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Complex behavior of ray in gaussian index profile periodically segmented waveguide

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Abstract. In this article, we present a numerical analysis concerning ray propagation in a multimode periodic segmented waveguide with a gaussian index segment profile. We show that this simple waveguide configuration exhibits a complex ray dynamics that can be regular or chaotic depending on the initial conditions.

PACS numbers: 05.45.Pq Numerical simulations of chaotic systems, 42.82.-m Integrated optics

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1. Introduction

Periodic Segmented Waveguide (PSW) is a waveguide formed by an array of high index segments embedded in a low index substrate. It has been shown that this kind of waveguides exhibits several interesting properties. They have been used in linear applications to make tapers or mode filters [1, 2, 3], by exploiting the fact that the mode size and the propagation constant can be adjusted by varying the duty-cycle (defined as the ratio of the high index segment over the period Λ_{PSW}). Elsewhere, they have been used to achieve efficient nonlinear guided wave interaction using the Quasi Phase Matching (QPM) or Balanced Phase Matching (BPM) schemes [4, 5, 6] that take advantage of a periodic reversal of the nonlinear coefficient associated with such waveguides. More recently they have been shown to exhibit interesting properties that can be useful for optical power limiting or managing applications [7, 8].

We have applied a ray analysis in the simple and ideal 2D case of a PSW characterized by a transverse gaussian index profile. The choice of the gaussian shaped index segment is motivated by the fact that such kind of profiles are naturally encountered in standard waveguide fabrication such as the usual proton exchange technic in $LiNbO_3$ [9]. It will be shown in this paper that the ray dynamics induced by a transverse gaussian index profile is a much more complex ray dynamics than that encountered with the usually considered parabolic index profile [10, 12, 13]. In particular, we show that the gaussian index profile is responsible for the emergence of a genuine chaotic behavior of the ray dynamics. Such a complex behavior is actually comparable to what has already been studied in a periodically perturbed waveguide [14, 15].

After a description of the system used, preliminary numerical results will then be discussed and finally, conclusions and perspectives will be drawn.

2. Numerical model

A schematic representation of a transverse gaussian index PSW is sketched in figure 1. A step index profile is assumed along the propagation direction z as it has been done in previous works [10]. Note that a smoother profile will not changed qualitatively our results. A ray path in a 2D medium can be described by the following equation in cartesian coordinates [16]:

$$\frac{d^2x}{dz^2} = \frac{1}{2\beta^2} \frac{dn^2(x)}{dx} \quad (1)$$

where, x and z are the transversal and longitudinal coordinates respectively, $n(x)$ is the transverse index profile, $\beta = n(x_0) \cos \theta_0$ is the invariant of the ray path, x_0 being the initial position of the ray and θ_0 being the incident angle of the ray respect to z axis. The ray path can be calculated by analyzing high and low index segments separately. We consider here dielectric waveguides characterized by a low index contrast $\delta n \approx 0.02 \ll 1$,

which allows us to neglect the reflections of rays at each interfaces. In the low index segment, $n(x) = n_s$ is constant, so that equation (1) is reduces to :

$$\frac{d^2x}{dz^2} = 0 \tag{2}$$

whose solution corresponds to (2) a straight ray. The high index segment is characterized by the following transverse gaussian index profile :

$$n(x) = n_s + \delta n e^{-\frac{x^2}{\omega^2}} \tag{3}$$

In the limit of usual classical assumption $\delta n \ll 1$ equation (1) then reads:

$$\frac{d^2x}{dz^2} = -\frac{2n_s\delta n}{\beta^2\omega^2} x e^{-\frac{x^2}{\omega^2}} \tag{4}$$

A numerical technic is required to integrate the ray path equation (4). The numerical integration is performed using the classical Runge Kutta (RK) technic [17]. Starting from initial conditions

$$\begin{aligned} x(0) &= x_0 \\ \frac{dx}{dz}(0) &= \theta_0, \end{aligned} \tag{5}$$

One can compute the ray path step by step by solving alternatively equation (2) and equation (4) by using the law refraction of each interface.

It is important to note that the gaussian shape profile makes equation (4) nonlinear respect to variable x , whereas for a parabolic index profile, equation (4) reduces to a linear equation [18]. For a parabolic PSW, the presence of a linear term in the RHS of equation (4) makes the system analogous to a parametric system [19] exhibiting parametric resonances. These resonances are responsible for a ray divergence whatever initials conditions, a feature that has already been pointed out in a previous work [10] and extended to gaussian beams [12, 13] where a similar instability may occur. For a gaussian index PSW, the situation is different because the nonlinear term in the RHS of equation (4) saturates the parametric instability and makes happen nonlinear resonance. Same phenomenon will arise for other index profiles as soon as the RHS of equation (4) is nonlinear respect to x such as, for example, exponential, erfc or sech profiles. The corresponding nonlinear resonance makes the ray dynamics much more complex as will be shown in the next section. It has to be mentioned that such a dynamics of ray propagation is comparable to the dynamics of an undamped nonlinear pendulum forced by a periodic motion of the suspension point. In complete analogy with a parametrically forced pendulum, we will show that a gaussian index PSW exhibits nonlinear resonances, frequency locking, as well as a chaotic behavior [11].

3. Results

The numerical technic outlined in the previous section has been used to compute a ray path in a gaussian profile PSW. It is worth noting that for a step [1, 20, 21] or a parabolic [10, 12] index transverse profile, some relevant waveguide properties

such as the propagation constant and the mode size, can be easily obtained from an equivalent uniform waveguide model. The refractive index difference δn_{eq} of the equivalent waveguide is defined as an average of high and low index segments:

$$\delta n_{eq} = \delta n \times \text{duty-cycle} \quad (6)$$

where, δn is the refractive index difference between the substrate and the guiding segments and the duty-cycle is defined as $\text{duty-cycle} = d/\Lambda$ where d is the segment length and Λ is the period (figure 1). The ray path of a PSW with a gaussian index profile and its equivalent uniform waveguide model are plotted on figure 2 [22]. One can note that the ray paths coincide so that the focusing period Z_p is almost identical for both waveguide. However, it can be shown that for some waveguide configurations the equivalent uniform waveguide approximation is no longer accurate. In figure 3, the focusing period Z_p is plotted as a function of the duty-cycle for a gaussian index PSW and its equivalent uniform waveguide. The waveguide parameters are the same used as in figure 2, except that the period is now $\Lambda = 45\mu\text{m}$. While the two curves are very close for most of the duty-cycle values, one can note some well pronounced differences for some particular values of duty-cycle. For duty-cycle taken between 0.55 and 0.6 the focussing period for the PSW tends to infinite, a feature which signals a divergence of the ray path. This is illustrated in figure 4, which represents the ray path for duty-cycle = 0.55. After a short propagation distance, the focusing period Z_p of the PSW deviate from the period corresponding to the uniform waveguide one, the amplitude oscillations of the ray increases and around $z = 3500\mu\text{m}$, the ray escapes from the guiding region. One can also note in figure 3 that the PSW curve exhibits some steps in which the focussing period Z_p remains constant for a given range of duty-cycle values. This important feature could not be describe with the equivalent uniform waveguide model. We are in the presence of a parametric nonlinear resonance which manifests itself through a period-locking (or frequency-locking) between two characteristic periods of the system: the segmentation period Λ and the focussing period Z_p . Period-locking occurs when the two periods (or the two frequencies) are commensurate with each other, i. e., when their ratio is a rational number $q = Z_p/\Lambda$. (One period is locked into being a rational number time q of the other period). Period-locking can be clearly identified for a ratio equal to 8, 7, 6 and 5 on figure 3. Other nonlinear resonances may also be identified for non integer values of q , but they are not clearly visible in figure 3. From a practical point of view interesting features may occurs in terms of waveguide dispersion associated with those nonlinear resonances. The average waveguide index has changed but Z_p and then the optical path length remain constant and, as a consequence, the transit time do not vary with the waveguide parameters. It means that is a waveguide dispersion free configuration.

With the help of the Poincaré's section, one can get an deeper insight of the ray dynamics. Poincaré sections are a convenient and widely used representation of a periodically perturbed nonlinear system. It consists of a projection of the trajectory (x, θ, z) onto the phase plane (x, θ) at positions $z = n\Lambda$, $n = 1, 2, 3, \dots$ [22]. Figures 5, 6 and 7 represent Poincaré sections for duty-cycle = 0.3, duty-cycle = 0.55 and

duty-cycle = 0.75 respectively for an ensemble of various initial conditions regularly distributed in phase space. Poincaré sections reveal that the dynamics is characterized by the coexistence of a regular or a chaotic behavior, depending on initial condition. A stable trajectory is constructed on a 3D torus known as a KAM (Kolmogorov-Arnold-Moser) torus which is characterized by a close curve circle on the Poincaré section. The corresponding value of q is irrational so that the ray densely covers the closed curve during its propagation. If an initial condition is taken on one of those closed curves circle, the ray path remains confined on the waveguide. On the other hand, initial conditions taken in the stochastic region surrounding the peripheral KAM torus generate an unstable trajectory that leads to a quick divergence of the ray because there is no more total internal reflection on a high index segment or the low index segment is too long and the ray cannot be recoupled into an high index segment. A set of stable islands results from a break up of one of the KAM tori in the case of a rational q value and it is the trace of a single trajectory. This corresponds to a subharmonic resonance, in which the period of the ray path Z_p is larger than the waveguide period Λ . For example, if the initial condition is taken in A (on figure 5), the next position at $z = \Lambda$ is in the other island (A1), the next one is in A2 at $z = 2\Lambda$. So that the ray path returns to the first island at $z = 8\Lambda$. After a while the points closely cover the set of 8 islands. Then, a rational q value generate $2q$ fixed points which corresponds to q pairs of hyperbolic and q elliptic points. Elliptic points refer to nonlinear resonances in which the period of the trajectory is synchronized with the period of the segmentation. For example, for duty-cycle = 0.3 (figure 5), duty-cycle = 0.55 (figure 6) and duty-cycle = 0.75 (figure 7), 8, 6 and 5 elliptic points are visible on the Poincaré sections, which respectively correspond to $q = 8$, $q = 6$ and $q = 5$. The corresponding islands arise from the same initial condition noted A on the figures. The KAM torus acts as a barrier that cannot be crossed by a ray trajectory which remains confined in a region delimited by the torus. Thus, the stability of a nonlinear resonance depends on whether the corresponding islands are surrounded or not by a KAM torus. Initial conditions noted A on figure 5 and figure 7 generate respectively 8 and 5 nonlinear resonances whose respective islands are surrounded by invariant tori. This makes the ray propagation more robust with respect to external perturbations. For instance, a small change of the duty-cycle values does not lead to a divergence of the ray. Conversely, the same initial condition noted A on figure 6 generates 6 nonlinear resonances represented by stable islands in the stochastic region. Those stable islands are not surrounded by a torus, so that for a small duty-cycle change the ray becomes unstable and escapes from the waveguide. This explains why the right part of the plateau duty-cycle = 0.55 on figure 3 is unstable.

4. Perspectives and conclusions

What has mainly interested us in this paper, concerns ray behavior in PSW characterized by a gaussian index segment profile. A simple ray propagation model has been used to

provide a preliminary numerical study of the waveguide properties. The ideal 2D case of investigation that was presented here concern first numerical results but, according to previous work of ray and wave propagation in a chaotic optical fiber [23, 24], PSW seem to be a promising and versatile system in which optical wave chaos could be observed and studied experimentally. The analysis shows a complex dynamic that can be found in usual nonlinear systems such as periodically forced nonlinear pendulum. For some waveguide parameters, the ray trajectory may be stable or chaotic depending on initial conditions. We also showed the limits of the equivalent uniform waveguide model, in that it is inherently unable to describe a nonlinear resonance. Interesting feature may also arise from the ray analysis of propagation, in particular regarding the properties of waveguide dispersion. However, deeper analysis has to be performed in order to fully investigate the potential of those waveguide. From a technological point of view, no major difficulty seems to exist to achieve an experimental study of our theoretical predictions considering the fact that the waveguide configuration proposed here is based on well known and widely used fabrication process.

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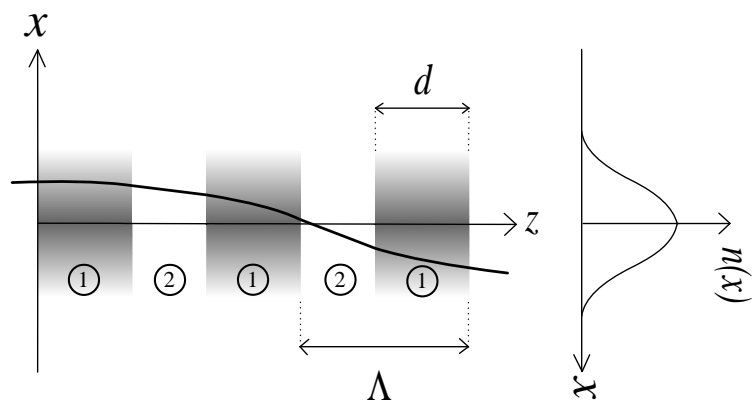


Figure 1. Schematic of the investigated longitudinally periodic waveguide and the transverse gaussian index profile

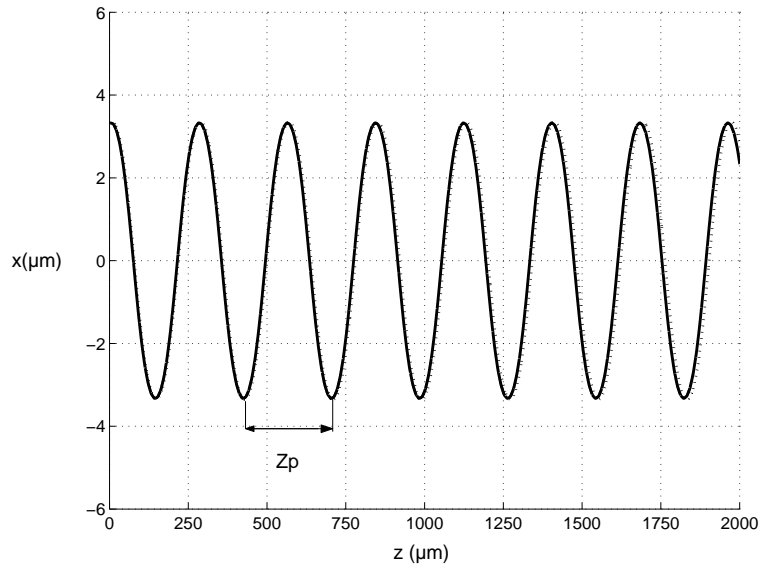


Figure 2. Ray path in a gaussian index segmented waveguide (solid line) and in the equivalent uniform waveguide (dotted line). Segmented waveguide parameters are the following ones : duty-cycle = 0.5, $\Lambda = 10\mu\text{m}$, $\delta n = 0.03$, $w = 4\mu\text{m}$ and the equivalent uniform waveguide parameters are $\delta n = 0.015$, $w = 4\mu\text{m}$. $Z_p = 275\mu\text{m}$ represent the focussing length which is almost identical for the both waveguides [22].

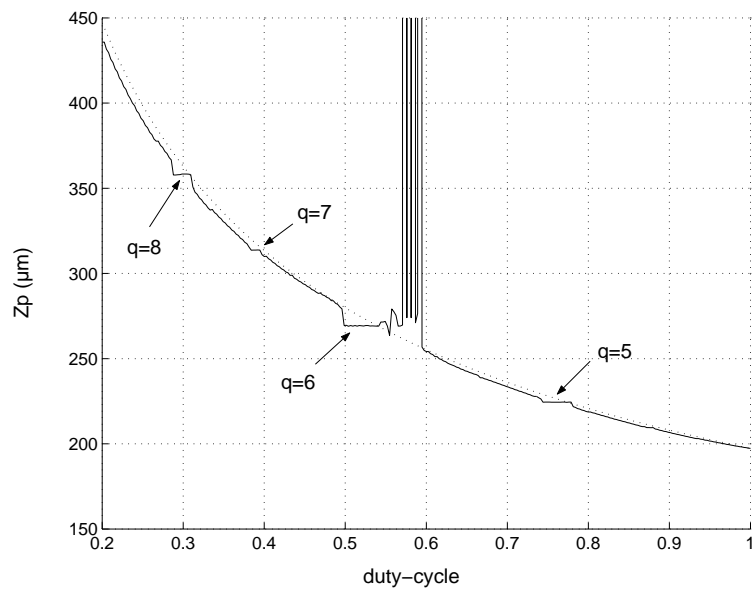


Figure 3. Focussing period Z_p as a function of duty-cycle for a gaussian index segmented waveguide (solid line) and its equivalent waveguide (dotted line). Same waveguide parameters used as in figure 2 except the period is now $\Lambda = 45\mu\text{m}$.

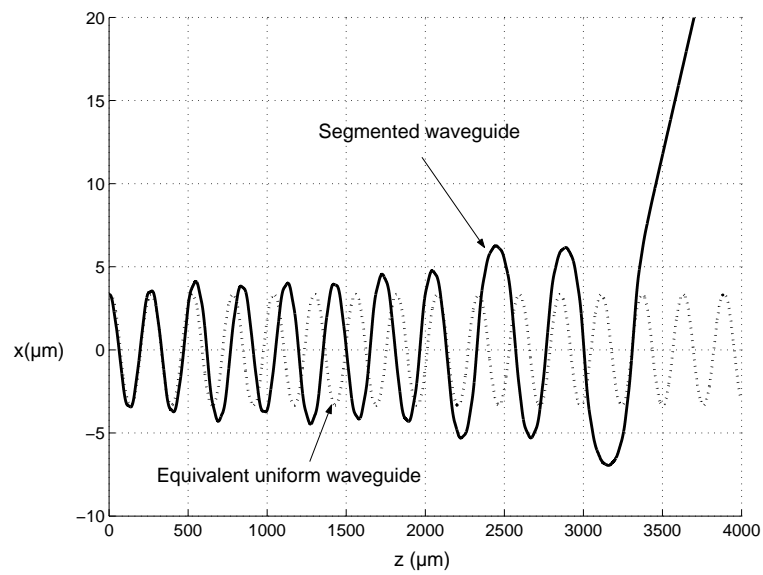


Figure 4. Ray path in a gaussian index PSW (solid line) corresponding to duty-cycle $= 0.55$ and the equivalent uniform waveguide (dot line). The focussing period Z_p of the PSW quickly shift from the uniform waveguide one and the ray diverge after some oscillations.

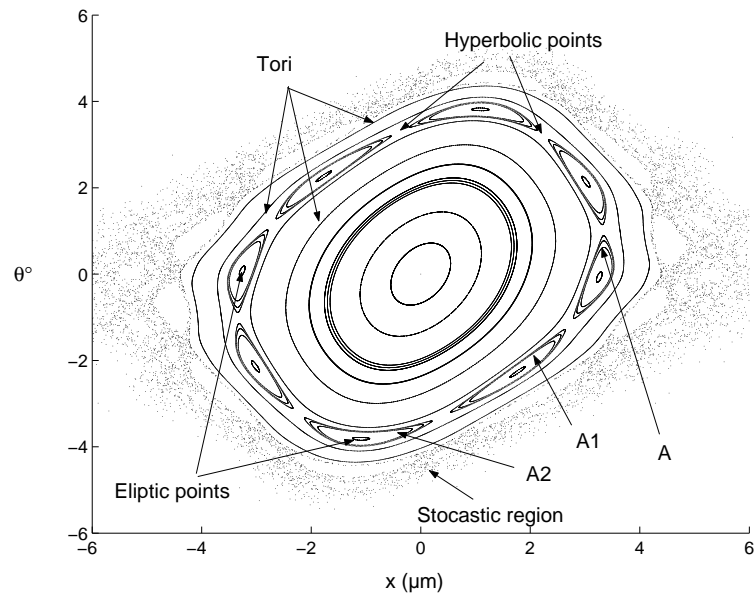


Figure 5. Poincaré section for duty-cycle = 0.3. The point A represent initial conditions used in figure 3. Initials conditions taken around the point A generate 8 fixed points which are surrounded by invariants tori.

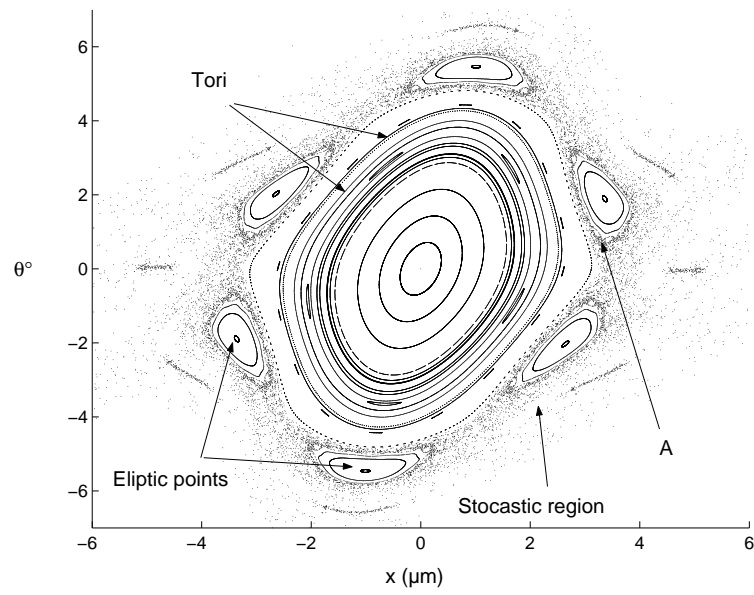


Figure 6. Poincaré section for duty-cycle = 0.55. The point A represent initial conditions used in figure 3. Initials conditions taken around the point A generate 6 fixed points which are not surrounded by invariants tori.

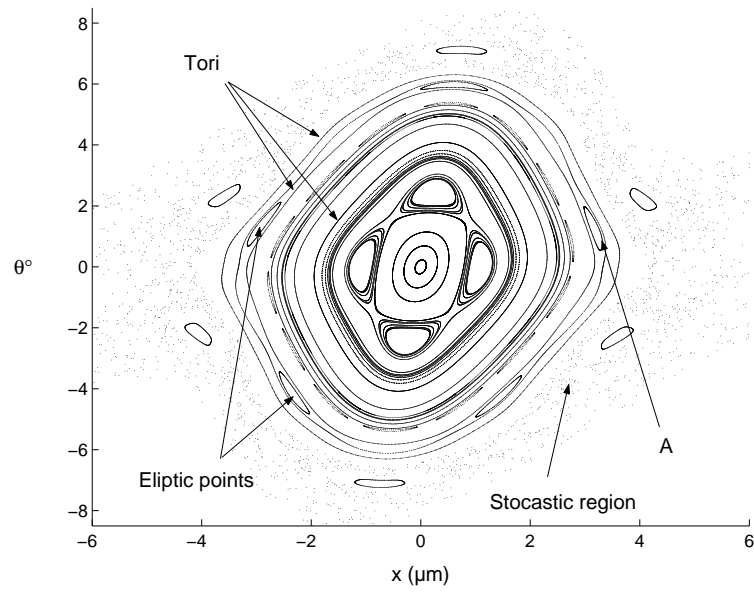


Figure 7. Poincaré section for duty-cycle = 0.75. The point A represent initial conditions used in figure 3. Initials conditions taken around the point A generate 5 fixed points which are surrounded by invariants tori.