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# Cantor Spiral Array for the Design of Thinned Arrays

S. Hebib, N. Raveu, *Member, IEEE*, and H. Aubert, *Senior Member, IEEE*

**Abstract**—This letter is focused on the design of thinned planar arrays achieving simultaneously the two following requirements: 1) a given beamwidth in the broadside direction and 2) a given peak side-lobe level in a specified sub-domain of the visible region. It is seen that Cantor spiral arrays are excellent candidates. Peak side-lobes of the order of  $-20$  dB and a beamwidth of  $0.6^\circ$  are obtained with only 200 radiating elements.

**Index Terms**—Fractal, spiral array, thinned array.

## I. INTRODUCTION

**T**HINNED planar antenna arrays are of great practical importance, especially for space applications where the minimization of the number of radiating elements in satellites is crucial. This letter is focused on the design of thinned arrays achieving simultaneously the two following requirements: 1) a given beamwidth in the broadside direction and, 2) a given peak side-lobe level in a specified subdomain of the visible range.

Among the many thinning strategies reported in the literature, Cantor Ring Arrays are promising solutions [1]. Such arrays consist in a uniform distribution of radiating elements on circles whose diameters are derived from a symmetric finite-stage polyadic Cantor set. Instead of adopting uniform distributions of elements on the circles, we propose here to locate elements at the intersection of the Cantor rings and the radials. We show that such configuration allows the design of Cantor-based arrays that reduce the number of radiating elements compared to classical ring arrays [2] composed of concentric rings with equal spacing between the rings. This is the first result reported here. Next, in order to reduce the number of radiating elements but keeping the beamwidth and peak side-lobe level relatively unchanged, we break the alignment of radiating elements in each radial by constituting spiral arms. Such “ring rotation technique” has been advantageously applied in [2] for reducing the peak side-lobe levels in arrays composed concentric rings with equal spacing between the rings.

We show here that this technique allows the design of Cantor spiral arrays that present the same performances than the original Cantor-based arrays in terms of beamwidth and side-lobe level, but with a significant reduction of 10% in the number of radiating elements. This is the second major result of the research work reported here.

## II. THE CANTOR SPIRAL ARRAY

Throughout this paper the arrays are composed of isotropic elements and equal amplitude signals are assumed at each el-

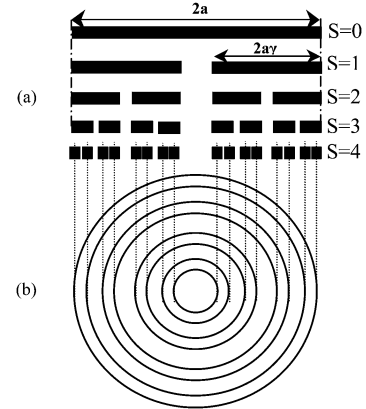


Fig. 1. (a) Symmetric polyadic Cantor set at stage of growth  $S = 1, 2, 3$ , and 4. (b) Resulting concentric rings at stage  $S = 4$ .

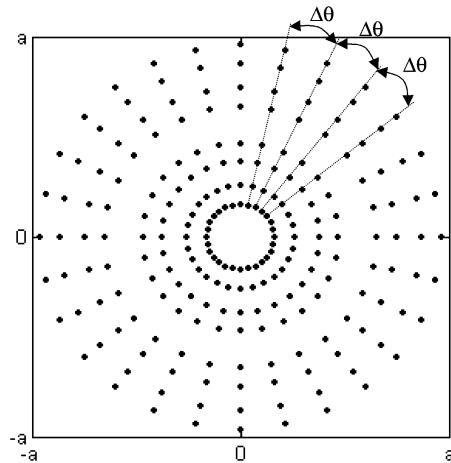


Fig. 2. Cantor-based ring array with elements located at the intersection of the Cantor rings and the equiangular radials.

ement. As illustrated in Fig. 1, in Cantor Ring Arrays [1] the elements are uniformly distributed on concentric circles whose diameters are derived from a symmetric finite-stage polyadic Cantor set (middle of each set). The descriptors of the Cantor set are the number of gaps at stage  $S$ , the stage of growth  $S$ , the reduction factor  $N$  and the length of the entire set  $L$ . The Cantor-based distribution of concentric rings is shown in Fig. 1(b) for  $N = 1, 2, 3, 4$ , and  $S = 4$ .

We propose to locate elements at the intersection of the Cantor rings and the equiangular radials. The resulting Cantor-based ring arrays present the same number of elements on all circles. Illustration of such arrays is given in Fig. 2 for 28 radiating elements.

Let us determine the Cantor-based ring array with a minimum number of radiating elements that satisfies the two following requirements:

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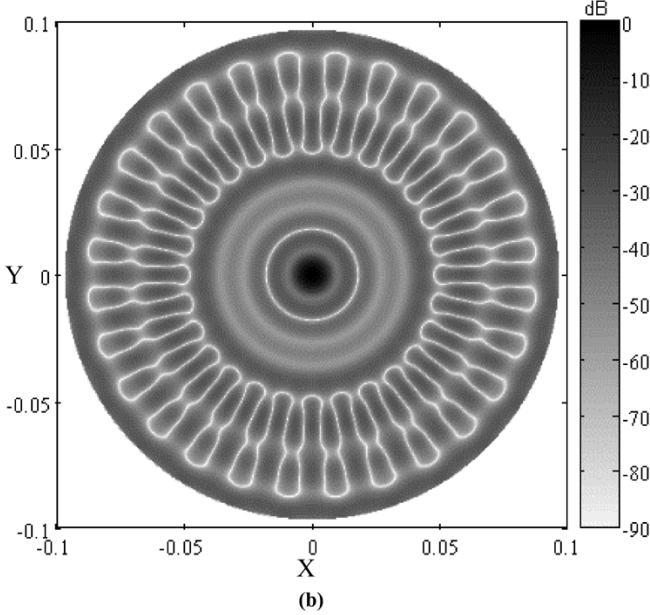
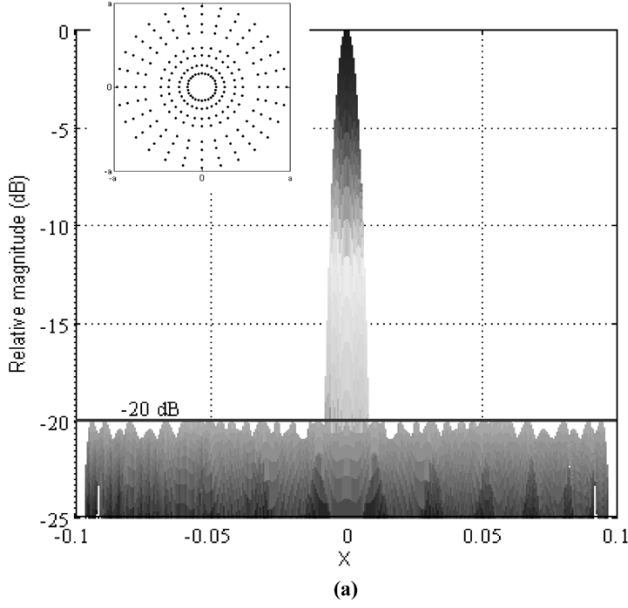


Fig. 3. Normalized array factor of the Cantor-based ring array shown in the insert of (a) with  $N = 1$ ,  $\gamma = 0.4367$ ,  $a = 56.23\lambda$  and  $S = 4$ : (a) side view and (b) top view.

- 1) a beamwidth of  $0.6^\circ$  in the broadside direction;
- 2) a peak side-lobe level of the order of  $-20$  dB in the range  $[-8.7^\circ, +8.7^\circ]$  around the broadside direction.

Genetic Algorithms (GAs) are used [3]–[5] to derive such thinned array. GAs have been chosen for their ability to converge toward a global solution, in the case of complex functions [6], [7]. The fitness function  $f$  is here given by

$$f = \frac{1}{|\text{peak side-lobe level (dB)}|}. \quad (1)$$

A random crossover point is applied with a probability of 0.7. A random mutation on one bit is performed with a probability of 0.1 (the mutation allows to avoid local minima). The GA reaches a solution for Cantor-based ring array with 224 sources,

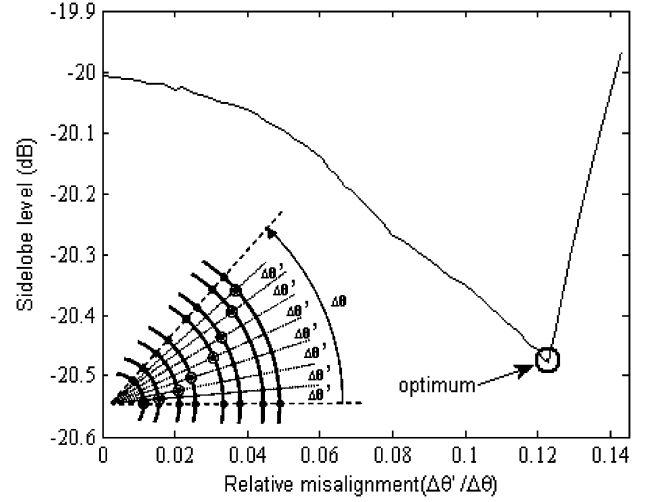


Fig. 4. Sidelobe level versus relative misalignment.

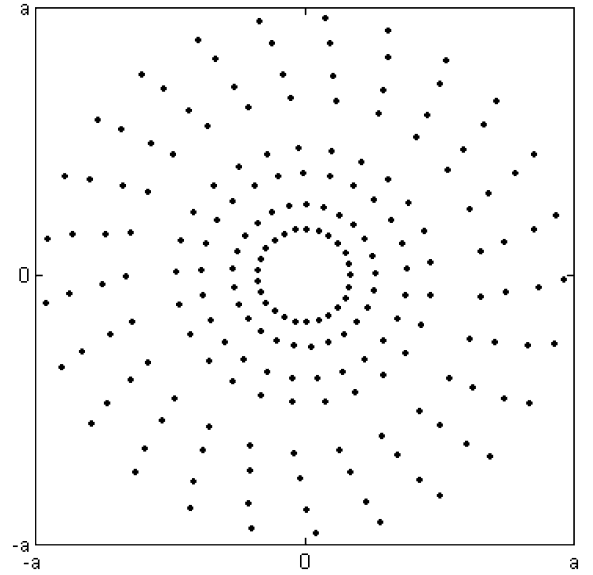


Fig. 5. Cantor spiral array.

$N = 1$ ,  $S = 4$ ,  $\gamma = 0.4367$  and  $a = 56.23\lambda$ , where  $\lambda$  designates the free-space wavelength. The resulting array factor is given in Fig. 3 ( $X$  and  $Y$  traduce the polar into cartesian coordinates normalized by  $90^\circ$ ). For comparison, classical ring arrays composed of concentric rings with equal spacing between the rings [2], with the same number of radiating elements (224) and the same beamwidth ( $0.6^\circ$ ) presents a peak side-lobe level of the order of  $-12$  dB, that is, 8 dB higher than one of the Cantor-based ring array.

Next, in order to reduce the number of radiating elements but keeping the beamwidth and peak side-lobe level relatively unchanged, we break the alignment of radiating elements in each radial. As shown in the insert of Fig. 4, elements of the original Cantor-based ring arrays are shifted by a constant angle  $\Delta\theta'$  so as to generate spiral arms (see Fig. 5). The peak side-lobe levels and beamwidth are determined for several normalized ratio  $\Delta\theta'/\Delta\theta$ , where  $\Delta\theta$  stands for the constant angle between two radials in the original Cantor-based ring array (see Fig. 2). In order to keep a relatively constant beamwidth, the

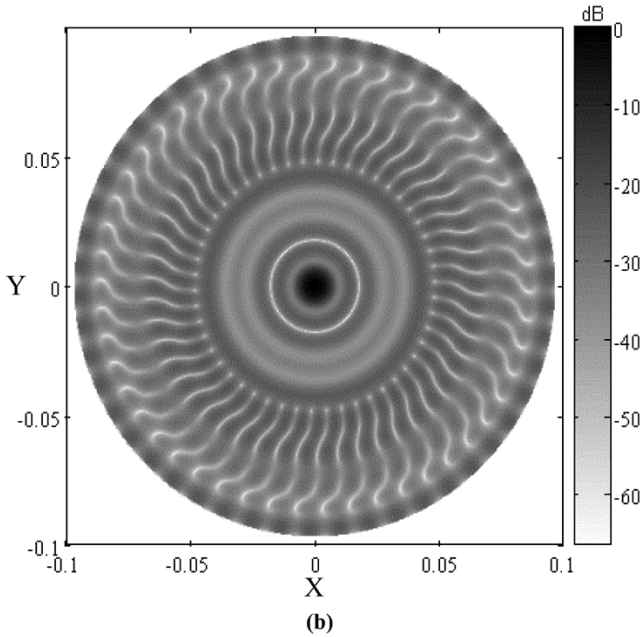
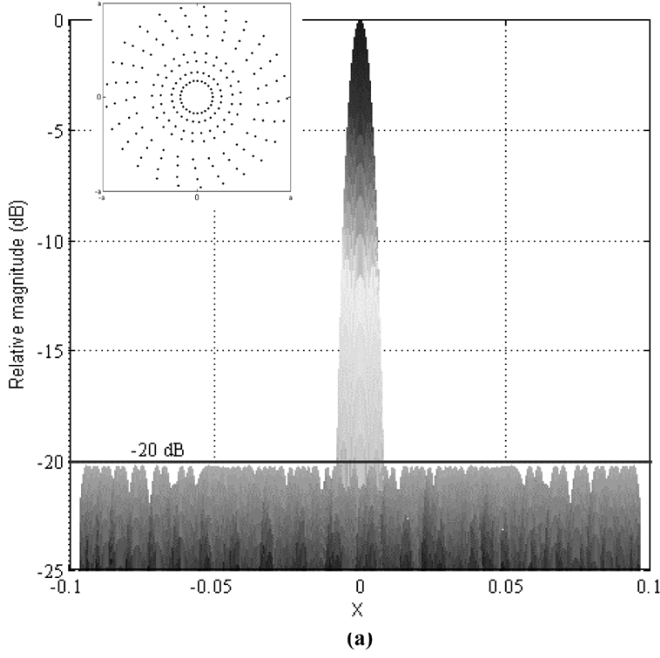


Fig. 6. Normalized array factor of the Cantor spiral array shown in the insert of (a) with  $N = 1$ ,  $S = 4$ ,  $\gamma = 0.43$ ,  $a = 55.7\lambda$  and  $\Delta\theta'/\Delta\theta = 0.13$ : (a) side view and (b) top view.

parameters  $\gamma$  and  $a$  have to be slightly tuned for each value of  $\Delta\theta'/\Delta\theta$ . As an example, consider the 224 radiating elements of

the Cantor-based ring array shown in the insert of Fig. 3 and determine the variation of the peak side-lobe level versus  $\Delta\theta'/\Delta\theta$  in a Cantor spiral array (see Fig. 4).

We observe that the Cantor spiral array such that  $\Delta\theta'/\Delta\theta = 0.123$  present a slight improvement in term of side-lobe level compared to the original Cantor-based ring array ( $-20.5$  dB instead of  $-20$  dB). Moreover we derive from GA that the Cantor spiral array with a beamwidth of  $0.6^\circ$  and a peak side-lobe level of the order of  $-20$  dB presents only 200 elements. This interesting thinned array, shown in Fig. 5, has the following parameters:  $N = 1$ ,  $S = 4$ ,  $\gamma = 0.43$ ,  $a = 55.7\lambda$  and  $\Delta\theta'/\Delta\theta = 0.13$ . Consequently, in terms of beamwidth and peak side-lobe level, this Cantor spiral array present the same performances than the original Cantor-based ring array, but with a significant reduction of 10% in the number of radiating elements. The normalized array factor of this thinned array is shown in Fig. 6 in the range of interest  $[-8.7^\circ, +8.7^\circ]$  around the broadside direction.

### III. CONCLUSION

For a given number of radiating elements and beamwidth in the broadside direction, Cantor-based ring arrays may present lower peak side-lobe level than one of the classical ring arrays composed of concentric rings with equal spacing between the rings. Moreover, in terms of beamwidth and peak side-lobe level, Cantor spiral arrays present the same performances than Cantor-based ring arrays with alignment of radiating elements, but with a significant reduction of 10% in the number of elements.

### REFERENCES

- [1] D. L. Jaggard and A. D. Jaggard, "Cantor ring arrays," *Microw. Opt. Technol. Lett.*, vol. 12, pp. 131–136, Jun. 1996.
- [2] J. W. Sherman and M. I. Skolnik, "Thinning planar arrays antennas with ring arrays," *IRE Int. Conv. Rec.*, vol. 11, pp. 77–86, Mar. 1963.
- [3] R. L. Haupt, "Thinned arrays using genetic algorithms," *IEEE Trans. Antennas Propag.*, vol. 42, no. 7, pp. 993–999, Jul. 1994.
- [4] S. E. El-Khamy, M. A. Lotfy, M. H. Ramadan, and A. A. El-Tayeb, "Thinned multi-ring arrays using genetic algorithms," in *Proc. Eighteenth Nat. Radio Sci. Conf., NRSC'2001*, vol. 1, Mansoura, Egypt, Mar. 27–29, 2001, pp. 113–121.
- [5] D. Marciano and F. Duràn, "Synthesis of antenna arrays using genetic algorithms," *IEEE Antennas Propag. Mag.*, vol. 42, no. 3, pp. 12–20, Jun. 2000.
- [6] R. L. Haupt and S. E. Haupt, *Practical Genetic Algorithms*, 2nd ed. New York: Wiley, 2004.
- [7] R. L. Haupt, "An introduction to genetic algorithms for electromagnetics," *IEEE Antennas Propag. Mag.*, vol. 37, no. 2, pp. 7–15, Apr. 1995.