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Olivier Martins, Nicolas Peltier, Stéphane Guédon, Sylvian Kaiser, Yves Maréchal, et al.. A New Methodology for Early Stage Thermal Analysis of Complex Electronic Systems. *Therminic 2009*, Oct 2009, Leuven, Belgium. hal-00423729

HAL Id: hal-00423729

<https://hal.science/hal-00423729v1>

Submitted on 12 Oct 2009

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7-9 October 2009, Leuven, Belgium

A New Methodology for Early Stage Thermal Analysis of Complex Electronic Systems

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Abstract-This paper presents a new methodology called Flex-CTM for Flexible Compact Thermal Modeling to build and to interface compact thermal models at different modeling levels. Each part of an electronic system is prepared to be Boundary Condition independent (BCI) such as to be plugged to other parts. Each part model is reduced to save memory and time consuming at the simulation stage. The resulting pluggable and reduced thermal model is called a micro-model. Therefore, a fast-to-simulate macro-model of a full microelectronic system can be obtained by assembling micro-models.

The Flex-CTM is found to have numerous advantages over both current resistive models (junction-to-case and junction-to-board) and Dynamic Compact Thermal Models. The first advantage of the methodology is that multi-source and dynamic simulations of an electronic system can be performed at any design level. The second one is the control of the accuracy. The third advantage is the Boundary Condition Independence property that allows architecture exploration. Finally and the most important, micro and macro-models can be shared by teams to be reused and completed.

Keywords - Compact Thermal Modeling, model coupling, Boundary Condition Independence, multi-level modeling.

I. INTRODUCTION

In microelectronics, device designers are increasingly miniaturizing the electronic components, to design smaller products and to add more features. This miniaturization is at the origin of a strong rise of the power density. In addition, the power density rise with the temperature elevation lead to strong electro-thermal phenomenon that can damage electronic components.

Hot spots on the components cause thermal and mechanical stresses which affect circuit reliability. Furthermore, thermal gradients within the die, due to local hot spots, involve delay errors in logical gates and thereby limit expected performances. As discussed before, the temperature rise leads to an overconsumption that reduces the autonomy of nomad systems. Moreover, high temperatures decrease the life time of a system. Combined with a strong electro-thermal loop phenomenon, the component can be damaged by thermal runaway.

In order to limit these risks, electronic engineers have to perform transient thermal simulations at an early stage of the design flow, and at several granularity levels (die, package, board). Many teams in a single company are in charge of building their own thermal model (package model, die

model, board model...). The different modelling scales do not allow to obtain a single fine multi-level model. The Flex-CTM methodology, allows to share each specific thermal model in such a way that each separated thermal model is fine and can be evaluated by setting its own environment and other model dependency to perform more realistic simulations.

To speed up the thermal characterization process, the models must be compact and accurate to run fast simulation allowing a maximum bias of 5%. To summarize the need, a model must meet the following four criteria. First, the models have to be dynamic to allow transient simulations. Second, multiple power sources can be applied to fit with real case exploration (e.g., architectural level, many dies in a package, many packages on a board, ...). Third, to save simulation time, the models must have a reduced number of unknowns. Fourth, to be pluggable and reusable in different use cases, the models have to be Boundary Condition Independent (BCI).

The paper is divided into 5 parts. First, a short background of existing thermal models is presented. Second, the Flex-CTM methodology is introduced explaining the build of elementary pluggable compact thermal models. Third, the modeling and the simulation of the whole system is explained. Fourth, a synthesis describes the interest of the methodology. Finally, the speed and accuracy performances of the methodology are evaluated for a typical co-simulation case.

II. BACKGROUND

Several models already exist to simulate the thermal behavior of an electronic system. First, numerical methods (for example, the Finite Element Method) split a volume into elementary units. According to the JEDEC convention, the numerical thermal model is also called detailed model. This kind of model is difficult to build if the geometries are complex because the build of a well adapted mesh becomes arduous. Moreover, these numerical models become huge and slow to simulate.

Pioneering DELPHI methodology has been introduced to generate smaller models, in terms of number of unknowns [1]. This is a fitting method that creates Compact Thermal Models (CTM). A CTM is made of a network of resistors between key points of a package (a junction and outers). Other fitting methodologies have been introduced to add capacitive terms in the DELPHI compact models [2], like the

European project PROFIT [3] and [4]. The building of these models takes a lot of time because many simulations of the detailed model must be performed in transient domain, varying heat transfer coefficients applied on exchange surfaces. In addition, the junction modeling in DELPHI-like models is quite rough because it is considered as a single uniform heat dissipation source.

Recently, HotSpot analytical models [5], have been introduced to enhance the thermal model capabilities. The static and dynamic behaviors are addressed by a resistive and capacitive network between blocks at different scale levels. These models are mostly specific for the die. The package and board models are quite coarse and valid for specific heat flux distribution.

None of the existing methods meet the whole specifications quoted previously (Introduction section §4). In the following section, the Flex-CTM methodology is introduced to build pluggable Flex-CTM (Flexible Compact Thermal Models). The Flex-CTM are exclusively conductive models; the convection and the radiation are considered as first order phenomena. A Flex-CTM is a thermal resistance and capacitance network between nodes and can be assimilated to an electrical RC network by thermoelectric analogy.

III. BUILD OF PLUGGABLE COMPACT THERMAL MODELS

Usually, thermal models are built by different actors at different integration levels and the classical methodology does not allow to easily reuse and couple these models. The Flex-CTM methodology begins with the creation of a micro-model for each part of the whole system. A micro-model is a BCI compact thermal model with external connections for power sources, imposed temperatures, convection, and other conductive models. This section explains how to build a micro-model.

Model splitting is the first stage of the methodology which breaks up the global geometrical electronic system into elementary homogeneous material parts. For instance, a BGA package (Figure 1) is split into four descriptions (die, substrate, encapsulant and bondwires). The interfaces of the elementary parts are classified into two groups: power sources (interface P) and exchange interface with the environment (interface E).

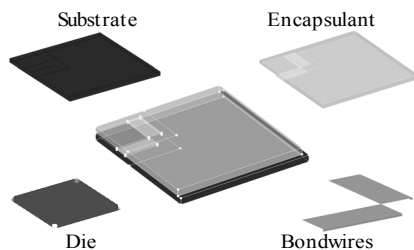


Figure 1: Geometrical and Physical Description of each part of a BGA Package

Second, the **extraction process** begins with the build of a numerical model of each part of the whole system, without any boundary condition. The model splitting step enables to mesh finely each model, without taking into account the geometrical constraints of the environment. Thus, more

accurate models of every part can be built in such way the discretization of the domain is adapted to a single material. The Finite Element Method (FEM) is chosen to retrieve the physical transfer functions at any meshing node. The transfer functions between nodes are written as a thermal admittance system. This system conjugates the thermal conductance sub-system G and the thermal susceptance sub-system C . The matrices G and C are square, symmetric and strictly definite positive. Equation (1) shows the computation of each element of the matrices G and C where k is the thermal conductivity of the material (in $\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$), ρ is its density (in $\text{kg}\cdot\text{m}^{-3}$) and C_p is its specific heat (in $\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$). Ω is the volume of the FEM meshing element. α_k is the form function according to the Galerkin method.

$$\begin{aligned} G(i, j) &= \iiint k \cdot \text{div}(\alpha_i) \cdot \text{div}(\alpha_j) d\Omega \\ C(i, j) &= \iiint \rho C_p \cdot \alpha_i \cdot \alpha_j d\Omega \end{aligned} \quad (1)$$

Moreover, the extraction process gives the geometrical properties of all the meshing nodes.

Third, the **node selection** step prepares the numerical model before being reduced. To perform it, a subset of external nodes which will be kept after reduction, has to be defined. The number of external nodes must be lower than the original one to obtain a small model after reduction. However, their number and their position have to ensure the most faithful transfer of the heat flux through the interface. So, the user has to define sub-sampled interfaces which are used in place of the original ones. The ne' nodes on this new interface are used to apply environmental and to measure the temperature at several points of the system. The user controls the trade-off between accuracy and the size of the reduced model for his study case, modifying the number of external nodes ne' . The replacement of the original interface by the sub-sampled one is performed through a coupling method. This coupling tool is detailed in the following section ("Whole System Modeling and Simulation"). The resulting model is the numerical model in which original interfaces (P and E) are substituted by sub-sampled interfaces (P' and E'), see Figure 2.

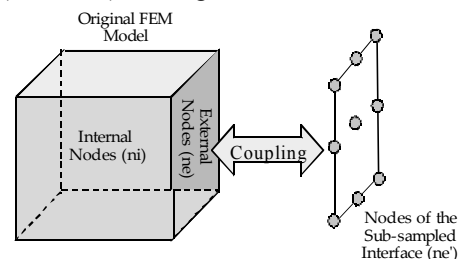


Figure 2: Node Selection

Fourth, the **reduction process** enables to decrease the dimension of a system, preserving its first order moments. The matrices G and C are ordered to place the nodes of respectively P' and E' at the ne' first columns. A Model Order Reduction (MOR) technique, based on projections on the Krylov subspace has been used to reduce the dimension [6], [7], [8].

The discreet expression of the detailed model in time domain has a static member and a dynamic member (2).

$$Gx + C\dot{x} = Bp \quad (2)$$

Where G is the thermal conductance subsystem, x the vector of the unknown temperatures, C the subsystem of thermal capacitances, B the input-output driver matrix, and p the vector of the discreet inputs-outputs.

G and C are sparse and symmetric definite positive $m \times n$ as dimension. The vector p and x are dense vector of size $m \times ne$.

To reduce the dimension of the system (2) the moment matching procedure will be applied in the frequency domain using the Laplace transformation.

$$\begin{aligned} Y(\omega) &= B^T(G + j\omega C)B \quad \text{if } R = G^{-1}B \text{ and } A = G^{-1}C \\ \text{then } Y(\omega) &= B^T(I + j\omega A)R \\ \text{and } T(\omega) &= Y^{-1}(\omega)p \end{aligned} \quad (3)$$

Where Y is the thermal admittance of the system, and T the vector of the unknown temperatures in the frequency domain.

The matrix B must be chosen to perform the average of temperatures and the average of power sources according to the P interface, elsewhere it is the identity. Therefore, Arnoldi algorithm is computed on A and R to extract the orthogonal bases U that spans the Krylov subspace.

$$K_{n'}(A, R) = \text{col-span}\{R, A^{-1}R, \dots, A^{(-n'+1)}R\} \quad (4)$$

Once U is calculated for a first number of moments ($n' = ne + ni'$), ni' starting at 10, the reduction is done by projecting G , C and B onto the base.

$$\begin{aligned} G'_{n'} &= U^T G U \\ C'_{n'} &= U^T C U \\ B'_{n'} &= U^T B \end{aligned} \quad (5)$$

The MOR technique is enhanced by adding a post-processing that controls the accuracy of the reduction. To control the accuracy, the matrix Y' of the transfer functions of admittances is calculated (6) in the frequency domain. Note that Y' is very fast to compute.

$$\begin{aligned} Y'(\omega) &= B'^T(I + j\omega A')R' \\ \text{where } R' &= G'^{-1}B' \text{ and } A' = G'^{-1}C' \end{aligned} \quad (6)$$

While the mean of the Y' (6) values are changing, (3), (4) then (5) are calculated increasing ni' . This loop allows to ensure to extract the optimum first order moments that match the Krylov subspace. The reduced model has ne' external nodes and N' unknowns where N' is approximately 10% of N .

At this stage, a micro-model of each part of the whole

system is built.

IV. WHOLE SYSTEM MODELING AND SIMULATION

This section deals with the coupling of micro-models to build a complete model of an electronic system. These micro-models are coupled together through their respective E' interface nodes. The interfaces of the models to connect are not supposed to have the same geometrical configuration. Therefore, the models cannot be coupled by just linking the interface nodes one by one. In the followings, the coupling method that allows to connect two micro-models is described.

Two micro-models (G_1, C_1) and (G_2, C_2) are assembled in a non-coupled system equation (7). Where G_k ($k = 1$ or 2) is the conduction matrix and C_k is the diffusion matrix of the micro-model k . T_k is the vector of temperature at each node of the micro-model k . F_k is the load vector at each node of the micro-model k .

$$\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \begin{bmatrix} \dot{T}_1 \\ \dot{T}_2 \end{bmatrix} + \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (7)$$

The distribution of the nodes of the two uncoupled micro-models is described Figure 3. The "e" index is for the external node block, and the "i" index is for the internal node block. The structure of the nodes of the two models is modified in order to isolate the external nodes of the model 2, see (8).

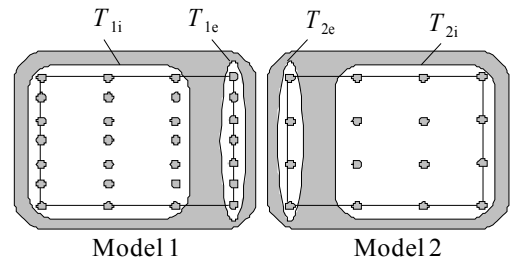


Figure 3: Nodes Distribution of two Uncoupled Models

$$\begin{aligned} T'_1 &= T_{1e} \cup T_{1i} \cup T_{2i} \\ T'_2 &= T_{2e} \end{aligned} \quad (8)$$

The objective of the coupling process is to create links ($G_{12}, G_{21}, C_{12}, C_{21}$) between two models as shown in equation (9).

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \dot{T}'_1 \\ \dot{T}'_2 \end{bmatrix} + \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} T'_1 \\ T'_2 \end{bmatrix} = \begin{bmatrix} F'_1 \\ F'_2 \end{bmatrix} \quad (9)$$

with

$$\begin{aligned} G_{11} &= G_1 \cup G_{2i} & G_{22} &= G_{2e} \\ C_{11} &= C_1 \cup C_{2i} & C_{22} &= C_{2e} \\ F'_1 &= F_1 \cup F_{2i} & F'_2 &= F_{2e} \end{aligned} \quad (10)$$

Where G_{2i} (resp. C_{2i}, F_{2i}) is the internal node sub-block of

G_2 (resp. C_2, F_2) and G_{2e} (resp. C_{2e}, F_{2e}) is the external node sub-block of G_2 (resp. C_2, F_2).

The link is realized by the method of Lagrange multipliers [9]. It transforms a minimization problem under constraints in a minimization problem without constraint. The Lagrange multipliers allow to solve the heat equation (7), ensuring the continuity of temperature at the interfaces of the two models (11).

$$\begin{aligned} \dot{T}'_2 &= X \dot{T}'_1 \\ T'_2 &= X T'_1 \end{aligned} \quad (11)$$

Where X is the cross coupling between the nodes of the two model interfaces.

This mathematical method applied to the thermal model coupling issue is described in [10]. The looking for the singular points of the Lagrangian gives the following matrix system:

$$\begin{bmatrix} C_{11} & C_{12} & -X^T & 0 \\ C_{21} & C_{22} & I & 0 \\ -X & I & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{T}'_1 \\ \dot{T}'_2 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} + \begin{bmatrix} G_{11} & G_{12} & 0 & -X^T \\ G_{21} & G_{22} & 0 & I \\ 0 & 0 & 0 & 0 \\ -X & I & 0 & 0 \end{bmatrix} \begin{bmatrix} T'_1 \\ T'_2 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

Eliminating λ_1, λ_2 and T'_2 in (12), leads to a new equation system (13) which describes the thermal behavior in transient state of the two coupled models.

$$\tilde{C} \dot{T}'_1 + \tilde{G} T'_1 = \tilde{F} \quad (13)$$

where

$$\begin{aligned} \tilde{C} &= C_{11} + X^T C_{21} + C_{12} X + X^T C_{22} X \\ \tilde{G} &= G_{11} + X^T G_{21} + G_{12} X + X^T G_{22} X \\ \tilde{F} &= F_1 + X^T F_2 \end{aligned} \quad (14)$$

In this methodology, the coupling matrix X is an application of the nodes of interface 2 to interface 1. For each node Node 2_j to be replaced in interface 2, the surrounding 4 nodes in interface 1 are identified (see Figure 4).

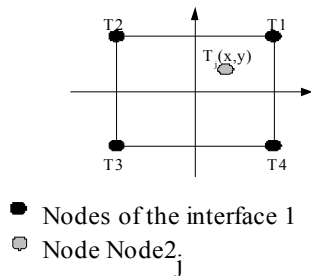


Figure 4: Neighbor Nodes Configuration for a Hexahedral Mesh of the Interface 1

Thus, the neighbor nodes of Node 2_j are weighted, computing the form factors at the coordinates of the node Node 2_j . Equation (15) shows the general form factors for a hexahedral mesh of first order.

$$\begin{aligned} \alpha_1(x, y) &= \frac{1+x}{2} \frac{1+y}{2} & \alpha_2(x, y) &= \frac{1-x}{2} \frac{1+y}{2} \\ \alpha_3(x, y) &= \frac{1-x}{2} \frac{1-y}{2} & \alpha_4(x, y) &= \frac{1+x}{2} \frac{1-y}{2} \end{aligned} \quad (15)$$

Then, the temperature T_j of the node Node 2_j is computed through equation (16).

$$\begin{aligned} T_j(x, y) &= \alpha_1(x, y).T1 + \alpha_2(x, y).T2 + \\ &\alpha_3(x, y).T3 + \alpha_4(x, y).T4 \end{aligned} \quad (16)$$

The equation (16) expresses the temperature T_j as the linear combination Λ_j of the second interface temperatures.

$$T_j = \Lambda_j.T'_1 \quad (17)$$

Finally, the coupling matrix X between the two models is built by all the line matrices Λ_j (18).

$$X = [\Lambda_1 \quad \Lambda_1 \quad \dots \quad \Lambda_{ne'_2}]^T \quad (18)$$

Where ne'_2 is the number of external nodes in the model 2. X is a rectangular matrix of size $ne'_2 \times (n_1 + ni_2)$, where n_1 is the number of nodes of the model 1 and ni_2 is the number of internal nodes of the model 2.

The Flex-CTM resulting from the coupling process is a RC network which can be solved with a Spice like transient simulator. Fixed potentials are applied on the model, to impose temperatures T_i on several external nodes of the Flex-CTM. Then, convection resistors R_{conv} (19) are plugged on the convection surface nodes nodes to impose heat transfer coefficients on surfaces of the system.

$$R_{conv} = \frac{1}{hS} \quad (19)$$

Where h is the heat transfer coefficient applied on the surface S of the model. Then, the power sources Q are applied on P' interface nodes and the Flex-CTM can be simulated in a transient scenario (see Figure 5).

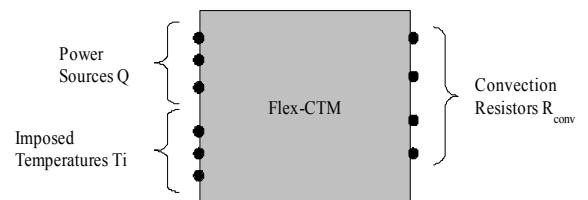


Figure 5: Simulation Condition Application

V. INTEREST OF THE FLEX-CTM METHODOLOGY

The building process of the Flex-CTM methodology is illustrated Figure 9. The models generated by the method present numerous advantages over the existing thermal models.

Flex-CTM models meet the specifications because they

are small, they can be reused and shared between the different design teams.

Precisely, the methodology homogenizes the models at different integration levels to facilitate the share of the models. In that way, a designer builds finely its own model of interest using other available micro-models of materials and environments.

Moreover, the Flex-CTM models contain few information of the original system. The geometrical and physical properties of the system are included implicitly in the model. Therefore, the models can preserve a confidentiality of the system properties.

Finally, the Flex-CTM have the “Flexible” property. Meaning that each micro-model of the Flex-CTM can be modified independently and replaced by another one.

VI. EVALUATION OF THE PERFORMANCES OF THE FLEX-CTM METHODOLOGY

The methodology is evaluated with a simple co-simulation test case. The studied system is a die in a Ceramic Pin Grid Array (CPGA) package, plugged on a Printed Circuit Board (PCB). The typical use is an integrator who wants to characterize a board for a given component. The integrator may have obtained the die and package information in datasheets or even their micro-models. The geometry of the IC package is described Figure 6.

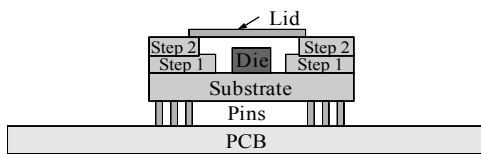


Figure 6: CPGA Geometrical Description

To evaluate the influence of the board on which the component will be plugged, the component is connected to two different boards (Figure 7).

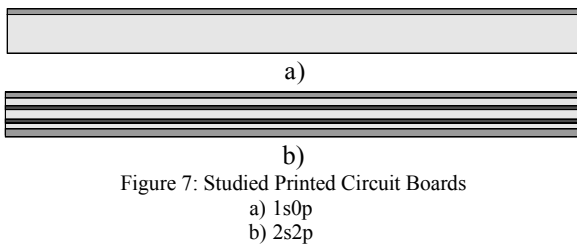


Figure 7: Studied Printed Circuit Boards

a) 1s0p
b) 2s2p

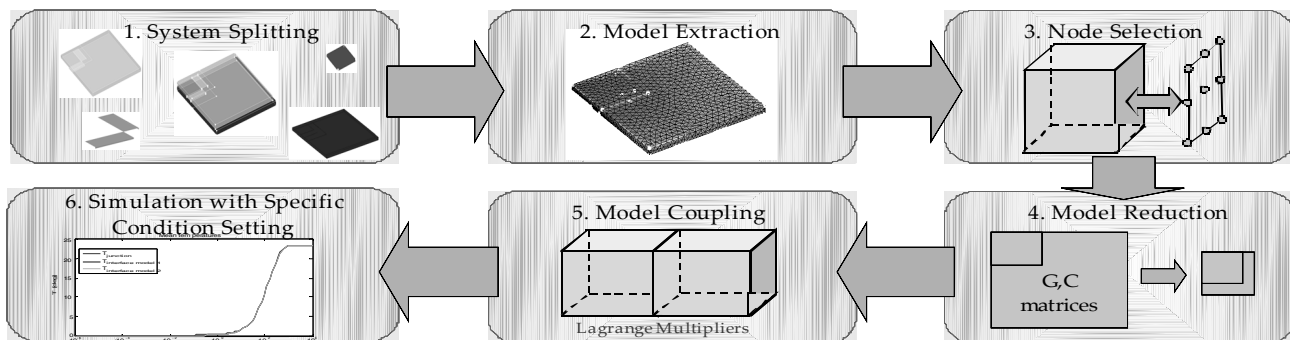


Figure 9: Building Process of the Flex-CTM Methodology

The model has been built with the typical values of the material thermal properties which can be found in the literature [11].

The air layer surrounding the die is assumed to be insulating due to its very weak thermal conductivity. So, in this case a unique die-to-package heat flow path through the bottom face of the die is considered. The CPGA is simulated applying a uniform power source of 1W on the junction surface.

A reference measure of the average temperature on the Die junction is obtained by simulating a FEM model of the full circuit. This measure is a reference to evaluate the methodology in terms of accuracy, model size and simulation time.

The first step of the Flex-CTM methodology splits the CPGA geometry into 4 descriptions according to the physical properties (see Figure 8).

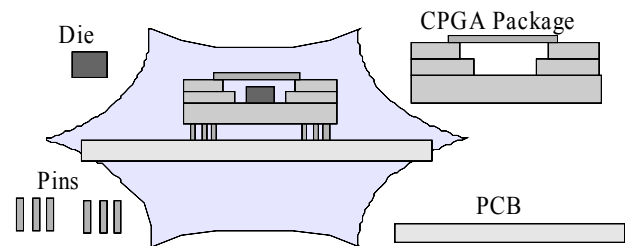


Figure 8: Model Splitting

The junction interface is on the top of the die and the model contains 3 coupling interfaces. The first one links the die to the package substrate, the second one links the package substrate to the pins and the last one links the pins to the PCB. The 4 (Die, Package, Pins, PCB) FEM models of the system are extracted in their respective numerical systems $G_{1,2,3,4}$ and $C_{1,2,3,4}$ with their related node properties. Concerning the heat transfer coefficients, they are identified using a commercial CFD tool. Then they are embedded in the CPGA Package and in the board models. The nodes belonging to the interfaces of the models are kept during the reduction process. TABLE I summarizes the number of nodes of each part at different steps of the methodology. The number in brackets indicates the number of external nodes kept of each micro-model.

TABLE I
 MODEL PARTS CHARACTERISTICS

Part	FEM Model	Sub sampled Model	Reduced Model
Die	70k (2600)	65k (227)	335 (227)
Package	147k (12k)	136k (356)	643 (356)
Pins	69k (3400)	66k (200)	208 (200)
PCB 1s0p	178k (900)	177k (100)	978 (100)
PCB 2s2p	178k (900)	177k (100)	822 (100)

Once the micro-models are available, they are coupled together through their corresponding coupling interfaces (E'). An assembly (die-package-pins) resulting of the coupling of the die, package and pins models is obtained. Then, to study the influence of the board, each PCB micro-model is connected successively to the assembly. So, two Flex-CTM of the whole system are built, one for each board.

Finally, a power step source of 1 W is uniformly distributed on the Junction interface P'. Reference simulations of the global FEM model are computed for each board and the corresponding Flex-CTMs are simulated under the same simulation conditions (Figure 10).

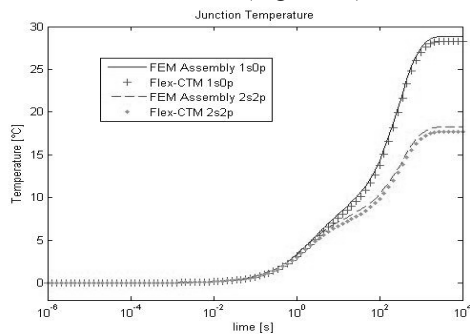


Figure 10: Original Model and Flex-CTM Temperatures

The comparisons in terms of accuracy, model size and simulation time between the original CPGA and the Flex-CTM are summarized in TABLE II. The first Flex-CTM needs about 4 hours to be built. The second needs only one half hour because the assembly die-package-pins is reused.

 TABLE II
 ORIGINAL AND FLEX-CTM MODELS PERFORMANCES

Model	Size	Simulation Time	Maximal Absolute Error
FEM Assembly 1s0p	530k	24 hours	0.57°C
Flex-CTM 1s0p	1583	12 seconds	
FEM Assembly 2s2p	530k	24 hours	0.52°C
Flex-CTM 2s2p	1427	12 seconds	

VII. CONCLUSIONS AND FUTURE WORK

The Flex-CTM methodology meets the needs of electronic engineers to perform a fast temperature analysis of a complex electronic system at different integration levels. Flex-CTM are BCI, so they can be reused whatever the environment is. Many power sources can be applied on

junction nodes allowing hot spot detection on a die. Moreover, Flex-CTM have a few node number, which allows multiple exploration or electro-thermal simulation in a short window of time. Finally, the methodology allows system designers to share their work at different integration levels.

The methodology has been evaluated with a simple test-case at the package modeling level. The results show that Flex-CTM meet the specifications required, specifically in terms of accuracy and simulation time saving.

The next step is now to enhance the methodology with an automated selection of the number and the position of nodes at the interfaces. This progress will ensure a higher accuracy and an optimal size of the Flex-CTM.

Besides, several test cases covering multi-level design aspects are to be run in order to go on further the validation and to better characterize the methodology.

Finally, a multi-source co-simulation test case will be studied to fit with a more realistic system.

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