - \Delta_2)^2 + q^2 - Q^{-2}}. \quad (7)$$

The amplitudes $A_{1,2}$ and frequencies $\delta\nu_{\text{res}}^{\pm}$ characterize resonant excitation of the system by an external source. Note that Eq. (7) coincide with Eq. (5) only in the lossless case $Q^{-1} = 0$. Otherwise, there are two different regimes of the excitation of coupled resonators, determined by the ratio between losses Q^{-1} and coupling q . If losses are small, $Q^{-1} < q$, two branches $\delta\nu_{\text{res}}^{\pm}$ demonstrate anti-crossing with the minimal frequency

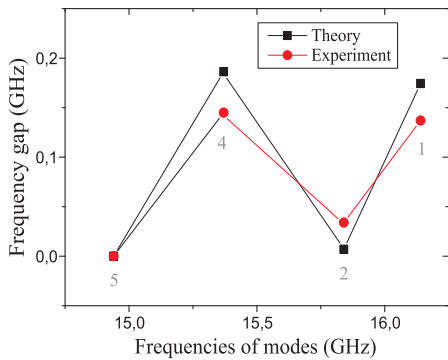


FIG. 6: (Color online.) Experimentally measured and theoretically calculated ($\text{Re}\sqrt{q^2 - Q^{-2}}$) minimal frequency gaps for pairs of interacting modes 1,2,4,5 presented in Fig. 1.

gap $\sqrt{q^2 - Q^{-2}}$, Fig. 4. If losses prevail over the coupling, so that $Q^{-1} > q$, the frequencies $\delta\nu^\pm$ merge in the interaction region $(\Delta_1 - \Delta_2)^2 \leq |q^2 - Q^{-2}|$, Fig. 5.

The amplitudes (6) as functions of the frequency ν and detuning $(\Delta_1 - \Delta_2)$ are shown at $Q^{-1} < q$ and $Q^{-1} > q$ in Figs. 4 and 5, in which the main features observed experimentally are seen (different values of Δ correspond to different frames of Figs. 2 and 3). To facilitate comparison with the experimental Figs. 2 and 3, the second, output resonator is driven in Fig. 4 ($\Delta_1 \equiv 0$, $\Delta_2 \equiv \Delta$), while the first resonator is driven in Fig. 5 ($\Delta_1 \equiv \Delta$, $\Delta_2 \equiv 0$). In the regime $Q^{-1} < q$, it is seen in Figs. 2 and 4 that in the interaction region (Figs 2b and c) fields in both resonators exhibit double-peaked spectra (level repulsion). Collective excitation of two resonators signifies formation of a quasi-extended necklace state there. Remarkably, away from the resonance (Figs. 2a and d) the first resonator is effectively excited at one of the resonant frequencies, close to $\delta\nu = -\Delta_1$, Fig. 4a) while the second resonator is equally excited at both the resonant frequencies $\delta\nu_{\text{res}}^\pm \simeq \Delta_{1,2}$ (Fig. 4b). In the regime $Q^{-1} > q$, both Figs. 3 and 5 show that the second resonator is excited with a single-peak spectrum (Fig. 5b), while the first one exhibits two peaks separated by a dark area driven with the frequency of the second resonator (Fig. 5a) [17].

The measured and calculated values of the frequency gap between coupled modes are presented in Fig. 6. The parameters of the system are:

$$\frac{\omega_0}{2\pi} \simeq 15.5\text{GHz}, \quad l_{\text{loc}} \simeq 12\text{mm}, \quad \omega_0\Gamma \simeq 7 \times 10^7\text{s}^{-1}, \quad (8)$$

and $v_g \simeq c/2.4$, whereas positions of the localized modes interacting in the regions 1–5 (Fig. 1) equal, respectively:

$$\begin{aligned} X_1 &\simeq 64 : 64 : 7 : 64 : 64 \text{ mm} , \\ X_2 &\simeq 117 : 192 : 64 : 128 : 235 \text{ mm} . \end{aligned} \quad (9)$$

Substituting values (8) and (9) into Eqs. (2) and (3) yields $Q_{1,2}^{-1}$ and q . We calculated minimal frequency gap

for interacting pairs 1,2,4,5 (for which $Q_1^{-1} \sim Q_2^{-1}$ [17]) as $\text{Re}\sqrt{q^2 - Q^{-2}}$ with $Q^{-1} = (Q_1^{-1} + Q_2^{-1})/2$, and compared with the measured gap from Fig. 1. Fig. 6 shows good agreement between the experiment and model.

In conclusion, we have observed level repulsion in the localization regime and have shown that it reflects the coupling of localization centers. The occurrence of anti-crossing or crossing of quasimodes as a sample configuration changes depends upon the ratio of the coupling strength between localized states and loss. These factors determine the statistics of nearest neighbor spacings and correlation in the widths of neighboring modes and thus the nature of wave propagation.

We thank B. Hu, J. Klosner, H. Rose, and Z. Ozinkowski for suggestions regarding the construction and implementation of the waveguide assembly. This research was sponsored by the Linkage International Grant of the Australian Research Council, National Science Foundation (DMR-0538350), PSC-CUNY, the Centre National de la Recherche Scientifique (PICS #2531 and PEPS07-20), and the Groupement de Recherches IMCODE.

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in the complex plane. Nonetheless, our simplified model with $Q_1 = Q_2$ describes quantitatively all the main fea-

tures of the coupling in this case.