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Doppler-tuned Bragg Spectroscopy of Excited Levels in He-Like Uranium: a discussion of the uncertainty contributions

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Abstract. We present the uncertainty discussion of a recent experiment performed at the GSI storage ring ESR for the accurate energy measurement of the He-like uranium $1s2p\ ^3P_2 \rightarrow 1s2s\ ^3S_1$ intra-shell transition. For this propose we used a Johann-type Bragg spectrometer that enables to obtain a relative energy measurement between the He-like uranium transition, about 4.51 keV, and a calibration x-ray source. As reference, we used the $K\alpha$ fluorescence lines of zinc and the Li-like uranium $1s^22p\ ^2P_{3/2} \rightarrow 1s^22s\ ^2S_{1/2}$ intra-shell transition from fast ions stored in the ESR. A comparison of the two different references, i.e., stationary and moving x-ray source, and a discussion of the experimental uncertainties is presented.

1. Introduction

We present the uncertainty discussion of a recent experiment performed at the GSI (Darmstadt, Germany) for the accurate energy measurement of the He-like uranium $1s2p\ ^3P_2 \rightarrow 1s2s\ ^3S_1$ intra-shell transition. This measurement allows, for the first time, to test two-photons Quantum Electrodynamics in He-like heavy ions. In this article we describe the techniques adopted in the measurement where x rays emitted from fast ions in a storage ring have been detected with a Bragg spectrometer. In particular, we study the contribution of the different uncertainties related to the relativistic velocity of the ions when a stationary or moving calibration x-ray source is considered. Additional details of such an experiment can be found in Ref. [1].

2. Description of the set-up

The experiment was performed at the GSI experimental storage ring ESR [2] in August 2007. Here, a H-like uranium beam with up to 10^8 ions was stored, cooled, and decelerated to an energy of 43.57 MeV/u. Excited He-like ions were formed by electron capture during the interaction of the ion beam with a supersonic nitrogen gas-jet target. At the selected velocity, electrons

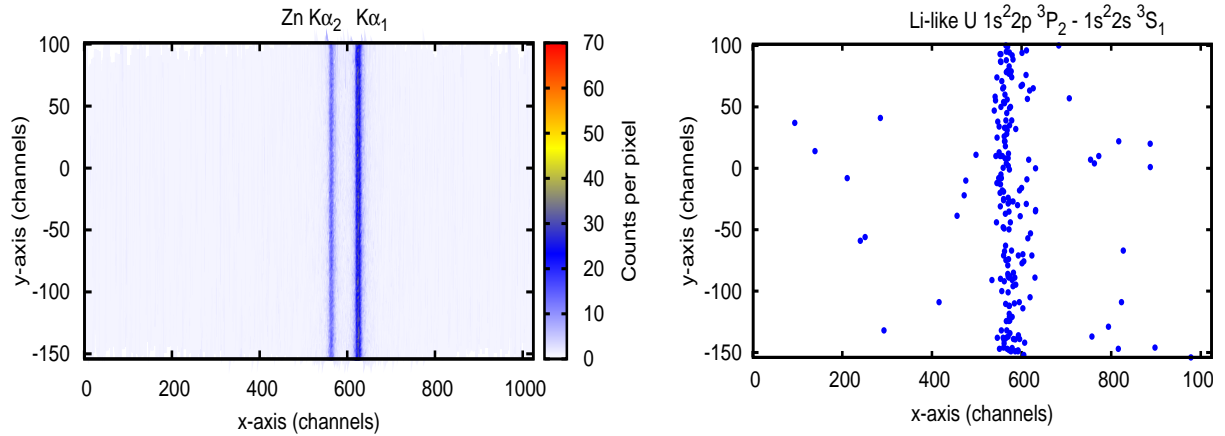


Figure 1. Reflection of the zinc $K\alpha$ (left) and Li-like uranium intra-shell $1s^22p^2P_{3/2} \rightarrow 1s^22s^2S_{1/2}$ transition (right) on the Bragg spectrometer CCD. The transition energy increases with the increasing of x-position. The slightly negative slope of the line is due to the relativistic velocity of the Li-like ions.

are primarily captured into shells with principal quantum number of $n \leq 20$, which efficiently populate the $n = 2 \ ^3P_2$ state via cascade feeding. This state decays to the $n = 2 \ ^3S_1$ state via an electric dipole (E1) intra-shell transition (branching ratio 30%) with the emission of photons of an energy close to 4.51 keV detected by a Bragg spectrometer.

The crystal spectrometer [3] was mounted in the Johann geometry in a fixed angle configuration allowing for the detection of x rays with a Bragg angle Θ around 46.0° . The spectrometer was equipped with a Ge(220) crystal cylindrically bent, with a radius of curvature $R = 800$ mm, and a newly fitted x-ray CCD camera (Andor DO420) as position sensitive detector. The imaging properties of the curved crystal were used to resolve spectral lines from fast x-ray sources nearly as well as for stationary sources [4]. For this purpose, it was necessary to place the Rowland-circle plane of the spectrometer perpendicular to the ion-beam direction. For a minimizing the systematic effects due to the ion velocity and alignment uncertainties, the observation angle $\theta = 90^\circ$ was chosen.

The value of the ion velocity v was selected such that the photon energy, E in the ion frame, was Doppler-shifted to the value $E_{\text{lab}} = 4.3$ keV in the laboratory frame, where $E_{\text{lab}} = E/[\gamma(1 - \beta \cos \theta)]$, with $\beta = v/c$ (c is speed of light) and $\gamma = 1/\sqrt{1 - \beta^2}$. This value of E_{lab} was chosen to have the He-like uranium spectral line position on the CCD close to the position of the 8.6 keV $K\alpha_{1,2}$ lines of zinc, which were observed in second order diffraction. The zinc lines were used for calibration and they were produced by a commercial x-ray tube and a removable zinc plate between the target chamber and the crystal. An image of the zinc $K\alpha$ lines from the Bragg spectrometer is presented in Fig. 1 (left side).

As an alternative method to the measurement of the He-like uranium intra-shell transition, we used a calibration line originating from fast ions, rather than one from the stationary source. For this purpose the $1s^22p^2P_{3/2} \rightarrow 1s^22s^2S_{1/2}$ transition in Li-like U at 4459.37 ± 0.21 eV [5, 6] was chosen. At the ESR, the Li-like ions were obtained by electron capture into He-like uranium ions. To match the energy of the He-like transition, an energy of 32.63 MeV/u was used to Doppler-shift the Li-like transition. An image of the Li-like uranium transition in the Bragg spectrometer is presented in Fig. 1 (right side).

Starting from the Bragg's law in differential form, $\Delta E \approx -E \Delta\Theta / \tan \Theta$, one obtains an approximate dispersion formula that is valid for small Bragg angle differences $\Delta\Theta$. Taking into account the relativistic Doppler effect, the measured value of the He-like U transition is given

by

$$E = E_0 \frac{n}{n_0} \frac{(1 - \delta(E_0)/\sin^2 \Theta_0)}{(1 - \delta(E)/\sin^2 \Theta)} \frac{\gamma(1 - \beta \cos \theta)}{\gamma_0(1 - \beta_0 \cos \theta)} \left(1 + \frac{\Delta x}{\tan \Theta_0 D}\right), \quad (1)$$

where n_0 and n are the diffraction order of the He-like U and reference lines, respectively, Θ_0 and $\Theta = \Theta_0 + \Delta\Theta$ the correspondent Bragg angle, Δx is the position difference of the spectral lines on the CCD along the dispersion direction and D is the crystal-CCD distance. $\delta(E)$ is the deviation of the index of refraction $n_r(E) = 1 + \delta(E)$ of the crystal material from the unity, which depends on the energy E of the reflected x ray ($\delta \sim 10^{-5} - 10^{-6}$ typically).

In the case $n = n_0$, the corrections due to the refraction index and other energy dependent corrections for curved crystals [7, 8] are negligible. In the case of a stationary calibration source, $\gamma_0 = 1$ and $\beta_0 = 0$.

The measured value of the He-like uranium transition energy and additional information can be found in Ref. [1]. In the following section we present in detail the analysis of the systematic uncertainties.

3. Evaluation of the experimental uncertainties

One of the main sources of uncertainty in the present experiment is the low amount of collected data (see Fig. 1, right). This limits the accuracy of the He- and Li-like uranium line position, i.e., the accuracy of Δx , which is proportional to the energy uncertainty, $(\delta E)_{\text{stat}} \propto \delta(\Delta x)$ (see Eq. (1)). In our specific experiment, characterized by the parameters listed in Table 1, numerical values of $(\delta E)_{\text{stat}}$ are presented in Table 2. Due to the high statistics in the stationary calibration source measurement, the Zn $K\alpha$ spectrum, $(\delta E)_{\text{stat}}$ is $\sim \sqrt{2}$ smaller than when the moving Li-like ion emission is used as a reference.

In the case of systematic uncertainties, three major sources dominate: the accuracy of the reference energy E_0 , of the ion velocity and of the observation angle θ . Similarly to the statistical uncertainty, the contribution of δE_0 is much smaller when Zn lines are used for calibration instead of the Li-like U transition (see Table 2). This is due to the high accuracy of the zinc $K\alpha$ transition energy, which in the case of $K\alpha_1$ is 8638.906 ± 0.073 eV [9], compared to the Li-like U transition accuracy of 0.21 eV [5, 6].

If on one hand, Doppler tuning of the photon energy in the laboratory frame produce two important systematic uncertainty contributions, due to the ions velocity and the observation angle; on the other hand, it allows for detecting the different spectral lines in the same narrow spatial region of the CCD detector, i.e., $\Delta x/D \ll 1$. This results in a drastic reduction of other systematic effects such as the influence of uncertainty of the crystal-CCD distance D , the accuracy of the CCD pixel size [10], the accuracy of the inter-plane distance of the crystal and effects from the optical aberrations in the Johann geometry set-up. Systematic uncertainties related to the relativistic velocity of the ions are treated in detail in the following subsections.

3.1. Ion velocity uncertainty

The ion velocity in the storage ring is imposed by the velocity of the electrons in the electron cooler [2]. This is related to the cooler voltage V by the simple relations

$$\gamma = 1 + \frac{eV}{m_e c^2}, \quad \beta = \frac{\sqrt{2 \frac{eV}{m_e c^2} + \left(\frac{eV}{m_e c^2}\right)^2}}{1 + \frac{eV}{m_e c^2}} \simeq \sqrt{2 \frac{eV}{m_e c^2}}, \quad (2)$$

where m_e and e are the mass and charge of the electron, respectively. The factor $eV/(m_e c^2)$ is in general very small, of the order of 4×10^{-2} in our specific case. The cooler voltage uncertainty δV propagates to the energy uncertainty via the parameters γ and β in Eq. (1). More precisely,

$(\delta\gamma)_V = e/(m_e c^2) \delta V$ and $(\delta\beta)_V \simeq [e/(2m_e c^2 V)]^{1/2} \delta V$. We note that $\delta\gamma/\delta\beta = \mathcal{O}(\sqrt{eV/(m_e c^2)})$. For this reason an observation angle of 90° , where the effect of $\delta\beta$ is minimal, was chosen. In the following formulas we will consider only the case $\theta = 90^\circ$.

The uncertainty of the cooler voltage has two principal sources: the accuracy of the absolute value and its linearity. The relation between the real voltage value V_{real} and the set value V can be written as $V = a + b V_{\text{real}}$, where the single uncertainties on the factors a and b has to be considered for our analysis. With this notation, for an observation angle of $\theta = 90^\circ$, the uncertainty propagation to the energy value is, for the case of a stationary calibration source,

$$\frac{(\delta E)_a}{E} = \frac{1}{\gamma} \left| \frac{\partial \gamma}{\partial a} \right| \delta a = \frac{e}{m_e c^2 \gamma} \delta a, \quad \frac{(\delta E)_b}{E} = \frac{1}{\gamma} \left| \frac{\partial \gamma}{\partial b} \right| \delta b = \frac{eV}{m_e c^2 \gamma} \delta b, \quad (3)$$

and

$$\frac{(\delta E)_a}{E} = \frac{\gamma_0}{\gamma} \left| \frac{\partial}{\partial a} \left(\frac{\gamma}{\gamma_0} \right) \right| \delta a = \frac{e^2}{m_e^2 c^4} \frac{|V - V_0|}{\gamma \gamma_0} \delta a, \quad \frac{(\delta E)_b}{E} = \frac{\gamma_0}{\gamma} \left| \frac{\partial}{\partial b} \left(\frac{\gamma}{\gamma_0} \right) \right| \delta b = \frac{e}{m_e c^2} \frac{|V - V_0|}{\gamma \gamma_0} \delta b, \quad (4)$$

in the case of a moving calibration source, where V_0 and γ_0 are the corresponding parameters and where the approximation $V \approx V_{\text{real}}$ has been applied. A reduction of the uncertainties is obtained when the moving calibration source is used. The uncertainty $(\delta E)_a$ due to the offset error of V is drastically decreased, by a factor $e|V - V_0|/(m_e c^2 \gamma_0) \ll 1$, here as the uncertainty due to the linearity is also reduced, but only by a factor $|V - V_0|/(V \gamma_0)$. Numerical values for our experiments are given in Table 1 and 2.

3.2. Observation angle uncertainty

If the effect of the uncertainty due to the ion velocity is minimal when $\theta = 90^\circ$, the effect of the uncertainty $\delta\theta$ of the observation angle itself is maximal. In the case $\theta = 90^\circ$, starting from Eq. (1), for a stationary calibration source we have

$$\frac{(\delta E)_\theta}{E} = \frac{1}{1 - \beta \cos \theta} \left| \frac{d}{d\theta} (1 - \beta \cos \theta) \right|_{\theta=90^\circ} \delta\theta = \beta \delta\theta, \quad (5)$$

and

$$\frac{(\delta E)_\theta}{E} = \frac{1 - \beta_0 \cos \theta}{1 - \beta \cos \theta} \left| \frac{d}{d\theta} \left(\frac{1 - \beta \cos \theta}{1 - \beta_0 \cos \theta} \right) \right|_{\theta=90^\circ} \delta\theta = |\beta - \beta_0| \delta\theta. \quad (6)$$

Table 1. Principal parameters of the ion beams for the He- and Li-like transition measurement (see text).

	He-like U	Li-like U
β	0.295578	0.257944
γ	1.046771	1.035026
$V(\text{Volt})$	23900	17898
θ	$90.00^\circ \pm 0.38^\circ$	
$a(\text{Volt})$	0 ± 10	
b	$1. \pm 2 \times 10^{-4}$	

Table 2. Different uncertainty contributions (in eV) when Li-like uranium and zinc transitions are used as the reference.

	Li-like U	Zn $K\alpha$
$(\delta E)_{\text{stat}}$	0.43	0.30
$(\delta E)_{E_0}$	0.21	0.04
$(\delta E)_a$	1×10^{-3}	0.08
$(\delta E)_b$	0.01	0.04
$(\delta E)_\theta$	0.11	0.88
$(\delta E)_{\text{TOT}}$	0.50	0.93

for a moving calibration source. Again, analogously to the calculation of the preceding subsection, a reduction of a factor $|\beta - \beta_0|/\beta$ in the uncertainty is obtained when x rays from fast ions are used for the calibration. In our experimental set-up, the value of $\delta\theta$ is principally due to the accuracy of the position of the gas-jet target with respect to the main axis of the spectrometer (± 0.5 mm). Numerical values for our experiments are in Table 1 and 2. A direct evaluation of deviation $\Delta\theta$ from 90° can be obtained via Eq. (6) with the measurement of the Li-like uranium energy 4460.12 ± 0.31 eV (statistical uncertainty only) using the zinc $K\alpha$ lines as reference, and its comparison with the literature value 4459.37 ± 0.21 eV [5, 6]. We estimated $\Delta\theta = -0.37^\circ \pm 0.18^\circ$ in agreement with the expected deviation (see Table 1).

4. Conclusions

We present the uncertainty discussion of an accurate energy measurement on He-like uranium intra-shell transition obtained via Bragg spectroscopy of Doppler-tuned x rays emitted from fast ions. We have evaluated and compared the systematic uncertainties when a stationary or moving calibration source is used. In particular in our experiment, the use of the x-ray emission of fast Li-like uranium ions as reference enables to reduce systematic uncertainties by a factor of about 4.

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