



**HAL**  
open science

# On the characterization of the regions of feasible trajectories in the workspace of parallel manipulators

Damien Chablat, Philippe Wenger

► **To cite this version:**

Damien Chablat, Philippe Wenger. On the characterization of the regions of feasible trajectories in the workspace of parallel manipulators. Tenth World Congress On The Theory Of Machines And Mechanisms, Jun 1999, Oulu, Finland. pp.1-6. hal-00428530

**HAL Id: hal-00428530**

**<https://hal.science/hal-00428530v1>**

Submitted on 28 Oct 2009

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# TENTH WORLD CONGRESS ON THE THEORY OF MACHINES AND MECHANISMS

Oulu, Finland, June 20-24, 1999

## ON THE CHARACTERIZATION OF THE REGIONS OF FEASIBLE TRAJECTORIES IN THE WORKSPACE OF PARALLEL MANIPULATORS

Damien Chablat<sup>1</sup> Philippe Wenger<sup>2</sup>

<sup>1</sup>McGill Centre for Intelligent Machines  
McGill University  
817 Sherbrooke Street West  
Montreal, Quebec, Canada H3A 2K6  
Chablat@cim.mcgill.ca

<sup>1</sup>INRIA Rocquencourt  
Domaine de Voluceau  
B.P. 105  
78 153 Le Chesnay France  
Philippe.Wenger@ircyn.ec-nantes.fr

<sup>2</sup>Institut de Recherche  
en Cybernétique de Nantes  
1, rue de la Noë  
44321 Nantes, France

*Key words :* Fully Parallel Manipulator, Singularity, Point-to-Point Trajectory, Continuous Trajectory, Octree

## 1 Introduction

It was shown recently that parallel manipulators with several inverse kinematic solutions have the ability to avoid parallel singularities [Chablat 1998a] and self-collisions [Chablat 1998b] by choosing appropriate joint configurations for the legs. In effect, depending on the joint configurations of the legs, a given configuration of the end-effector may or may not be free of singularity and collision. Characterization of the collision/singularity-free workspace is useful but may be insufficient since two configurations can be accessible without collisions nor singularities but it may not exist a feasible trajectory between them.

The goal of this paper is to define the maximal regions of the workspace where it is possible to execute trajectories. Two different families of regions are defined : 1. those regions where the end-effector can move between any set of points, and 2. the regions where any continuous path can be tracked. These regions are characterized from the notion of aspects and free-aspects recently defined for parallel manipulators [Chablat 1998b]. The construction of these regions is achieved by enrichment techniques and using an extension of the octree structures to spaces of dimension greater than three. Illustrative examples show the interest of this study to the optimization of trajectories and the design of parallel manipulators.

## 2 Preliminaries

### 2.1 Kinematics

The input vector  $\mathbf{q}$  (the vector of actuated joint values) is related to the output vector  $\mathbf{X}$  (the vector of configuration of the moving platform) through the following general equation :

$$F(\mathbf{X}, \mathbf{q}) = 0 \quad (1)$$

Vector  $(\mathbf{X}, \mathbf{q})$  will be called *manipulator configuration* and  $\mathbf{X}$  is the platform configuration and will be more simply termed *configuration*. Differentiating equation (1) with respect to time leads to the velocity model

$$\mathbf{A}\mathbf{t} + \mathbf{B}\dot{\mathbf{q}} = 0 \quad (2)$$

With  $\mathbf{t} = [w, \dot{\mathbf{c}}]^T$ , for planar manipulators ( $w$  is the scalar angular-velocity and  $\dot{\mathbf{c}}$  is the two-dimensional velocity vector of the operational point of the moving platform),  $\mathbf{t} = [\mathbf{w}]^T$ , for spherical manipulators and  $\mathbf{t} = [\mathbf{w}, \dot{\mathbf{c}}]^T$ , for spatial manipulators ( $\dot{\mathbf{c}}$  is the three-dimensional velocity vector and  $\mathbf{w}$  is the three-dimensional angular velocity-vector of the operational point of the moving platform).

Moreover,  $\mathbf{A}$  and  $\mathbf{B}$  are respectively the direct-kinematics and the inverse-kinematics matrices of the manipulator. A singularity occurs whenever  $\mathbf{A}$  or  $\mathbf{B}$ , (or both) can no longer be inverted. Three types of singularities exist [Gosselin 1990]:

$$(1) \quad \det(\mathbf{A}) = 0 \quad (2) \quad \det(\mathbf{B}) = 0 \quad (3) \quad \det(\mathbf{A}) = 0 \quad \text{and} \quad \det(\mathbf{B}) = 0$$

## 2.2 Parallel and serial singularities

Parallel singularities occur when the determinant of the direct kinematics matrix  $\mathbf{A}$  vanishes. The corresponding singular configurations are located inside the workspace. They are particularly undesirable because the manipulator can not resist any effort and control is lost.

Serial singularities occur when the determinant of the inverse kinematics matrix  $\mathbf{B}$  vanishes. By definition, the inverse-kinematic matrix is always diagonal: for a manipulator with  $n$  degrees of freedom, the inverse kinematic matrix  $\mathbf{B}$  can be written as  $\mathbf{B} = \text{Diag} [\mathbf{B}_{11}, \dots, \mathbf{B}_{jj}, \dots, \mathbf{B}_{nn}]$ , each term  $\mathbf{B}_{jj}$  being associated with one leg. A serial singularity occurs whenever at least one of these terms vanishes. When the manipulator is in a serial singularity, there is a direction along which no Cartesian velocity can be produced.

## 2.3 Point-to-point trajectory

There are two major types of tasks to consider : point-to-point trajectories and continuous trajectories. We consider in this section point-to-point trajectories.

**Definition :** A point-to-point trajectory  $T_{pp}$  is defined by a set of  $p$  configurations in the workspace :  $T_{pp} = \{\mathbf{X}_1, \dots, \mathbf{X}_i, \dots, \mathbf{X}_p\}$ . By definition, no path is prescribed between any two configurations  $\mathbf{X}_i$  and  $\mathbf{X}_j$ .

**Hypothesis :** In a point-to-point trajectory, the moving platform can not move through a parallel singularity.

Although it was shown recently that in some particular cases a parallel singularity could be crossed [Nenchev 1997], hypothesis 1 is set for the most general cases.

A point-to-point trajectory  $T_{pp}$  will be feasible if there exists a continuous path in the Cartesian product of the workspace by the joint space which does not meet a parallel singularity and which makes the moving platform pass through all prescribed configurations  $\mathbf{X}_i$  of the trajectory  $T_{pp}$ .

## 2.4 Continuous Trajectory

**Definition :** A continuous trajectory  $T_c$  is defined by a parametric curve in the operational space as :  $T_c = \lambda([0, 1])$  where  $\lambda$  is a continuous function defined on  $[0, 1]$  and differentiable by parts on this interval.

**Hypothesis :** In a continuous trajectory, the moving platform cannot meet a parallel singularity nor a serial singularity.

A continuous trajectory  $T_c$  will be feasible if there exists a continuous path in the Cartesian product of the workspace by the joint space which is free of parallel and serial singularities and which makes the platform move along the continuous trajectory  $T_c$ .

**Remark :** A fully parallel manipulator with several inverse kinematic solutions can change its joint configuration between two prescribed configurations. Such a manoeuvre may enable the manipulator to avoid a parallel singularity (Figure 1). More generally, the choice of the joint configuration for each configuration  $\mathbf{X}_i$  of the trajectory  $T_{pp}$  can be established by any other criteria like stiffness or cycle time [Chablat et al. 1998]. Note that a change of joint configuration makes the manipulator run into a serial singularity, which is not redhibitory for the feasibility of point-to-point trajectories.

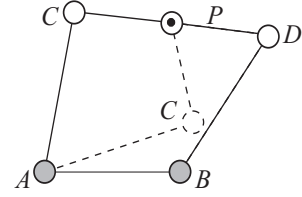


Fig. 1: Singular (solid) and a regular (dotted) configurations (the actuated joints are  $A$  and  $B$ )

### 3 The N-connected regions

**Definition :** The  $N$ -connected regions of the workspace are the maximal regions where any point-to-point trajectory is feasible. For manipulators with multiple inverse and direct kinematic solutions, it is not possible to study the joint space and the workspace separately. First, we need to define the *regions of manipulator reachable configurations* in the Cartesian product of the workspace by the joint space  $W.Q$ .

**Definition :** The regions of manipulator reachable configurations  $R_j$  are defined as the maximal sets in  $W.Q$  such that

- $R_j \in W.Q$ ,
- $R_j$  is connected,
- $R_j = \{\mathbf{X}, \mathbf{q}\}$  such that  $\det(\mathbf{A}) \neq 0$

In other words, the regions  $R_j$  are the sets of all configurations  $(\mathbf{X}, \mathbf{q})$  that the manipulator can reach without meeting a parallel singularity and which can be linked by a continuous path in  $W.Q$ .

**Proposition :** A trajectory  $T_{pp} = \{\mathbf{X}_1, \dots, \mathbf{X}_p\}$  defined in the workspace  $W$  is feasible if and only if :

$$\forall \mathbf{X} \in \{\mathbf{X}_1, \dots, \mathbf{X}_p\}, \quad \exists \mathbf{q}_i \in Q, \exists R_j \quad \text{such that } (\mathbf{X}_i, \mathbf{q}_i) \in R_j$$

In other words, for each configuration  $\mathbf{X}_i$  in  $T_{pp}$ , there exists at least one joint configuration  $\mathbf{q}_i$  and one region of manipulator reachable configurations  $R_j$  such that the manipulator configuration  $(\mathbf{X}_i, \mathbf{q}_i)$  is in  $R_j$ .

**Proof :** *Indeed, if for all configurations  $\mathbf{X}_i$ , there is one joint configuration  $\mathbf{q}_i$  such that  $(\mathbf{X}_i, \mathbf{q}_i) \in R_j$  then the trajectory is feasible because, by definition, a region of manipulator reachable configurations is connected and free of parallel singularity. Conversely, if for a given configuration  $\mathbf{X}_i$ , it is not possible to find a joint configuration  $\mathbf{q}_i$  such that  $(\mathbf{X}_i, \mathbf{q}_i) \in R_j$ , then no continuous, parallel singularity-free path exists in  $W.Q$  which can link the other prescribed configurations.*

**Theorem :** The N-connected regions  $W_{Nj}$  are the projection  $\mathbf{\Pi}_W$  of the regions of manipulator reachable configurations  $R_j$  onto the workspace:

$$W_{Nj} = \mathbf{\Pi}_W R_j$$

**Proof :** *This results is a straightforward consequence of the above proposition.*

The N-connected regions cannot be used directly for planning trajectories in the workspace since it is necessary to choose one joint configuration  $\mathbf{q}$  for each configuration  $\mathbf{X}$  of the moving

platform such that  $(\mathbf{X}, \mathbf{q})$  is included in the same region of manipulator reachable configurations  $R_j$ . However, the N-connected regions provide interesting global information with regard to the performances of a fully parallel manipulators because they define the maximal regions of the workspace where it is possible to execute any point-to-point trajectory.

A consequence of the above theorem is that the workspace  $W$  is N-connected if and only if there exists a N-connected region  $W_{Nj}$  which is coincident with the workspace :

$$W_{Nj} = W$$

## 4 The T-connected regions

**Definition :** The *T-connected regions* of the workspace are the maximal regions where any continuous trajectory is feasible.

When the mobile platform moves along a continuous trajectory, a fully parallel manipulator cannot pass through a serial or a parallel singularity. The aspects  $\mathbf{A}_{ij}$  [Chablat 1998b] characterize the maximal domains without serial and parallel singularity.

We recall below the definition of an aspect :

- $\mathbf{A}_{ij} \subset W \cdot Q$ ;
- $\mathbf{A}_{ij}$  is path-connected;
- $\mathbf{A}_{ij} = \{(\mathbf{X}, \mathbf{q}) \subset W \cdot Q \text{ such that } \det(\mathbf{A}) \neq 0 \text{ and } \det(\mathbf{B}) \neq 0\}$

**Proposition :** A continuous trajectory  $T_c = \{\mathbf{X}_\lambda, \lambda \in [0, 1]\}$  defined in the workspace  $W$  is feasible if and only if :

$$\forall \lambda \in [0, 1], \exists \mathbf{q} \in Q \text{ such that } (\mathbf{X}_\lambda, \mathbf{q}) \in \mathbf{A}_{ij}$$

In other words, for all configurations  $\mathbf{X}_\lambda$  in  $T_c$ , there exists at least one joint configuration  $\mathbf{q}$  such that the manipulator configuration  $(\mathbf{X}_\lambda, \mathbf{q})$  is in  $\mathbf{A}_{ij}$ .

**Proof :** *Indeed, if for all configurations  $\mathbf{X}_\lambda$ , there is one joint configuration  $\mathbf{q}$  such that  $(\mathbf{X}_\lambda, \mathbf{q}) \in \mathbf{A}_{ij}$  then the trajectory is feasible because, by definition, an aspect is connected and free of serial and parallel singularity. Conversely, if for a given configuration  $\mathbf{X}_\lambda$ , it is not possible to find a joint configuration  $\mathbf{q}$  such that  $(\mathbf{X}_\lambda, \mathbf{q}) \in \mathbf{A}_{ij}$ , then no continuous singularity-free path exists in  $W \cdot Q$  which can link the other prescribed configurations.*

**Theorem :** The T-connected regions  $W_{Tj}$  are the projection  $\mathbf{\Pi}_W$  of the aspect  $\mathbf{A}_{ij}$  onto the workspace:

$$W_{Tj} = \mathbf{\Pi}_W \mathbf{A}_{ij}$$

**Proof :** *This results is a straightforward consequence of the above proposition.* A consequence of the above theorem is that the workspace  $W$  is T-connected if and only if there exists a T-connected region  $W_{Tj}$  which is coincident with the workspace :

$$W_{Tj} = W$$

## 5 Example: A Two-DOF fully parallel manipulator

For more legibility, a planar manipulator is used as illustrative example in this paper. This is a five-bar, revolute ( $R$ )-closed-loop linkage, as displayed in figure 2. The actuated joint variables are  $\theta_1$  and  $\theta_2$ , while the Output values are the  $(x, y)$  coordinates of the revolute center  $P$ . The passive joints will always be assumed unlimited in this study. Lengths  $L_0 = 7$ ,  $L_1 = 8$ ,  $L_2 = 5$ ,

$L_3 = 8$ , and  $L_4 = 5$  define the geometry of this manipulator entirely, in certain units of length that we need not specify. The actuated joints are not unlimited  $\theta_1, \theta_2 \in [0, \pi]$ . In order to point out that the workspace of a parallel manipulator may not be N-connected even when there is no obstacle obstructions, it is assumed here that the manipulator is free of self-collisions. Examples with collision analyses have been investigated and can be found in the PhD thesis of the first author of this paper [Chablat 1998c].

The workspace is shown in figure 3. We want to know whether this manipulator can execute any point-to-point motion or continuous trajectory in the workspace. To answer this question, we need to determine the the N-connected regions and the T-connected regions.

## 5.1 N-connected regions

It turns out, although there is no obstacles obstructions, that the workspace of the manipulator at hand is not N-connected, e.g. the manipulator cannot move its platform between any set of configurations in the workspace. In effect, due to the existence of limits on the actuated joints, not all joint configurations are accessible for any configuration in the workspace. Thus, the manipulator may lose its ability to avoid a parallel singularity when moving from one configuration to another. This is what happens between points  $\mathbf{X}_1$  and  $\mathbf{X}_2$  (Figure 3). These two points cannot be linked by the manipulator although they lie in the workspace which is connected in the mathematical sense (path-connected) but not N-connected. In fact, there are two separate N-connected regions which do not coincide with the workspace and the two points do not belong to the same N-connected region (Figures 4 and 5). We know that the workspace of serial manipulators is always N-connected when there is no collision [Wenger 1991]. The example treated in this section shows that this is false when the manipulator is parallel.

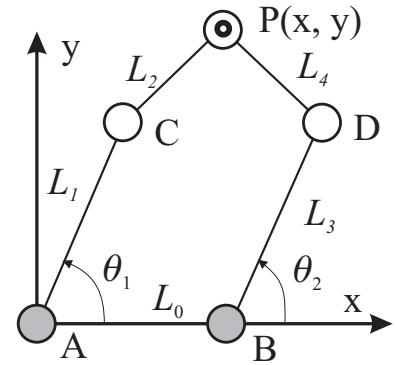


Fig. 2: A two-dof fully parallel manipulator

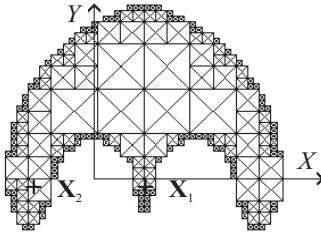


Fig. 3: The workspace

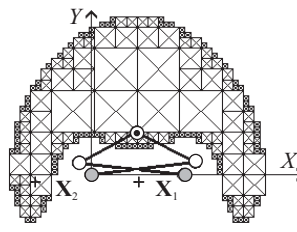


Fig. 4: The first N-connected region of the workspace

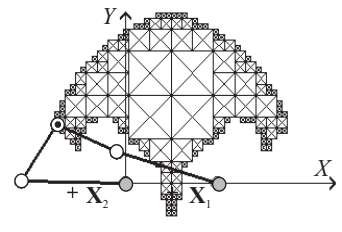


Fig. 5: The second N-connected region of the workspace

## 5.2 T-connected regions

The T-connected regions are the projection onto the workspace of the aspects. These regions are smaller than the N-connected regions. According to the joint configuration of the manipulator, i.e. according to the choice of the inverse kinematic solution, the singularity-free regions differ. Figures 6, 7, 8 and 9 depict the T-connected regions corresponding to  $\det(\mathbf{A}) > 0$ . Their union forms the first N-connected region (Figure 4). Figures 10, 11, 12 and 13 show the T-connected regions such that  $\det(\mathbf{A}) < 0$  and their union is the second N-connected region (Figure 5).

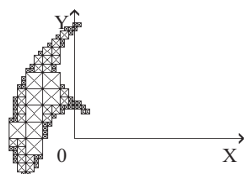


Fig. 6: Joint configuration 1

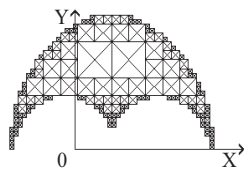


Fig. 7: Joint configuration 2

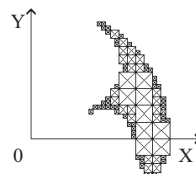


Fig. 8: Joint configuration 3

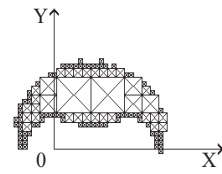


Fig. 9: Joint configuration 4

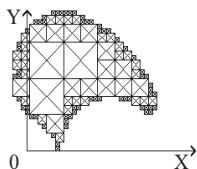


Fig. 10: Joint configuration 5

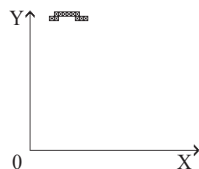


Fig. 11: Joint configuration 6

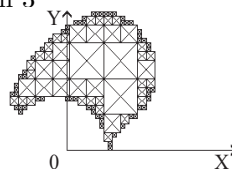


Fig. 12: Joint configuration 7

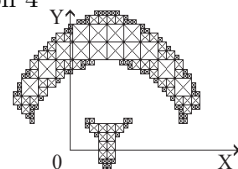


Fig. 13: Joint configuration 8

## 6 Conclusions

In this paper, the N-connected and the T-connected regions have been defined to characterize the maximal regions of the workspace where any point-to-point and continuous trajectories are feasible, respectively.

The manipulators considered in this study have multiple solutions to their direct and inverse kinematics. The N-connected regions were defined by first determining the maximum path-connected, parallel singularity-free regions in the Cartesian product of the workspace by the joint space. The projection of these regions onto the workspace were shown to define the N-connected regions. The T-connected regions were defined by the projection onto the workspace of the aspects, i.e. the maximal serial and parallel singularity-free domains in the Cartesian product of the workspace by the joint space.

The N-connectivity and the T-connected analysis of the workspace are of high interest for the evaluation of manipulator global performances as well as for off-line task programming.

## 7 References

- Chablat, D. and Wenger, Ph., "Working Modes and Aspects in Fully-Parallel Manipulator", IEEE Int. Conf. On Robotics and Automation, pp. 1964-1969, 1998.
- Chablat, D. and Wenger, Ph., "Moveability and Collision Analysis for Fully-Parallel Manipulators", 12th CISM-IFTOMM Symposium, RoManSy, 1998.
- Wenger, Ph., Chedmail, P., "Ability of a Robot to Travel Through its Free Workspace", The Int. Jour. of Robotic Research, Vol. 10:3, June 1991.
- Gosselin, C. and Angeles, J., "Singularity analysis of closed-loop kinematic chains", IEEE Trans. On Robotics And Automation, Vol. 6, No. 3, June 1990.
- Nanchev, D.N., Bhattacharya, S., and Uchiyama, M., "Dynamic Analysis of Parallel Manipulators under the Singularity-Consistent Parameterization", Robotica, Vol. 15, pp.375-384,1997.
- Chablat, D., Wenger, Ph. , Angeles, J., "The isoconditioning Loci of A Class of Closed-Chain Manipulators", IEEE Int. Conf. On Robotics and Automation, pp. 1970-1975, May 1998.
- Chablat, D., "Domaines d'unicité et parcourabilité pour les manipulateurs pleinement parallèles", PhD thesis, Nantes, November 1998.