Modelling the influence of process parameters on the densification of granular media under horizontal vibrations.

Sébastien Nadler, Olivier Bonnefoy, Jean-Marc Chaix, Ahmed Raihane, Gérard Thomas, Jean-Louis Gelet

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S. NADLER (1), **O. BONNEFOY** (1)(*), **J.-M. CHAIX**(2), **A. RAIHANE**(1), **G. THOMAS**(1), **J.L. GELET**(3)

(1) *Ecole Nationale Supérieure des Mines de Saint Etienne, Centre SPIN – LPMG -UMR CNRS 5148, 158 Cours Fauriel - 42023 Saint-Étienne Cedex 2, France*
(2) *Laboratoire SIMAP, UMR 5266 Grenoble INP-CNRS-UJF BP 75 - F-38402 Saint Martin d'Hères Cedex, France*
(3) *Ferraz-Shawmut, 6 rue Vaucanson, 69720 Saint Bonnet de Mure, France*
(*) bonnefoy@emse.fr

Abstract:

Numerical and experimental studies have been undertaken to analyze the compaction of a granular media submitted to sinusoidal horizontal vibrations. We characterize especially the influence of the dimensionless acceleration Γ of the vibrations and of the size of the box. For low value of Γ, the surface layers are compacted and the bottom layers remain at their initial density. The jamming effect of the bottom layers is explained by the influence of the pressure in these layers, which constraints the grains in their displacements. For high values of Γ, the bottom layers get compacted, the surface layers are fluidized and the density is decreased. We have also noticed that the compaction intensity depends on the size of the box: when the size of the box is lowered, the compaction becomes smaller.

Key-Words:

granular matter ; compaction ; horizontal vibrations ; numerical simulation ; Discrete Element Method.

1. Introduction

Vibration-based processes are widely used in industry to densify granular packing. In our application, we focus on electrical fuses made out of a ceramic box containing a silver/copper blade with sand grains in-between. The electrical properties depend on the density and homogeneity of the granular medium. In this study, we aim at characterizing and understanding the rheological behavior of the granular medium, when submitted to sinusoidal horizontal vibrations.

The literature shows many experimental studies of compaction induced by vertical vibrations. The Rennes group [Philippe 2001, Philippe 2003] has studied the influence of vertical taps, and modeled the compaction phenomenon by a Monte-Carlo algorithm, that allows a rearrangement of the grains after each vertical tap. A few articles related to horizontal vibrations report the presence of convection patterns. For instance, [Medved] notices and depicts the presence of two counter-rotating convection rolls in the upper part of the bed. Numerical studies led by [Liffman] have developed a hybrid hard sphere model and particle dynamics scheme to describe the influence of horizontal vibrations on the rheology of 2D granular media. In our laboratory, vertical and horizontal vibrations have been studied by [Rouèche] and [Raihane] respectively, both with an experimental point of view.

Hereafter, we shall focus on understanding the effect of horizontal vibrations on the packing and on the granular motion. Experiments led on sand grains in a rectangular box [Raihane] have shown that the density is maximized for a critical value of the dimensionless acceleration Γ. This existence of a maximum reveals two opposite trends that will be discussed. A numerical approach using a soft sphere DEM commercial software (PFC3D by Itasca) will be adopted in this study. To analyze the densification process, we shall try to establish correlations between the pressure field, the acceleration and the local density of the granular packing. Then, we shall depict the influence of the size of the box on the compaction of the system and compare numerical results with real-world experiments in order to evaluate their relevance.

2. Materials and methods

2.1. Experimental set-up

The sand packing is placed in a rectangular box (Figure 1). A sinusoidal horizontal displacement is then imposed: $x = A.\sin(\omega t)$. $\Gamma = \frac{A\omega^2}{g}$ is defined as the dimensionless acceleration where g the gravity field $(g \approx 9.8\ m.s^{-2})$ $w = 2\pi f$ the pulsation. Five points of view of the transparent PMMA box can be chosen: North, South, East, West and upside. The internal dimensions of the box are denoted Lx, Ly, and H_{sand} is the height of the sand bed. The volume size distribution of the grains can be roughly approximated by a Gaussian curve with a mean diameter of 450 µm and a standard deviation of 100 µm. The grain shape appears to be quite rounded. In Table 1 are listed most of the parameters imposed in the experiments and used in our modelling.

2.2. Modelling method

Results of modelling works found in literature are usually difficult to reproduce because the set of parameters is not extensively indicated. Indeed, there is always a difference between experimental parameters and model parameters, the latter being chosen to keep the calculation time reasonable (Table 1). Additionally, we have transformed the Lagrangian description natively used by the DEM software in order to compare the modelling results with experimental ones.

Lagrangian description.

The modelling is based on a series of operations applied to an assembly of spheres presenting a uniform numeric particle size distribution (PSD), randomly placed in a parallelepiped, and packed under gravity. The PSD has been chosen polydisperse in order to prevent the packing from crystallizing. The mean diameter in the model has been chosen twice that of the experimental one to keep the particles number and hence the calculation time at “human scale”. Sinusoidal horizontal vibrations are imposed to the walls. The position, velocity, and stress tensor associated to each sphere are computed at each time step with Newton’s law of motion. The collisions are described by the Hertz model and friction during contact is approximated by Coulomb’s law[1]. The average velocity of a sphere over one period of vibration is then calculated as the ratio of displacement to time. To reduce the calculation time, we reduced the spheres number by using periodic boundary conditions, the unit cell length being denoted Ly (Figure 2). The influence of the periodic cell size Ly has been analyzed, notably on the maximal grains vertical velocity on North side (Figure 4). Simulations have shown that, the finite size effect vanishes when Ly exceeds 10 grains diameters. For larger values, the maximal velocity becomes independent of the unit cell size: the box behaves as if the transverse dimension Ly is infinite, *i.e.* without the influence of the lateral East and West walls.

Eulerian description.

We have created a grid related to the box, with cells indexed by a set (i,j,k $\in$ N) of coordinates corresponding to a space position. The Lagrangian representation of the software (related to the spheres) can be transformed into an Eulerian representation by averaging the velocities on each case of the grid. To avoid the fluctuations due to the granular nature of the medium,

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[1] *Sliding occurs if Fs>=µ Fn where Fn is the normal force acting on the particle and Fs is the tangential force.*

the data (pressure, velocity, density) have been averaged on a series of oscillations periods. The pressure is calculated by taking the third of the trace of the stress tensor given by our software (≈isostatic pressure).

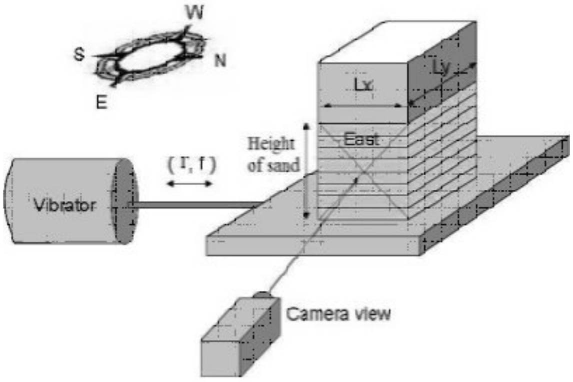

Figure 1. Schematic experimental setup

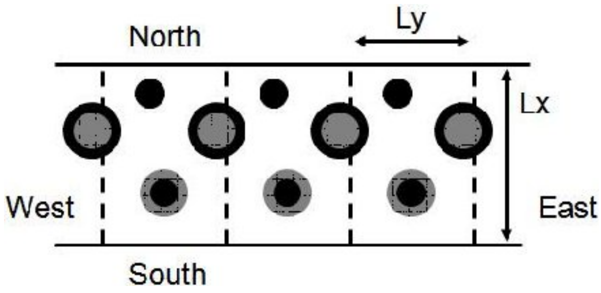

Figure 2. Schematic description of a periodic box (upside view)

Table 1. Typical parameters of the model

Parameter	Real Value	Model Value	Comment, ≠Model/Real
Mean diameter [mm]	≈ 0.45	≈0.90	Bigger sphere *
Rigidity [$N.m^{-3/2}$]	2.0 10^8	2.0 10^5	Smaller rigidity*
Density [$Kg\ m^{-3}$]	2700	2700	
Shear coefficient [Pa]	4.0 10^{10}	4.0 10^7	α Young modulus *
Poisson coefficient [ ]	0.25	0.25	
Friction (wall/ball) [ ]	0.3	0.3	Powder rheometer value
Friction (ball/ball) [ ]	0.7	0.7	Powder rheometer value
Lx [mm]	40	40	
Ly [mm]	80	∞	
H_{sand} [mm]	40	55	
			**Value of the model chosen to minimize the calculation time*

3. Results

3.1 Convective pattern

For f=50 Hz and an acceleration above a critical value, the rheological behavior of the granular medium can be mainly described by two counter-rotating convection rolls in the upper region as described on Figure 2.

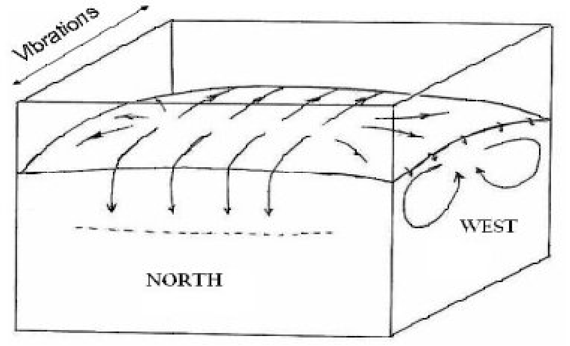

Figure 3. Counter-rotating convection rolls observed experimentally when f=50 Hz and Γ>1.5.

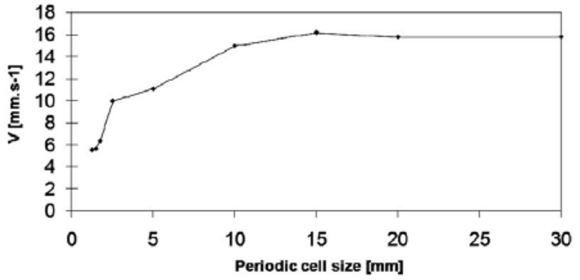

Figure 4. Influence of the periodic cell size on the maximal velocity on north side

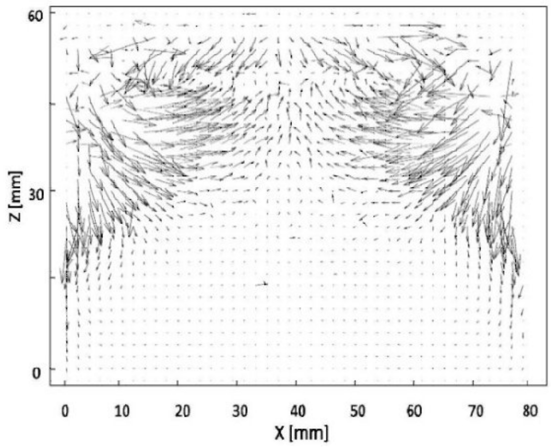

Figure 5. East side view, velocity filed measured experimentally by Particle Image Velocimetry technics. Observation of counter-rotating convection rolls for Γ=4.2 and f=50 Hz. Typical velocity range : 0-30 mm.s^{-1}

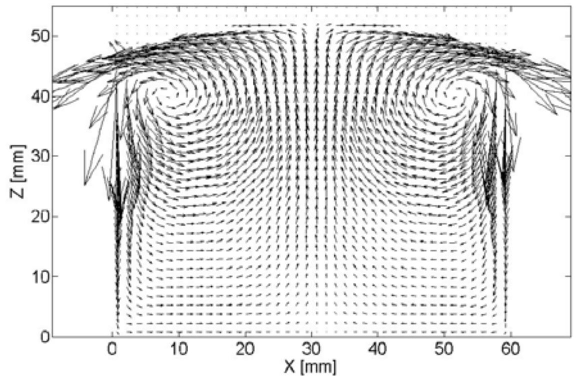

Figure 6. East side view, modelling's mean velocity field , Γ=3.2 and f=50 Hz. Typical velocity range: 0-20 m m.s^{-1}. Velocities are averaged over 300 cycles of vibrations

Schematically, this experimental observation can be interpreted as follows: the vibrations create an intermittent gap along the North or South walls and the granular medium.
By gravity, the grains can fall down into this gap. Then, they penetrate into the bulk, pushed horizontally by the walls and go upwards where the pressure is less important. Finally, when they arrive at the top of the pile, the grains may fall down again and the loop is closed (*Cf.* [Raihane]). Figure 5 shows the experimental velocity field as measured by an ultrafast CCD camera and a PIV software. The convective pattern of the experimental (Figure 5) and numerical (Figure 6) velocity field are similar. The numerical velocity field has been averaged over 300 cycles of vibrations, it explains its symmetry and smoothness by comparison with the more chaotic experimental pattern.

3.2 Influence of the acceleration on the density

Experimental results

[Raihane] measured the global dynamic density of the packing fraction, when the acceleration undergoes two consecutive increase-decrease quasi-static cycles (Figure 7). His results showed three phases during the first cycle of this vibration procedure. At the beginning, the densification is really important, with a density rise of approximately 5%. Afterwards, a loss of density is observed and can be correlated to the fluidization of the surface layers at high acceleration values. When the acceleration decreases, the density is continuously increasing and overpasses the initial density.

Then, if we increase and decrease another time the acceleration, we stay on the same branch of density: we obtain a reversible cycle. The reader can find more materials on reversibility and irreversibility of vibrated granular media in [Nowak].

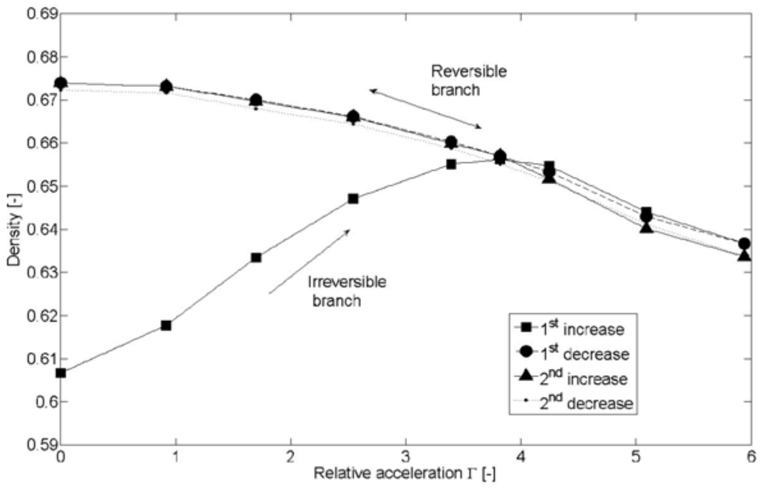

Figure 7. Experimental variation of the dynamic density with continuous evolution of Γ. From Ahmed Raihane's Ph .D. (in press)

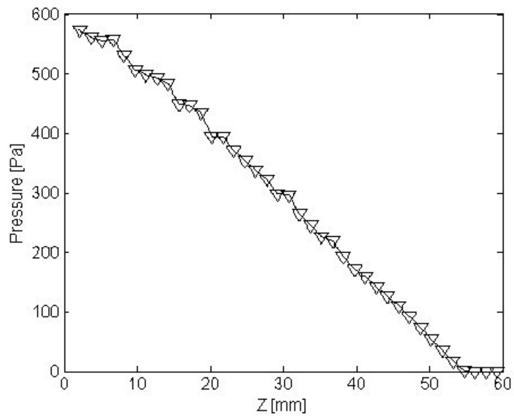

Figure 8. Profile of pressure for a static initial packing

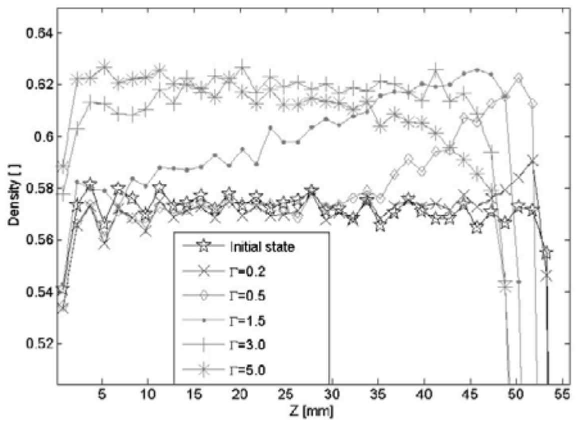

Figure 9. Computed evolution of the density vs. height for Γ between 0 and 5 in dynamic mode; f=50 Hz.

Modelling results

Initial state.

An assembly of 32000 spheres, of initial density Ci=0.57, is created. The initial state can be characterized by the pressure (Figure 8) and the density (Figure 9) profiles. The pressure appears to be proportional to the depth and reaches a maximum of 580 Pa for a 55 mm depth. This value is of the same order of magnitude as the theoretical approximation of hydrostatic pressure for thin bed packing: $P \approx C\rho gh \approx 810$ Pa (where C is the density of the packing).

During vibrations.

We have recorded the dynamic density profile after 100 cycles of vibrations. At this stage, a steady state has been attained. We can observe two opposite phenomena:

For low acceleration (Γ<1.5), the density at the bottom of the box remains equal to the density of the initial state because, in this region, the compressive stress is high compared to the shear stress, which makes the Coulomb criterion not reached.

For high accelerations (Γ>3.0), the shear stress is so high, that the Coulomb criterion is reached, even in the deeper layers. Therefore, the grains can move and the system compacts even at the bottom of the box. In the upper layers, the pressure is much smaller than the shear stress induced by the vibrations, the granular medium becomes fluid: the density decreases.
Relaxed state.
Experimentally and numerically, we can observe a further densification, when one stops the vibrations. The density increase ranges between in the jammed bottom layers and 3% in the fluidized top layers of the packing.
Our modelling results are in agreement with [Raihane] experimental data and enable an explanation of the observed phenomena. For high accelerations, the bottom layers are compacted and the surface layers are fluidized. For low accelerations the gain of density is essentially due to the compaction of the surface layers, the bottom layers are jammed by the pressure.

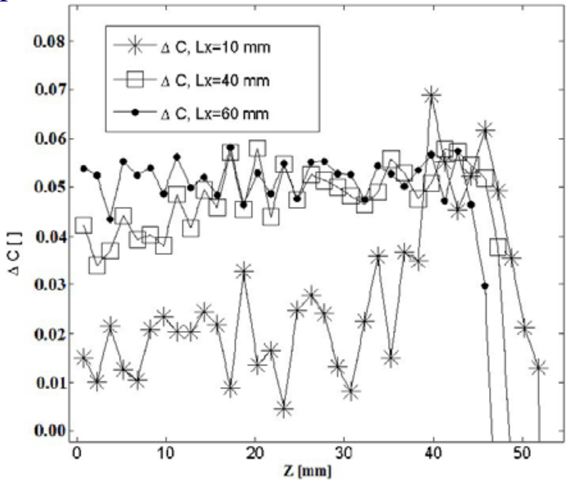

Figure 10. Relative evolution of the density for Lx=10 mm, 40 mm, 60 mm: modelling result. Γ=3, f=50 Hz.

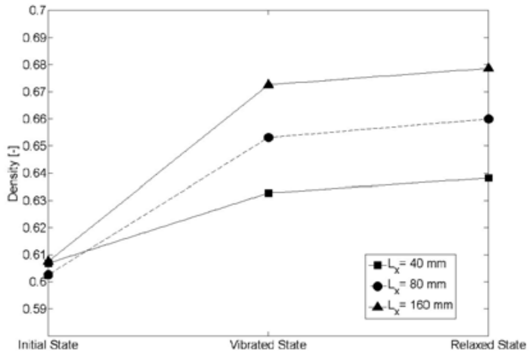

Figure 11. Influence of the size Lx of the box, Γ=1.5, f=50 Hz.

3.3 Influence of the length of the box

Many parameters have an incidence on the system behavior. Here we focus on the influence of the size of the box on the system. Effectively, the experimenter has to be cautious when he considers the influence of Γ on the density of the system (Figures 7 and 9): these results are only valid given one box dimension.
Experiments and modelling have been made to evaluate the influence of the length Lx (Figure 10 and 11). The modelling shows that for small values of Lx (Lx=10 mm), only the upper layers are compacted, whereas for greater values of Lx (Lx=40 mm, Lx=60 mm), all the packing is compacted. This behavior can be explained by a higher constraint on the displacement of the grains in small boxes.
If we compare Figure 9 and Figure 10, we can see that increasing Lx has got a similar effect as imposing a bigger value to Γ: the mobility of the grains increases and it enables to compact the bottom layers.
Experimentally, at Γ =1.5, the density increases with the size of the box (Figure 11). The modelling suggests that the compaction of the bottom layers occurs only if Lx is high enough. This behavior explains the experimental evolution of density with Lx.

4. Conclusion and prospect

Two phenomena have been described in this work: a densification of the deepest layers for high values of Γ, a densification of the surface layers for low value of Γ whereas the bottom layers remains solid. As seen above, parameters as the size Lx has an important influence on the compaction of the system. We plan to study the influence of other elements as the Ly size, the granular distribution, the coefficient of friction. We will also investigate the transient states of compaction: the media needs some time before acquiring a final density, the evolution of the density during these transient states will be examined.

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