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## Environmental policy, education and growth with finite lifetime: the role of the abatement technology

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# ENVIRONMENTAL POLICY, EDUCATION AND GROWTH WITH FINITE LIFETIME: THE ROLE OF THE ABATEMENT TECHNOLOGY

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## ABSTRACT

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This note shows that the assumptions about the abatement technology modify the impact of the environmental taxation on the long-run growth driven by human capital accumulation à la Lucas (1988), when lifetime is finite.

Whereas no impact of the environmental policy on long-run growth is found when pollution originates from final output and abatement is an activity requiring final output to reduce net emissions, this note demonstrates that a tighter environmental tax enhances human capital accumulation when it is assumed that abatement services are produced with physical capital.

*Keywords* : Growth; Environment; Overlapping generations; Human capital; Finite Lifetime; Abatement;

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## 1 INTRODUCTION

The influence of the environmental policy on long-run growth driven by human capital accumulation à la Lucas (1988) has been studied by several authors. They highlighted the importance of assumptions about the influence of pollution on the productivity of education (Gradus and Smulders, 1996; van Ewijk and van Wijnbergen, 1995), about endogenous labor supply (Hettich, 1998), about preferences for schooling (Grimaud and Tournemaine, 2007), about finite lifetime (Pautrel, 2008a) or about the negative impact of pollution on health (Pautrel, 2008b, 2009), on the outcome of the environmental taxation on long-run human capital accumulation.

All these works give insightful results on the way the deterioration of the environment affects growth when the channel of transmission is education. Unfortunately, some of these results are very sensitive to the source of pollution: when pollution originates from final output rather than physical capital, Hettich (1998) with endogenous labor supply and Pautrel (2008a) with finite lifetime do not find any influence of the environmental policy on education in the long-run, anymore.

This note argues that, in the case of finite lifetime, such a result occurs when abatement is considered as an activity requiring an amount of final output to reduce net emissions. Assuming rather that abatement services are produced in a specific sector using a part of the physical capital stock, this note finds a positive impact of the environmental policy on human capital accumulation while the source of pollution is final output.

## 2 THE MODEL

Let's consider a Yaari (1965)-Blanchard (1985) overlapping generations model with human capital accumulation and environmental concerns. Time is continuous. Each individual born at time  $s$  faces a constant probability of death per unit of time  $\beta \geq 0$ . Consequently his life expectancy is  $1/\beta$ . When  $\beta$  increases, the life span decreases. At time  $s$ , a cohort of size  $\beta$  is born. At time  $t \geq s$ , this cohort has a size equal to  $\beta e^{-\beta(t-s)}$  and the constant population is equal to  $\int_{-\infty}^t \beta e^{-\beta(t-s)} ds = 1$ . There are insurance companies and there is no bequest motive.

The expected utility function of an agent born at  $s \leq t$  is:<sup>1</sup>

$$\int_s^\infty [\log c(s, t) - \zeta \log \mathcal{P}(t)] e^{-(\varrho+\beta)(t-s)} dt \quad (1)$$

where  $c(s, t)$  denotes consumption in period  $t$  of an agent born at time  $s$ ,  $\varrho \geq 0$  is the rate of time preference and  $\zeta > 0$  measures the weight in utility attached to the environment.

The representative agent can increase his stock of human capital by devoting time to schooling, according to Lucas (1988):

$$\dot{h}(s, t) = B [1 - u(s, t)] h(s, t) \quad (2)$$

where  $B$  is the efficiency of schooling activities,  $u(s, t) \in ]0, 1[$  is the part of human capital allocated to productive activities at time  $t$  for the generation born at  $s$  and  $h(s, t)$  is the stock of human capital at time  $t$  of an individual born at time  $s$ . We assume that the human capital of the agent when he is born,  $h(s, s)$ , is inherited from the dying generation (Song, 2002). Because the mechanism of intergenerational transmission of knowledge is complex, we consider that newborns inherit from the dying generation the average aggregate human capital stock, that is  $h(s, s) = H(s)$  (population being equal to unity).<sup>2</sup>

Households face the following budget constraint:

$$\dot{a}(s, t) = [r(t) + \beta] a(s, t) + u(s, t) h(s, t) \omega(t) - c(s, t) \quad (3)$$

where  $a(s, t)$  is the financial wealth in period  $t$  and  $\omega(t)$  represents the wage rate per effective unit of human capital  $u(s, t) h(s, t)$ . In addition to the budget constraint, there exists a transversality condition which must be satisfied to prevent households from accumulating debt indefinitely:

$$\lim_{v \rightarrow \infty} [a_{s,v} e^{-(r+\beta)(v-t)}] = 0$$

The representative agent chooses the time path for  $c(s, t)$  and his working time  $u(s, t)$  by maximizing (1) subject to (2) and (3). It yields

$$\dot{c}(s, t) = [r - \varrho] c(s, t) \quad (4)$$

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<sup>1</sup>We use logarithmic utility for the sake of simplicity. Our results remain valid when the intertemporal elasticity of substitution of the consumption is different from unity. Proof upon request.

<sup>2</sup>Assuming that  $h(t, t) = \eta H(t)$  with  $\eta \in ]0, 1[$ , like Song (2002), would not modify our qualitative results. Proof upon request.

Integrating (3) and (4) and combining the results gives the consumption at time  $t$  of an agent born at time  $s$ :

$$c(s, t) = (\varrho + \beta) [a(s, t) + \omega(s, t)] \quad (5)$$

where  $\omega(s, t) \equiv \int_t^\infty [u(s, \nu)h(s, \nu)w(\nu)] e^{-\int_t^\nu [r(\zeta) + \beta] d\zeta} d\nu$  is the present value of lifetime earning. It also gives the equality between the rate of returns to human capital and the effective rate of interest (the interest rate on the debt  $r$  plus the insurance premium  $\beta$  the agent has to pay when borrowing (see Blanchard and Fisher, 1989)):

$$\frac{\dot{w}(t)}{w(t)} + B = r(t) + \beta \quad (6)$$

Due to the simple demographic structure, all individual variables are additive across individuals. Consequently, the aggregate consumption equals

$$C(t) = \int_{-\infty}^t c(s, t) \beta e^{-\beta(t-s)} ds = (\varrho + \beta) [K(t) + \Omega(t)] \quad (7)$$

where  $\Omega(t) \equiv \int_{-\infty}^t \omega(s, t) \beta e^{-\beta(t-s)} ds$  is aggregate human wealth in the economy. The aggregate stock of physical capital is defined by

$$K(t) = \int_{-\infty}^t a(s, t) \beta e^{-\beta(t-s)} ds$$

and the aggregate human capital is

$$H(t) = \int_{-\infty}^t h(s, t) \beta e^{-\beta(t-s)} ds, \quad (8)$$

In the economy, a government taxes polluting firms and uses the revenues from the tax to provide abatement services that improve the environmental quality. His budget is balanced at each date (see below).

There are two production sectors that operate under perfect competition: one produces final output denoted  $Y$ , the other produces abatement services denoted  $D$ . The final output is produced with the following technology:

$$Y(t) = A_y (\phi(t) K(t))^{\alpha_k} H_y(t)^{\alpha_h} E(t)^{1-\alpha_k-\alpha_h}, \quad \text{with } \phi, \alpha_k, \alpha_h \in ]0, 1[ \quad (9)$$

The parameter  $A_y > 0$  is a productivity scalar,  $H_y(t) \equiv \left[ \int_{-\infty}^t u(s, t) h(s, t) \beta e^{-\beta(t-s)} ds \right]$  is the amount of the aggregate stock of human capital used in output production,  $\phi(t)K(t)$  is the part of the physical capital stock used in output production,  $E(t)$  represents the emissions of pollution. Firms in the final output sector support a tax  $\tau > 0$ , implemented by the government, on each unit of pollutant emissions they create. They maximize profit  $Y(t) - r(t)\phi(t)K(t) - w(t)H_y(t) - \tau E(t)$  by equating factor rewards to marginal productivity:

$$r(t) = \alpha_k Y(t) / (\phi(t)K(t)), \quad \text{and} \quad w(t) = \alpha_h Y(t) / H_y(t) \quad (10)$$

and by equating the marginal cost of the pollutant emissions to their marginal productivity:

$$\tau E(t) = (1 - \alpha_k - \alpha_h) Y(t) \quad (11)$$

The higher the output production, the higher the flow of polluting emission, for a given level of environmental tax. Putting (11) in (9), we can express the final output as a function of the physical capital stock, the human capital stock and the environmental tax rate:

$$Y(t) = \mathcal{A}(\tau, \alpha_k, \alpha_h) (\phi(t)K(t))^\alpha H_y(t)^{1-\alpha} \quad (12)$$

where  $\mathcal{A}(\tau, \alpha_k, \alpha_h) \equiv (A_y (1 - \alpha_k - \alpha_h)^{1-\alpha_k-\alpha_h})^{1/(\alpha_k+\alpha_h)} \tau^{-(1-\alpha_k-\alpha_h)/(\alpha_k+\alpha_h)}$  is a decreasing function of  $\tau$ . From (10), factor rewards  $r(t)$  and  $w(t)$  reduce with the environmental tax.

The abatement sector produces abatement services aimed at curbing the emissions of pollution. To keep things simple, we follow Michel and Rotillon (1995) considering that only physical capital is used in the abatement sector with the following constant-returns technology:<sup>3</sup>

$$D(t) = A_D (1 - \phi(t)) K(t), \quad \text{with} \quad A_D > 0 \quad (13)$$

The government purchases the abatement services  $D(t)$  at a price  $P_D(t)$ , defined by profit maximization ( $P_D(t) = r(t)/A_D$ ) and publicly provides them to the economy. Its budget being balanced at each date the revenue of the environmental tax funds the abatement services expenditures

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<sup>3</sup>Constant returns to scale are required to enable the abatement activities to rise in the long-run at the common rate of growth (see below).

$\tau E(t) = P_D(t)D(t)$ . From equations (10),(11), (13) and the expression of  $P_D(t)$ , we obtain the allocation of physical capital to output production:

$$\phi = \left( \frac{1 - \alpha_h}{\alpha_k} \right)^{-1} \quad (14)$$

Because  $1 - \alpha_k - \alpha_h > 0$ ,  $\phi \in ]0, 1[$ . It is constant and depends on the technology parameters.

The stock of pollution, denoted by  $S(t)$ , evolves according to two opposite forces. On the one hand, it increases in the net flow of pollution, the pollutant emissions to abatement services ratio  $E(t)/D(t)$ . On the other hand, it decreases due to a natural rate of decay  $\zeta > 0$ , such that:

$$\dot{S}(t) = f \left( \frac{E(t)}{D(t)} \right) - \zeta S(t), \quad \text{with } f(\cdot) > 0, f'(\cdot) > 0, f''(\cdot) > 0 \quad (15)$$

### 3 THE GENERAL EQUILIBRIUM AND THE BALANCED GROWTH PATH

The final output is used either to consume, either to invest in physical capital. Therefore, the market clearing condition is:

$$Y(t) = C(t) + \dot{K}(t).$$

with  $\dot{K}(t) = dK(t)/dt$ .

Differentiating (8) with respect to time and using the fact that  $u(s, t) = u(t)$ ,<sup>4</sup> the aggregate accumulation of human capital is:

$$\dot{H}(t) = B [1 - u(t)] H(t) \quad (16)$$

and differentiating (7) with respect to time gives

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{c}(s, t)}{c(s, t)} - \frac{1}{C(t)} [\beta C(t) - \beta c(t, t)] \quad (17)$$

Aggregate consumption growth differs from individual consumption growth by the term into brackets  $-\beta C(t) + \beta c(t, t)$  which represents what Heijdra and Ligthart (2000) called the “*generational turnover effect*”. This effect appears because at each date a cross-section of the existing population

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<sup>4</sup>Using (10), the equalization of the rates of returns given by equation (6) implies that the rate of returns to human capital is independent of  $s$ , therefore all individuals allocate the same effort to schooling:  $u(s, t) = u(t)$ .



dies (reducing aggregate consumption growth by  $\beta C(t)$ ) and a new generation is born (adding  $\beta c(t, t)$ ). Because new agents born without financial assets, their consumption  $c(t, t)$  is lower than the average consumption  $C(t)$  and therefore the “*generational turnover effect*” reduces the growth rate of the aggregate consumption.

Using the expression of  $dK(t)/dt$ ,  $d\Omega(t)/dt$  and equation (4) we obtain:

$$\dot{C}(t)/C(t) = r(t) - \varrho - \beta(\varrho + \beta)K(t)/C(t) \quad (18)$$

The *generational effect* rises with the probability to die  $\beta$ : on one hand, agents die at a higher frequency (that increases the generational turnover) and on the other hand the propensity to consume out of wealth  $\varrho + \beta$  increases due to the shorter horizon.

From previous results, the dynamics of the model is written as:<sup>5</sup>

$$\begin{aligned} \dot{x}(t) &= \{[\alpha(1 - \tau) - 1](\phi b(t)u(t))^{1-\alpha} - \varrho - \beta(\varrho + \beta)x(t)^{-1} + x(t)\}x(t) \\ \dot{b}(t) &= \{B[1 - u(t)] - (\phi b(t)u(t))^{1-\alpha} + x(t)\}b(t) \\ \dot{u}(t) &= \{\alpha^{-1}[B - \beta] - (1 - \tau)(\phi b(t)u(t))^{1-\alpha} - \dot{b}(t)/b(t)\}u(t) \end{aligned} \quad (19)$$

where  $x(t) \equiv C(t)/K(t)$  and  $b(t) \equiv H(t)/K(t)$ .

Along the balanced growth path,  $C$ ,  $K$ ,  $H$ ,  $D$ ,  $E$  and  $Y$  evolve at a common positive rate of growth (denoted  $g^*$ , where a  $\star$  means “*along the BGP*”) and the allocation of human capital across sectors is constant. As a consequence, along the balanced growth path  $\dot{x} = \dot{b} = \dot{u} = 0$ ,  $x = x^*$ ,  $b = b^*$ ,  $u = u^*$  and  $g^* > 0$ .

From (19),  $\dot{u} = 0$  gives the equality between the returns to physical capital (the effective interest rate) and the returns to human capital:

$$\alpha(1 - \tau)(\phi b^*u^*)^{1-\alpha} + \beta = B \quad (20)$$

that defines  $\phi b^*u^*$  as an increasing function of  $\tau$ , denoted by

$$\mathcal{R}(B, \tau) \equiv \left( \frac{B - \beta}{\alpha(1 - \tau)} \right)^{1/(1-\alpha)} \quad (21)$$

Evaluating  $\dot{x} - \dot{b}$  (from 19) along the BGP and using equation (21), we obtain  $x^* = \frac{\beta(\varrho + \beta)}{Bu^* - \beta - \varrho}$ . Because  $x^* > 0$ , we impose  $u^* > (\beta + \varrho)/B$ : the growth rate along the BGP can not exceed the

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<sup>5</sup>Demonstration upon request.

maximum feasible rate.<sup>6</sup> Furthermore,  $\dot{b}$  evaluated along the BGP and equation (21) give  $x^* = \mathcal{R}(B, \tau)^{1-\alpha} + B(u^* - 1)$ . Equating the two expressions of  $x^*$  we find that there exists  $u^* \in \left] \frac{\varrho + \beta}{B}, 1 \right[$  solving  $\Gamma(u; \tau) = 0$  where  $\Gamma(u; \tau)$  is defined as follows

$$\Gamma(u; \tau) \equiv [Bu - \beta - \varrho] \times \{ \mathcal{R}(B, \tau)^{1-\alpha} + B(u - 1) \} - \beta(\varrho + \beta).$$

Because  $\Gamma(\cdot)$  is a continuous increasing function of  $u$  with  $\Gamma(0; \tau) < 0$  and  $\Gamma(1; \tau) > 0$ , there exists a unique  $u^*$ . Finally, because  $\Gamma(\cdot)$  is an increasing function of  $\tau$ , from the implicit function theorem,  $u^*$  is a decreasing function of  $\tau$  denoted by  $\mathcal{U}(\tau)$  with  $\mathcal{U}'(\tau) < 0$ .<sup>7</sup>

From (16), the growth rate of the economy along the BGP is:<sup>8</sup>

$$g^* = B(1 - \mathcal{U}(\tau)). \quad (22)$$

**Proposition 1.** *If we assume that abatement activities are produced with a part of the physical capital stock, a tighter environmental tax enhances the BGP human capital accumulation à la Lucas (1988) when lifetime is finite, while the source of pollution is final output.*

*Proof.* It comes directly from  $\mathcal{U}'(\tau) < 0$  and equation (22). ■

This result may be explained as follows. When we consider abatement as an activity requiring an amount of final output, a tighter environmental tax has the following effects (see Hettich, 1998). First, there is a “crowding-out effect” on aggregate consumption and physical capital investment: *Ceteris Paribus* the tighter environmental taxation initially increases abatement (because abatement equals the revenue of the environmental tax  $\tau E$ ) and the remaining part of final output (used for consumption and the accumulation of physical capital) is crowded-out. Second, there is a “factor-reallocation effect” between physical capital and human capital leading the output production to become more intensive to human capital because the after-tax interest rate reduces with the higher tax. Furthermore, with

<sup>6</sup>The ratio  $(\beta + \varrho)/B$  is lower than unity because we impose that the growth rate of the economy along the BGP is positive. Using the fact that  $r + \beta = B$  (see equation 20) and that  $\dot{C}/C = r - \varrho - \beta(\varrho + \beta)K/C > 0$  implies  $r > \varrho$ , we obtain  $B > \varrho + \beta$ .

<sup>7</sup>Note that the BGP equilibrium is saddle-path stable. Proof upon request.

<sup>8</sup>Combining equations from (19) evaluated along the BGP, we find that the human capital to physical capital ratio  $H/K$  and the aggregate consumption to physical capital ratio  $C/K$  along the balanced growth path are increasing in the environmental tax:  $\phi b^* = \left( \frac{B - \beta}{\alpha(1 - \tau)} \right)^{1/(1 - \alpha)} \mathcal{U}(\tau)^{-1} > 0$  and  $x^* = \frac{\beta(\varrho + \beta)}{B\mathcal{U}(\tau) - \beta - \varrho} > 0$

finite lifetime, the “*generational turnover effect*” that disconnects the after-tax interest rate and the aggregate consumption rate of growth generates a third impact (see Pautrel, 2008a).<sup>9</sup>

When pollution originates from the physical capital stock, the overall impact is positive for human capital accumulation (the positive “*factor re-allocation effect*” and “*generational turnover effect*” are higher than the negative “*crowding-out effect*”). When final output is the source of pollution, the “*factor re-allocation*” and the “*generational turnover effect*” are reduced because the wage rate diminishes besides the interest rate and the substitution between the physical capital and the human capital is limited. That is the reason why, even with finite lifetime, the different effects offset to give no impact of the environmental tax on the long-run accumulation of human capital.

In this note, we assumed that abatement is produced with physical capital. As a result, the aforementioned initial “*crowding-out effect*” on the aggregate consumption and the physical capital investment does not exist anymore. That explains why, there exists a positive impact of the environmental taxation on long-run growth, while the source of pollution is output.

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<sup>9</sup>Indeed, with finite lifetime, the aggregate consumption rate of growth differs from the individual consumption rate of growth  $r - \rho$ , by the “*proportionnal*” difference between average consumption and consumption by newly born households  $[C(t) - c(t, t)]/C(t)$  (see equation 17). And this difference, that reduces the aggregate consumption rate of growth, depends positively on the aggregate consumption to physical capital ratio  $x$ . Because the substitution of the physical capital by the human capital increases the aggregate consumption to physical capital ratio  $x$ , the aggregate consumption rate of growth (when lifetime is finite) is higher than its initial value when the environmental tax rises. In order to restore the equalization of the growth rates along the BGP, the physical capital rate of growth increases. *Ceteris Paribus*, the human capital to physical capital ratio  $b$  diminishes and as a result the after-tax interest rate falls: agents allocate a part of their resources from final output to human capital accumulation ( $u$  diminishes) and the interest rate back to its initial value. If the substitution between the two types of capital is enough, a tighter environmental tax enhances human capital accumulation along the BGP.

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