Accounting for the effect of local punching at the bar/ specimen interface in SHPB experiments

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Abstract. The object of this work is to provide a displacement correction for the local punching due to an axial loading at the end of a bar. For this purpose, an analytical calculation of the dynamic 3-D indentation of an elastic isotropic semi-infinite bar is carried out. It provides a first-order correction for the displacement obtained through the 1-D wave analysis commonly used in SHPB processing. This correction applies to the displacements as they are calculated at both ends of a specimen sandwiched between the two bars of a SHPB device. It improves the results obtained for the dynamic behavior of the specimen, in particular at early instants of loading.

1. INTRODUCTION

The SHPB device, called also Kolsky apparatus, is widely used to measure the stress-strain behavior of materials at high strain rates of loading. This behavior is derived from the measured forces and velocities at the specimen faces. These quantities are obtained through the knowledge of the axial strains that are recorded by strain gauges glued on the lateral faces of the input and output bars (see formulas (1) underneath). These four quantities (forces and speeds) provide redundant measurements allowing for the use of inverse methods to investigate the specimen behavior [1]. A particular and simple use of such methods is to compute (using a 1-D transient calculation) the initially elastic response of the specimen. This method [2] was proposed for an optimized determination of the relative position of the origin of the three waves involved in the SHPB. It has been used for years in our laboratory where we have observed that the best value of the Young's modulus of the specimen was always smaller (in particular with small diameter specimens) than the expected or known value. We have suspected that this systematic error was due to an imperfect measurement of the average strain of the specimen. The bar face in contact with the specimen (when it has a smaller diameter than the bar) does not remain plane as it is non-uniformly loaded, i.e. submitted to a local "elastic punching". The corresponding displacement field is restricted to the impact zone and is not recorded by the strain gauges located far from the bar end.

We are interested in the evaluation of the non-planar deformation of the loaded surface induced at the end of a bar by a central axial load. The elementary 1-D theory of wave propagation in bars, and more advanced theories that consider wave dispersion due to lateral inertia effects in the bars [3, 4, 5], do not tackle this problem. They rather investigate wave propagation at large distances from the bar end [6]. As suggested by some authors [7], the determination of the local displacement at a bar end is of practical importance for the use of the SHPB device. This displacement has been determined through a study described in details in [8]. Because the restricted number of permitted pages, only the basis and the general ideas are recalled in the present paper.

Numerical and experimental studies are also carried out. They provide estimations of the consequence of the proposed displacement correction on the dynamic response of a specimen. The

correction is especially significant at the beginning of the loading where it permits an estimation of the Young's modulus of the tested material.

2. FORMULATION OF THE PROBLEM

2.1 Experiment

In a typical SHPB device (for compression testing), three waves are involved during the experiment: an incident compressive wave $\varepsilon_i(a, t)$ generated by the impact of the striker, a reflected (tensile) wave $\varepsilon_r(a, t)$ due to the lower impedance of the specimen (both recorded at gauge A – abscissa a) of the input bar), and a transmitted compressive wave $\varepsilon_t(b, t)$ recorded at gauge B – abscissa b) of the output bar).

By use of wave theory, the waves measured at A and B are obtained at the bar-specimen interfaces where they are denoted $\varepsilon_i(t)$, $\varepsilon_r(t)$ and $\varepsilon_t(t)$ [9].

The measured forces and velocities are:

$$\begin{array}{ll} F_i(t) = A_i E(\varepsilon_i(t) + \varepsilon_r(t)) & F_o(t) = A_o E \varepsilon_t(t) \\ V_i(t) = -c_{0i}(\varepsilon_i(t) - \varepsilon_r(t)) & V_o(t) = -c_{0r}\varepsilon_t(t) \\ \text{at the input interface} & \text{at the output interface} \end{array} \tag{1}$$

Where c_{0i} and c_{0r} are the velocities of the longitudinal waves and A_i and A_o are the cross-sectional areas of the input and output bars, respectively.

In Figure 1, the sample is shown together with the output bar, with a deformed configuration of the bar end under a distributed force $F(t)$. The bar end displacement derived from relations (1) is $u(t)$. The additional displacement $p(t)$ resulting from the local elastic 3-D axisymmetric deformation of the bar is referred to as the elastic punching. It appears when the diameter of the specimen is smaller than that of the bar. The determination of the elastic punching $p(t)$ provides a correction for the displacement $u(t)$ given by SHB formulas, allowing for the measurement of the real displacement at the specimen-bar interface: $d(t) = u(t) + p(t)$.

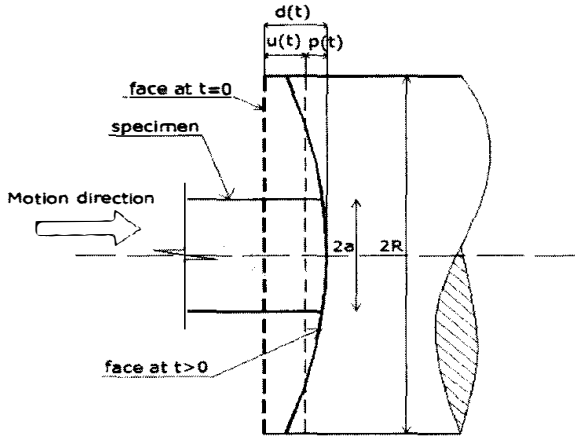

Figure 1. Schematic view of displacements at the output bar end during a SHPB experiment.

2.2 Theoretical problem

Following the usual SHPB practice, the friction between the sample and the bar faces is neglected and the uniaxial stresses within the specimen are assumed to be uniformly distributed through the cross section.

Based on these two assumptions, the problem becomes that of a semi-infinite bar with traction-free lateral faces, axi-symmetrically loaded at its end by a uniform distribution of time dependant

normal stress. The resulting force $F(t)$ is assumed to be known, as obtained through relations (1) in the case of a SHPB testing.

The problem can be divided in two parts. At early instants, the waves that are produced by the loading are the same as they would be in a semi-infinite solid, as long as the reflected waves at the free cylindrical surface of the bar are not superposed with them. The solution of the first part is then valid for a very short time (if compared with the SHPB test duration). An explicit solution is proposed in [8]. A short time later, the radial oscillations near the bar end tend to produce a quasi-static state in the radial direction while the early axial waves produced by the loading are already far from the end of the bar. The solution of the corresponding problem provides a sufficient correction for SHPB applications.

3. SOLUTION OF THE PROBLEM

Following the notations of figure 1, the corrected displacement for any point under the sample is defined by the relation $d(t) = u(t) + p(t)$ where $p(t)$ represents the "elastic punching".

A trick used to solve this problem is based on the superposition principle. Two loadings are applied simultaneously at the end of the bar. These loadings are chosen so that they result in opposite forces $F(t)$ and $-F(t)$. (Figure 2). In order to determine the correction $p(t)$, the dynamic problem is transposed into the static configuration of a self-equilibrated end-loading problem of a semi-infinite cylinder. This configuration is able to provide at any time t the mean value of the correction $p(t)$ to be added to the measured displacement $u(t)$.

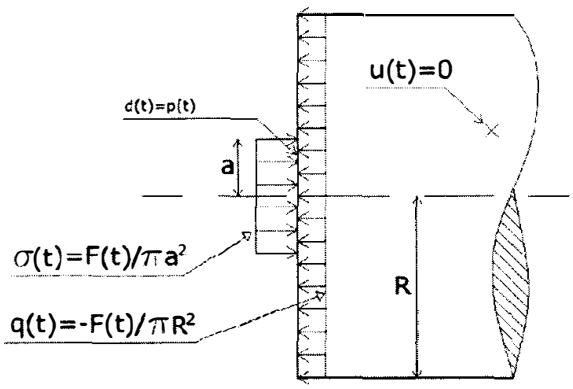

Figure 2. Determination of the local displacement $p_{t\geq t_1}(t)$ by superposition of states of stress.

The method and subsequent calculations are detailed in [8]. The exact analytical solution obtained is not easy to use for SHPB applications.

In a second step, it is then searched (as also described in [8]) for an approximate expression in a closed-form that will be more convenient in the frame of SHPB practice. This step leads to the following formula.

$$p = \frac{4}{3\pi^2}\frac{1-\nu}{\mu}\frac{F}{a\rho_a}[2\rho_a - (\rho_a^2+1)E(\rho_a) - (\rho_a^2-1)K(\rho_a)] \tag{2}$$

Where,

p is the average displacement under the loaded area, a is the radius of the loaded area,

ρ_a is the ratio of the radius of the loaded area versus that of the bar (r/a)

F is the applied (known) force, μ and ν are Lamé's coefficients

Functions $E(\rho_a)$, $K(\rho_a)$ are well defined in the literature. They have the following expressions:

$$E(\rho_a) = \int_0^{\frac{\pi}{2}} \sqrt{1-\rho_a^2\sin^2\theta}\, d\theta; \quad K(\rho_a) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-\rho_a^2\sin^2\theta}}$$

4. NUMERICAL SIMULATION

A dynamic numerical simulation is performed with Abaqus explicit code, for a cylinder of diameter 50 mm submitted to uniform distributions of pressure over diameters of 15 and 25 mm, due to the same force magnitude. The evolution of the loading force with time is given in the insert (Figure 3). Figure 3 shows the average displacement as calculated with formula (2) and as obtained numerically, and finally the 1D displacement at large distance from the impact zone as it is derived from the standard 1-D wave theory: $u(t) = \frac{1}{\pi R^2 \rho c_0} \int_0^t F(\tau) d\tau$.

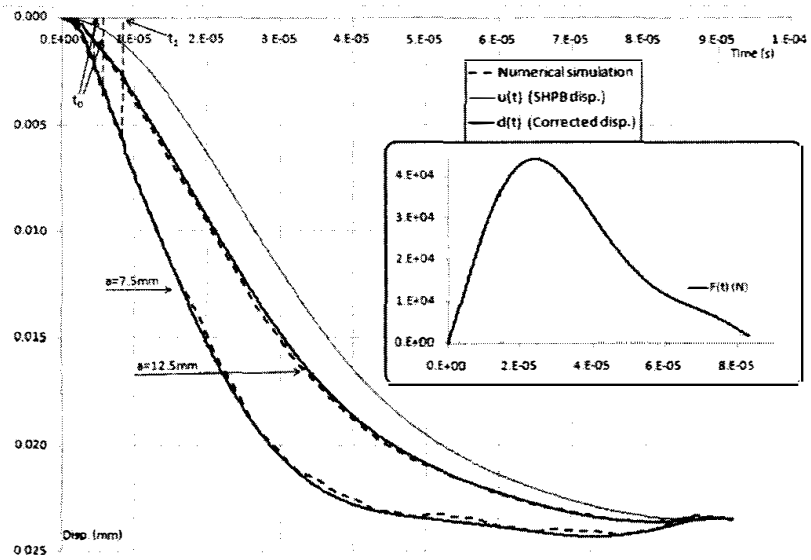

Figure 3. Numerical, analytical and 1D average displacements under disks of pressure of diameters 15 and 25 mm, applied on a 50 mm diameter cylinder, according to the same time-dependant function.

This simulation confirms that the closed-form solution is almost equal to the one given by the numerical simulation. The test lasts 85 μS and simulates the behavior of a brittle material. We purposely have chosen a short loading duration to conveniently visualize the obtained solutions. In the present case, it appears that the amplitude of the correction is in the range of the displacement itself up to a force equal to 40 kN (250 or 90 Mpa for a specimen of 15 mm or 25 mm respectively). The case of a 25 mm diameter specimen loaded by the same resulting force indicates that the correction for punching becomes rather small in this case (a/R greater than 0.5). The distance between the corrected displacement $d(t)$ and the 1-D displacement $u(t)$ given by the SHPB, corresponds to the elastic punching $p(t)$. It vanishes when a = R.

5. PRACTICAL IMPLEMENTATION OF THE DISPLACEMENT CORRECTION

In Figure 1 a scheme of the shape of one bar face is displayed (for any given time). If we consider the common case of two identical bars and if we denote by l_0 the initial length of the specimen, its current length becomes $l_0 - 2[u(t) - p(t)]$. The value $l_0 - 2u(t)$ given by the SHPB device is therefore under-estimating the length of the specimen. Consequently the real strain of the specimen is less than that given by the classical SHPB analysis.

Note that Eq. 2 may be written in the form:

$$p(t) = \frac{4}{3\pi^2} \frac{1-\nu}{a\rho_a \mu} \left[2\rho_a - (\rho_a^2 + 1)E(\rho_a) - (\rho_a^2 - 1)K(\rho_a) \right] F(t) = K_p F(t) \qquad (3)$$

Where K_p is function of the specimen, elastic properties and diameter of the bar Therefore K_p is constant all along the experiment and from (3) we can deduce that $1/K_p$ is homogeneous to a spring stiffness.

The punching effect is then equivalent to that produced by a spring of stiffness $1/K_p$. (smaller than that of the bars) acting between the bars and the specimen. It explains that SHPB processing without punching corrections leads to an underestimated value of the Young's modulus of the specimen.

Taking into account punching, the classical formulas (1), remain unchanged concerning the forces. The formulas, at the input interface (left) and at the output interface (right), are:

$$F_i(t) = A_i E(\varepsilon_i(t) + \varepsilon_r(t)) \qquad F_o(t) = A_o E \varepsilon_t(t)$$
$$V_i(t) = -c_{0i}(\varepsilon_i(t) - \varepsilon_r(t)) + \frac{dF_i(t)}{dt} K_{pi} \qquad V_o(t) = -c_{0r}\varepsilon_t(t) + \frac{dF_o(t)}{dt} K_{po} \tag{4}$$

In the common particular case of identical input and output bars and a cylindrical specimen in equilibrium, formulas giving the stress are unchanged.

Formulas giving the nominal strain are corrected in the following manner:

$$\varepsilon_{corr.} = \varepsilon_{SHPB} + \frac{2K_p \sigma(t) S_s}{l_s} \tag{5}$$

where S_s and l_s are the cross-sectional area and the length of the specimen, respectively.

6. EXPERIMENTAL ILLUSTRATION AND APPLICATIONS

In the processing procedure of SHPB, we use [10], the waves are first transported to the bar ends, taking into account the dispersion. The incident wave being known, we assume that the specimen has a purely 1-D elastic behavior. Taking the real specimen dimensions (area, length and density), a 1-D elastic calculation with a chosen Young's modulus, using the incident wave at bar end as input data, builds the reflected and transmitted waves as they would be produced by the elastic specimen. This calculation is very fast (it appears instantaneous). The "best" Young's modulus is then chosen to find the best match in shape between simulated waves and the beginning of (shifted) real ones. This procedure, called "elastic simulation" is described in [2], and is used for a more precise shift for the waves to be processed. This shift correction is always very small (a few microseconds) and has no influence on the dispersion effects.

The important point, for the present paper, is that it allows for the measurement of the apparent elastic modulus. When used without taking account of the punching correction, it leads to an underestimated value of Young's modulus.

This is illustrated in Figures 4 to 6 below.

It is observed that the expected real modulus of steel corresponds to waves that are different of real ones and that a (two times) lower modulus produces waves in good agreement with the real ones. This is due to the punching effect. When a test is processed with a specimen that has the same diameter than the bar, there is no punching effect and one recovers the expected modulus.

Figure 6 shows that the punching correction is worth being done. It particularly applies when the specimen diameter is smaller than that of the bars and when the total strain of the test is in a reduced range, as it is the case when low strain-rates are required. In the present test the average strain-rate was around 100/s.

7. CONCLUSION

The problem of the local punching of a bar axially loaded at one end has been investigated. When a known axial stress is dynamically applied on a circular central part of a bar end, it induces an axial displacement which varies along the radius of the bar. This displacement, depending on the

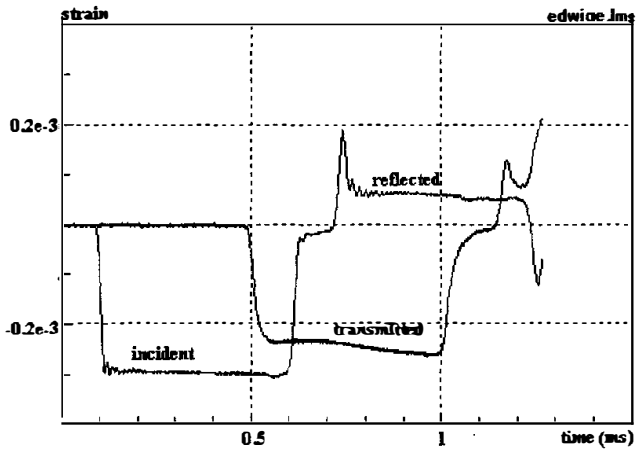

Figure 4. Basic recorded waves of a real test.

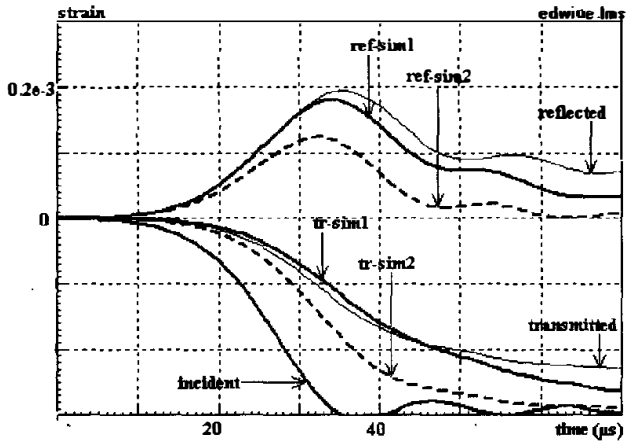

Figure 5. Waves at bar ends.

(Steel bars, diameter 20 mm, striker speed 2.8 m/s, steel specimen: length 6.5 mm, diameter 5.1 mm)

"incident", "reflected", "transmitted" (thin lines) show real waves at bar ends.
"tr-sim1" and "ref-sim1" show simulated transmitted and reflected waves produced by a purely elastic specimen with Young's modulus = 1.10^{11} Pa
"tr-sim2" and "ref-sim2" show simulated transmitted and reflected waves produced by a purely elastic specimen with Young's modulus = 2.10^{11} Pa (as expected)

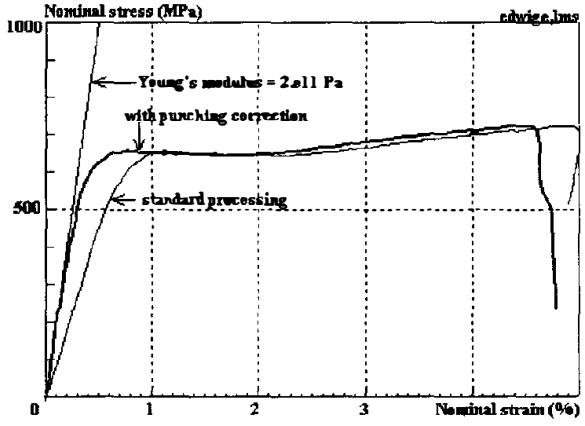

Figure 6. Nominal stress-strain curve: influence of the punching correction.

mechanical parameters of the bar and on both parameters of the loading (stress and diameter), is closely approximated by a simplified formula.

When applied to SHPB testing, it allows us to calculate the local punching induced by a specimen smaller than the bars. Consequently, it improves the measurement of material properties at low strains, and in particular provides an evaluation of the Young's modulus of elastic materials.

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