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¹ Why climate sensitivity may not be so unpredictable?

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7 Different explanations have been proposed as to why the range of climate
8 sensitivity predicted by GCMs have not lessened substantially in the last decades,
9 and subsequently if it can be reduced. One such study (*Why is climate sen-*
10 *sitivity so unpredictable?*, Roe and Baker, 2007 [①]) addressed these questions
11 using rather simple theoretical considerations and reached the conclusion that
12 reducing uncertainties on climate feedbacks and underlying climate processes
13 will not yield a large reduction in the envelope of climate sensitivity. In this
14 letter, we revisit the premises of this conclusion. We show that it results from
15 a mathematical artefact caused by a peculiar definition of uncertainty used
16 by these authors. Applying standard concepts and definitions of descriptive
17 statistics to the exact same framework of analysis as Roe and Baker, we show
18 that within this simple framework, reducing inter-model spread on feedbacks
19 does in fact induce a reduction of uncertainty on climate sensitivity, almost
20 proportionally. Therefore, following Roe and Baker assumptions, climate sen-
21 sitivity is actually not so unpredictable.

1. Introduction

22 Uncertainties in projections of future climate change described in the last Assessment
23 Report of the IPCC (IPCC, 2007 [a]) are high, as illustrated by the broad range of
24 climate sensitivity – defined as the global mean temperature increase for a doubling
25 of CO₂ – simulated by general circulation models (GCMs). Attempts to explain this
26 fact have focused mainly on uncertainties in our understanding of the individual physical
27 feedback processes (especially associated to clouds), difficulties to represent them faithfully
28 in GCMs, nonlinearity of some processes and complex interactions among them giving rise
29 to a chaotic behaviour of the climate system (Randall et al. [2007^a]). A review of these
30 explanations can be found in Bony et al., 2006 [a]. Nevertheless, in this letter, we leave
31 aside all these considerations to focus our interest solely on the explanation proposed by
32 Roe and Baker, 2007 [a] (RB07) which somewhat differ from the above-mentioned. This
33 study uses the framework of feedback analysis, which has often been used to describe the
34 relationship between physical processes involved in global warming and climate sensitivity
35 (see for instance Lu and Cai, 2008 [a], Dufresne and Bony, 2008 [a], Soden and Held,
36 2006 [a]). The feedback analysis framework assumes a linear approximation of radiative
37 feedbacks, resulting in a simple relationship between a global feedback gain f and climate
38 sensitivity ΔT . In this classic setting, the main originality of RB07 approach consists
39 in analyzing explicitly the way uncertainties on f , due to a limited understanding of
40 their underlying physical processes, propagates into uncertainties on ΔT : assuming f
41 is a random variable with mean \bar{f} and standard deviation σ_f , RB07 uses this simple
42 probabilistic model to highlight several fundamental properties of uncertainty propagation

43 from feedbacks to climate sensitivity. The most prominent conclusion of this analysis is
44 that reducing uncertainties on f does not yield a large reduction in the uncertainty of
45 ΔT , and thus that improvements in the understanding of physical processes will not yield
46 large reductions in the envelope of future climate projections. This conclusion, if true,
47 would clearly have crucial implications for climate research and policy.

48 In section 2, we revisit the premises of RB07 conclusion. We highlight that it is the
49 result of a peculiar way of defining uncertainty. Moreover, we show in section 5 that
50 this conclusion is a pure mathematical artefact with no connection whatsoever to climate.
51 Since the basic question of uncertainty definition appears to be at stake, section 3 briefly
52 recalls widely used definitions and elementary results on uncertainty and its propagation
53 as they can be found in Descriptive Statistics textbooks. In section 4, we apply these
54 standard concepts and definitions to the exact same framework of analysis as RB07. We
55 show that within this simple framework, reducing inter-model spread on feedbacks does
56 in fact induce a reduction of uncertainty on climate sensitivity, almost proportionally.
57 Finally, section 6 concludes.

2. Overview of RB07 approach

58 RB07 uses the feedback analysis framework. Denoting ΔT_0 the Planck temperature
59 response to the radiative perturbation and f the feedback gain (referred to as feedback
60 factor in RB07), they obtain:

$$61 \quad \Delta T = \frac{\Delta T_0}{1 - f} \quad (1)$$

62 RB07 then assumes uncertainty on Planck response to be neglectible so that the entire
63 spread on ΔT results from the uncertainty on the global feedback gain f . To model

64 this uncertainty, RB07 assumes that f follows a gaussian distribution with mean \bar{f} , stan-
 65 dard deviation σ_f and implicit truncation for $f > 1$ (implications of this truncation are
 66 discussed in appendix 1). Then, they derive an exact mathematical expression of the
 67 distribution of ΔT through equation (1). This simple probabilistic climatic model is then
 68 used by RB07 to analyze the way uncertainties on f , due to a limited understanding of
 69 their underlying physical processes, propagates into uncertainties on ΔT . Their analysis
 70 highlights two fundamental properties of uncertainty propagation:

- 71 • Amplification: The term in $\frac{1}{1-f}$ in equation (1) amplifies uncertainty on feedbacks, all
 72 the more intensely as \bar{f} is close to (though lower than) one. Small uncertainties on feed-
 73 backs are thus converted in large uncertainties on the rise of temperature.
- 74 • Insensitivity: Quoting RB07, “*reducing uncertainty on f has little effect in reducing*
 75 *uncertainty on ΔT ”, also stated as “the breadth of the distribution of ΔT is relatively*
 76 *insensitive to decreases in σ_f .”*

77 We fully subscribe to the first property and elaborate further on it in section 4. However,
 78 we are puzzled by the second property, that is, the claimed insensitivity of uncertainty
 79 on ΔT to uncertainty on feedbacks. The reason why one may find this second assertion a
 80 priori puzzling, is that it intuitively seems to be at a contradiction with the first property
 81 highlighted. Indeed, if small uncertainties on f are amplified into large uncertainties
 82 on ΔT , it suggests that a strong dependency exists between both uncertainties, rather
 83 than no or little dependency. We therefore dig into the details of RB07 argumentation
 84 regarding this assertion. To get to that conclusion, it appears that RB07 actually focus
 85 on the probability $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$ that ΔT lies in the interval $[4.5^\circ C, 8^\circ C]$ in

86 response to a sustained doubling of the CO_2 concentration. This interval is defined as
87 immediately above the range obtained with the CMIP3/AR4 GCMs (IPCC, 2007 [a]).
88 They study graphically how this probability fluctuates with the level of uncertainty on
89 feedbacks, by plotting for several values of σ_f the obtained cumulative distribution of
90 ΔT . Doing this graphical analysis, they observe that the probability of large temperature
91 increase $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$ is insensitive to σ_f . This observation is easily verifiable:
92 we replicated RB07 cumulative distribution chart in figure 1c, and we computed several
93 values of $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$ for $\bar{f} = 0.65$ and σ_f ranging from 0.10 to 0.20, finding
94 it to fluctuate between 0.18 and 0.20. Therefore, in agreement with RB07, it is fair to
95 say that the probability of large temperature increase (i.e. $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$) is quite
96 insensitive to σ_f in this domain. However, concluding from this observation that “*the*
97 *breadth of the distribution of ΔT is relatively insensitive to decreases in σ_f ” and that
98 “*reducing uncertainty on f has little effect in reducing uncertainty on ΔT ” implicitly
99 assumes two very different definitions of uncertainty: while on the side of feedback the
100 uncertainty is measured by standard deviation σ_f , on the side of sensitivity the probability
101 $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$ is used as a metric of uncertainty. As will be developed in section 3,
102 standard deviation is a standard, consensual uncertainty metric but the probability to lie
103 in a fixed interval is not. While under this peculiar double definition of uncertainty RB07
104 conclusion holds, it is fair to ask whether it would still hold with a different uncertainty
105 metric for ΔT ; second, whether the probability to lie in a fixed interval can be considered
106 an acceptable measure of distribution breadth; and third, what are the implications of**

107 using such an asymmetric definition of uncertainty. The following sections attempt to
108 answer these questions.

3. Standard measurement and propagation of distribution spread

109 To investigate the first question, which relates to the basic issue of uncertainty definition,
110 we briefly recall a few standard definitions and concepts, as they can be found almost
111 identically in most Descriptive Statistics textbooks. For details, the reader can refer for
112 instance to Barlow, 1989 [@], Van der Vaart, 2000 [@], Reinard, 2006 [@], James and
113 Eadie, 2006 [@] to mention but a few such textbooks.

114 Descriptive Statistics primary purpose is to provide metrics summarizing a sample of
115 observations and similarly, in probabilistic terms, metrics summarizing the probability
116 density function (pdf) underlying them. Technically, the correspondance between both
117 is simply that a sample summary is an *estimator* (a function of the data) which esti-
118 mates a distribution summary *estimand* (a parameter). In the present case, we study
119 continuous random variables thus we are rather concerned about pdf metrics than sam-
120 ple metrics, even though these pdfs actually aim at fitting a sample of observations, in
121 that case CMIP3/AR4 GCMs simulations (Meehl et al. [2007]). Descriptive Statistics
122 usually group metrics under three categories: location, scale and shape parameters. The
123 so-called location parameters are meant to identify the center of a distribution. Most
124 common location measures are mean, mode and median. The so-called scale parameters,
125 also referred to as dispersion, variability, variation, scatter or spread measures, describe
126 how far from the above-defined center possible values covered by the distribution tend
127 to be. This second group of metrics is the one we are interested in for our discussion,

128 as it is concerned with the measurement of distribution spread. Most common measures
 129 are standard deviation, interquartile range (IQR), range or median absolute deviation
 130 (MAD), more rarely full width at half maximum (FWHM). Variance and coefficient of
 131 dispersion should also be mentioned though they are not expressed in the same unit as the
 132 variable. Above mentioned references give complete mathematical expressions, properties,
 133 strengths and limitations of these. We underline a property of particular interest to our
 134 discussion: above mentioned measures of spread are invariant in location and linear in
 135 scale. In other words, denoting S any particular measure of spread amongst those listed
 136 above, X a random variable and $Y = aX + b$ then:

$$137 \quad S_Y = |a| \cdot S_X \quad (2)$$

138 Further, in the general case of a dependency of the type $Y = \phi(X)$:

$$139 \quad S_Y \simeq |\phi'(M_X)| \cdot S_X \quad (3)$$

140 where ϕ' represents the first derivative of ϕ and M is a location parameter. This linear
 141 approximation is commonly used to combine errors on measurements, though generally
 142 in its multivariate formulation, and is thus sometimes referred to as the error propagation
 143 framework. It may also be used to study the way uncertainty on some input variable(s)
 144 propagates into uncertainty on an output obtained from a determinist function, as in
 145 section 4.

4. Standard uncertainty propagation in RB07 feedback model

146 We now analyse the dependency between uncertainty on feedbacks and uncertainty on
 147 climate sensitivity in RB07 model. Denoting $S_{\Delta T}$ a measure of climate sensitivity spread,

148 S_f a measure of feedback spread and M_f a measure of feedback location, the uncertainty
 149 propagation recalled in equation (3) can be applied straightforward to equation (1), lead-
 150 ing to:

$$151 \quad S_{\Delta T} \simeq \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \quad (4)$$

152 Note that Equation (4) holds for any choice of pdf for feedback factor f and thus applies
 153 more generally than in the particular case of a truncated gaussian pdf chosen by RB07.
 154 Equation (4) also provides a simple relationship between $S_{\Delta T}$, S_f and M_f which translates
 155 into the following two properties:

156 • Amplification: In agreement with RB07 first above recalled result, for a fixed level
 157 of feedback uncertainty S_f , the level of sensitivity uncertainty $S_{\Delta T}$ is amplified when
 158 feedback M_f approaches one. Since estimates of feedback parameters in CMIP3/AR4
 159 models (Soden and Held, 2006 [@], Randall et al. [2007^a]) suggest M_f is close enough
 160 to one ($M_f \simeq 0.65$) and hence yields substantial amplification, it seems that “*the climate*
 161 *system is operating in a regime in which small uncertainties in feedbacks are amplified in*
 162 *the resulting climate sensitivity uncertainty*”, to quote RB07.

163 • Proportionality: In disagreement with RB07 second above recalled result, for a fixed level
 164 of average feedback M_f , the level of climate sensitivity uncertainty $S_{\Delta T}$ is proportional
 165 to the level of feedback uncertainty S_f ($S_{\Delta T} \simeq 9.8 S_f$ for $M_f \simeq 0.65$). This simple
 166 relationship between both uncertainties is intuitive. Indeed, when $S_f = 0$, feedbacks are
 167 determinists and ΔT also is, considering no other source of uncertainty in the climate
 168 system, hence $S_{\Delta T} = 0$. As values of f get increasingly scattered, resulting values of
 169 climate sensitivity also get more scattered proportionally (figure 1a and 1b).

170 This proportionality has general validity in the sense that it holds for any above-recalled
 171 standard spread measure and for any distribution of f . However, it is an approximation for
 172 small values of S_f . We therefore find it relevant to investigate how this linear dependency
 173 is affected when S_f increases. To perform this analysis, we exhibit more precise results
 174 on uncertainty propagation in RB07 model. First, when spread is measured by IQR, an
 175 exact relationship holds for any value of S_f and any distribution of f (appendix 2):

$$176 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} S_f \cdot \left\{ 1 - \frac{w_f}{1-M_f} S_f - \frac{1-w_f^2}{4(1-M_f)^2} S_f^2 \right\}^{-1} \quad (5)$$

177 where w_f measures the asymmetry of f distribution. Hence, when $S \equiv \text{IQR}$, the dependency
 178 between $S_{\Delta T}$ and S_f is always overlinear when $w_f \geq 0$, eg when f has a symmetric or right
 179 skewed distribution. When it is left skewed, the dependency is sublinear for small values
 180 of S_f but eventually becomes overlinear when S_f is large enough. Second, when spread
 181 is measured by standard deviation, a second order Taylor expansion of equation (1) leads
 182 to a more accurate approximation (appendix 3):

$$183 \quad S_{\Delta T} \simeq \frac{\Delta T_0}{(1-M_f)^2} S_f \cdot \left\{ 1 + \frac{2w_f}{1-M_f} S_f + \frac{k_f-1}{(1-M_f)^2} S_f^2 \right\}^{\frac{1}{2}} \quad (6)$$

184 Again, overlinearity prevails when $w_f \geq 0$ or S_f large enough, which is connected to the
 185 convexity of the dependency between ΔT and f . Third, when S is standard deviation
 186 and f distribution is log-normal, an exact formula holds for any S_f :

$$187 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \cdot \left\{ 1 + \left[\frac{S_f}{1-M_f} \right]^2 \right\} \quad (7)$$

188 and is again overlinear. Finally, overlinear relationships can also be derived when the
 189 distribution of f is assumed to be gamma or beta (equations (12) and (14) in appendix
 190 4).

191 To summarize the above discussion, its main outcome is rather intuitive and has actually
 192 few to do with climate: if the spread of feedback factor values decreases, the resulting
 193 spread of climate sensitivity values also decreases. Secondly, the dependency is as follows:
 194 it is linear for small feedback spreads and tends to get overlinear for larger values. Last,
 195 the proportionality coefficient in the dependency sharply increases as feedback intensifies.

5. Properties of the probability to lie in a fixed interval

196 We now focus on whether the probability to lie in a fixed interval can be considered
 197 an acceptable measure of distribution breadth, as implicitly done by RB07 to reach their
 198 main conclusion. We approach this question very generally: let X be a continuous random
 199 variable with location M_X , spread S_X and pdf p_X . Let $[a, b]$ be a fixed interval near but
 200 above the center ($M_X < a < b$). Then, when $S_X \rightarrow 0$ the variable becomes determinist
 201 ($X = M_X$) and it results that $\mathbb{P}(X \in [a, b])$ equals to zero since $M_X \notin [a, b]$. When
 202 $S_X \rightarrow +\infty$ the distribution covers such a wide range of values that the probability to
 203 exceed any given threshold slowly increases towards 0.5 (figure 2b). In particular $\mathbb{P}(X >$
 204 $a) \rightarrow 0.5$ and $\mathbb{P}(X > b) \rightarrow 0.5$, hence $\mathbb{P}(X \in [a, b]) = \mathbb{P}(X > a) - \mathbb{P}(X > b) \rightarrow 0$
 205 (appendix 5). Hence the dependency between $\mathbb{P}(X \in [a, b])$ and S_X is characterized by
 206 a non monotonous function that increases, flattens and then decreases to zero (figure
 207 2a). In light of this non monotonous dependency, it is difficult to hold $\mathbb{P}(X \in [a, b])$
 208 as a valid measure for the width of X distribution. Further, the observed insensitivity
 209 of $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$ to feedback spread S_f , which lead authors to their conclusion,
 210 happens to proceed directly from the above described dependency: this flattening of the

211 dependency is a pure mathematical artefact which systematically manifests under these
212 definitions, and has nothing to do with climate.

213 Finally, if one still wants to stick to this peculiar, asymmetric definition of uncertainty, it
214 has to be noted that in RB07 model, even though the dependency is flat in the domain
215 $S_f \in [0.1, 0.2]$, the dependency is strong for $S_f < 0.1$ when $M_f \approx 0.65$ and subsequently
216 leads to a steep decrease of $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$ to zero (figure 1d). In fact, since
217 feedback current estimates suggest $S_f \simeq 0.09$ and $M_f \approx 0.65$ (Soden and Held, 2006 [©],
218 Randall et al. [2007^a]), the domain of strong dependency may actually already be reached
219 to date.

6. Conclusion

220 Developments in section 5 suggest that, while the probability $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$
221 may be of interest practically, this metric is irrelevant to describe “*the breadth of the dis-*
222 *tribution of climate sensitivity*” which was RB07 explicit intent. To address this question,
223 any measure of distribution spread chosen amongst those classically used in Descriptive
224 Statistics and recalled in section 3, appear to us more appropriate. With such measures of
225 spread, we showed in section 4 that in RB07 framework, when the spread of feedback pa-
226 rameter S_f decreases, the resulting spread of climate sensitivity $S_{\Delta T}$ values also decreases.
227 Further, we also highlighted that in this framework, the decrease is approximately linear
228 for S_f small and tends to be overlinear (i.e. to be steeper) for larger values of S_f owing
229 to the convexity of the dependency between ΔT and f .

230 Other than the definition issue discussed here, the relevance of RB07 simplified model to
231 describe the dependency between climate sensitivity and feedbacks may also be discussed

232 but this was beyond the scope of this article. In any case, if one holds this model to be
 233 accurate, a decrease of the spread on feedback will lead to a decrease of the uncertainty
 234 on climate sensitivity and a narrowing of the envelope of future climate projections. If
 235 enough studies are undertaken to better understand and assess the physical processes
 236 involved in the different feedbacks, neither are doomed to remain at their current level.

Appendix

1 – Implications of the truncation

237
 238 Since the linear feedback model of RB07 implicitly assumes $f \leq 1$, the gaussian distribu-
 239 tion $\mathcal{N}(\bar{f}, \sigma_f)$ proposed by RB07 is implicitly truncated for $f > 1$ – otherwise equation
 240 (1) would produce negative values of ΔT . This truncation has several implications. First,
 241 σ_f (resp. \bar{f}) does not exactly match standard deviation (resp. mean) of the truncated
 242 distribution. For instance, when $(\bar{f}, \sigma_f) = (0.75, 0.25)$ the standard deviation of f equals
 243 0.18 and its mean equals 0.67. Second, it introduces some negative skewness in the dis-
 244 tribution of f (-0.39 in the same example) which becomes more and more asymmetric as
 245 σ_f and \bar{f} increases. Finally, since the truncated gaussian pdf is finite and non zero in the
 246 vicinity of $f = 1$, the obtained pdf of climate sensitivity behave as a Pareto distribution
 247 in $\mathcal{O}(\Delta T^{-2})$ for high values, and hence does not have a finite mean, nor a finite variance.
 248 Hence, the truncated gaussian model of RB07 forbids the use of standard deviation as
 249 a measure of climate sensitivity spread, which explains the use of IQR in figure 1. For
 250 the purpose of RB07 which is to study climate sensitivity spread, assuming a parametric
 251 distribution of f – such as log-normal, gamma or beta – which leads to finite mean and
 252 deviation for sensitivity and exact mathematical expressions of the dependency between

the deviation of ΔT and the deviation of f (appendix 3), would be in our view more convenient. However, the results on the dependency between $S_{\Delta T}$ and S_f presented in section 4 are general and also hold under RB07 gaussian assumption. Therefore, RB07 truncated gaussian is in our view mathematically inconvenient, but it does not affect uncertainty propagation: for a gaussian distribution just as for any other, the spread dependency is approximately linear for small spreads and overlinear otherwise, as equation (4) and (5) demonstrate and as figure 1b illustrates.

2 – Exact uncertainty propagation equation for IQR

If X is a continuous random variable X , we denote X_α its α -quantile, $S_X = X_{0.75} - X_{0.25}$ its interquantile range, $M_X = X_{0.50}$ its median and $w_X = \frac{X_{0.75} + X_{0.25} - 2X_{0.50}}{X_{0.75} - X_{0.25}}$ a dimensionless, quantile-based metric of asymetry. We thus have $X_{0.75} = M_X + \frac{1}{2}S_X(1 + w_X)$ and $X_{0.25} = M_X - \frac{1}{2}S_X(1 - w_X)$. Since when Φ is a diffeomorphism, we also have $[\Phi(X)]_\alpha = \Phi(X_\alpha)$, hence from (1):

$$\begin{aligned} S_{\Delta T} &= \Delta T_{0.75} - \Delta T_{0.25} = \frac{\Delta T_0}{(1-f_{0.75})} - \frac{\Delta T_0}{(1-f_{0.25})} = \frac{\Delta T_0}{(1-f_{0.75})(1-f_{0.25})} S_f \\ &= \frac{\Delta T_0}{(1-M_f)^2} S_f \cdot \left\{ 1 - \frac{w_f}{1-M_f} S_f - \frac{1-w_f^2}{4(1-M_f)^2} S_f^2 \right\}^{-1} \end{aligned}$$

3 – Second order term in uncertainty propagation equation

Assuming $Y = \phi(X)$, we analyse the way the approximation of the relationship between both spread measures S_Y and S_X is modified when a second order term is introduced in the Taylor development of ϕ about M_X :

$$Y \simeq \phi(M_X) + \phi'(M_X)(X - M_X) + \frac{1}{2}\phi''(M_X)(X - M_X)^2 \quad (8)$$

272 When the chosen spread measure S is standard deviation, calculations can be performed
 273 explicitly:

$$274 \quad S_Y \simeq |\phi'(M_X)| \cdot S_X \cdot \left\{ 1 + \left[\frac{\phi''(M_X)}{\phi'(M_X)} w_X \right] S_X + \left[\frac{\phi''(M_X)^2}{4\phi'(M_X)^2} (k_X - 1) \right] S_X^2 \right\}^{\frac{1}{2}} \quad (9)$$

275 Equation (9) shows that non linear terms in the resulting relationship between S_Y and S_X
 276 depends on the shape of the distribution $p(x)$ through its skewness w_X (a dimensionless
 277 measure of assymetry) and kurtosis k_X (a dimensionless measure of peakedness), and on
 278 the shape of function ϕ through the curvature factor $\frac{\phi''(M_X)}{\phi'(M_X)}$ (the rate of increase of the
 279 slope in M_X). A remarkable consequence of equation (9) is that when X distribution is
 280 symmetric ($w_X = 0$) and since kurtosis always exceeds one (Jensen inequality) hence the
 281 dependency of S_Y to S_X is always over linear. Actually, sublinearity would require quite
 282 special conditions: a distribution $p(x)$ with low kurtosis and high skewness, simultaneously
 283 with a function ϕ characterized by strong curvature with sign opposite to skewness.

284 Applying equation (9) to model (1), it follows:

$$285 \quad S_{\Delta T} \simeq \frac{\Delta T_0}{(1-M_f)^2} S_f \cdot \left\{ 1 + \frac{2w_f}{1-M_f} S_f + \frac{k_f-1}{(1-M_f)^2} S_f^2 \right\}^{\frac{1}{2}} \quad (10)$$

286 **4 – Exact uncertainty propagation equations for standard deviation**

287 Since the domain of value of f in RB07 model is $] - \infty, 1]$, we assume single tailed
 288 distributions defined on this support to avoid a truncation and make mathematical
 289 developments more convenient. For several usual distributions, the relationship be-
 290 tween $S_{\Delta T}$ and S_f can thus be explicited. Assuming a log-normal distribution with pdf
 291 $\frac{1}{(1-f)\sigma\sqrt{2\pi}} \exp \left[-\frac{(\ln(1-f)-\mu)^2}{2\sigma^2} \right]$, mean $M_f = 1 - e^{\mu + \frac{\sigma^2}{2}}$ and variance $S_f^2 = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$ we

292 obtain $S_{\Delta T}^2 = \Delta T_0^2 \cdot e^{-2\mu + \sigma^2} (e^{\sigma^2} - 1)$. Recombining :

$$293 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \cdot \left\{ 1 + \left[\frac{S_f}{1-M_f} \right]^2 \right\} \quad (11)$$

294 Assuming a gamma distribution with pdf $(1-f)^{k-1} \frac{\exp(-(1-f)/\theta)}{\Gamma(k)\theta^k}$, mean $M_f = 1 - \theta k$ and
295 variance $S_f^2 = \theta^2 k$, we obtain $S_{\Delta T}^2 = \Delta T_0^2 \cdot [\theta^2(k-1)(k-2)]^{-1}$. Recombining :

$$296 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \cdot \left\{ 1 - \left[\frac{S_f}{1-M_f} \right]^2 \right\}^{-1} \cdot \left\{ 1 + \left[\frac{S_f}{1-M_f} \right]^2 \right\}^{-\frac{1}{2}} \quad (12)$$

297 Assuming a beta distribution with pdf $\frac{\Gamma(2k)}{\theta\Gamma(k)^2} (1 - \frac{1-f}{\theta})^{k-1} (\frac{1-f}{\theta})^{k-1}$ on $[1 - \theta, 1]$, mean
298 $M_f = 1 - \frac{\theta}{2}$ and variance $S_f^2 = \theta^2[8k + 4]^{-1}$, we obtain $S_{\Delta T}^2 = \Delta T_0^2 \cdot [k(2k-1)] \cdot [\theta^2(k -$
299 $1)^2(k-2)]^{-1}$. Recombining :

$$300 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \cdot \left\{ 1 - \left[\frac{S_f}{1-M_f} \right]^2 \right\}^{\frac{1}{2}} \left\{ 1 - 2 \left[\frac{S_f}{1-M_f} \right]^2 \right\}^{\frac{1}{2}} \left\{ 1 - 3 \left[\frac{S_f}{1-M_f} \right]^2 \right\}^{-1} \left\{ 1 - 5 \left[\frac{S_f}{1-M_f} \right]^2 \right\}^{-\frac{1}{2}} \quad (13)$$

301 5 – Dependency between spread and probability weight of an interval

302 Assume X_1 is a random real variable with pdf $p_1(x)$, cdf $P_1(x)$, center M_1 and spread
303 $S_1 > 0$. Let $[a, b]$ be a fixed interval near but above the center (eg $M_1 < a$). For $\lambda > 0$,
304 we introduce $X_\lambda = \lambda(X_1 - M_1) + M_1$, which has pdf $\frac{1}{\lambda} p(\frac{x-M_1}{\lambda} + M_1)$, cdf $P(\frac{x-M_1}{\lambda} + M_1)$,
305 center M_1 and spread λS_1 . To analyse the dependency between the probability of a real
306 variable to fall in $[a, b]$ and the spread of its underlying distribution, we study $F(\lambda; a, b) =$
307 $\mathbb{P}(X_\lambda \in [a, b])$. F can be expressed using the cdf of X_λ :

$$308 \quad \begin{aligned} F(\lambda; a, b) &= P(\frac{b-M_1}{\lambda} + M_1) - P(\frac{a-M_1}{\lambda} + M_1) \\ F(0; a, b) &= P(-\infty) - P(-\infty) = 0 \quad \text{since } M_1 < a < b \\ F(+\infty; a, b) &= P(M_1) - P(M_1) = 0 \end{aligned} \quad (14)$$

309 Since $F(0; a, b) = F(+\infty; a, b) = 0$, and $F \geq 0$, then F reaches a maximum, and it
310 has the general pattern mentioned in the text. It is also straightforward to obtain that
311 $F(\lambda; a, b) \sim \frac{(b-a)p_1(M_1)}{\lambda^2}$ for large λ .

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References

- IPCC (2007), *Climate Change 2007: The Physical Basis*. Cambridge University Press.
- Barlow, R. J. (1989), *A Guide to the Use of Statistical Methods in the Physical Sciences*. John Wiley and Sons.
- Bony, S. et al (2006), How well do we understand and evaluate climate feedback processes? *J. Climate*. 19:34453482.
- Bony, S., Webb, M., Stevens, B., Bretherton C., Klein, S., and Tselioudis, G. (2008) CFMIP-GCSS Plans for Advancing Assessments of Cloud-Climate Feedbacks *Gewex News*.
- Dufresne, J. L., and Bony, S. (2008), An assessment of the primary sources of spread of global warming estimates from coupled atmosphere-ocean models *J. Climate*.
- James, F., Eadie, W. T., (2006), *Statistical Methods in Experimental Physics*. World Scientific.

- 333 Lu, J. and Cai, M. (2008), A new framework for isolating individual feedback processes
334 in coupled general circulation climate models. Part I: formulation, *Climate Dynamics*.
335 DOI 10.1007/s00382-008-0425-3
- 336 Meehl, G. A., C. Covey, T. Delworth, B. McAvaney, J. F. B. Mitchell, M. Latif, R. J.
337 Stouffer, and K. E. Taylor, 2007: The WCRP CMIP3 multi-model dataset: A new era
338 in climate change research, *Bull. Am. Meteorol. Soc.*, **8**, 1383-1394.
- 339 Randall, D.A., et al., 2007^a: Climate models and their evaluation. *Climate Change 2007:*
340 *The Scientific Basis. Contribution of Working Group I to the Fourth Assessment Report*
341 *of the Intergovernmental Panel on Climate Change*, S. Solomon, D. Qin, M. Manning,
342 Z. Chen, M. Marquis, K. B. Averyt, M. Tignor, and H. L. Miller, Eds., Cambridge
343 University Press, Cambridge, United Kingdom and New York, NY, USA, chap. 8.
- 344 Reinard, J. C. (2006), *Communication Research Statistics*. SAGE.
- 345 Roe, G. H., and Baker, M. B. (2007), Why Is Climate Sensitivity So Unpredictable?,
346 *Science*. Vol. 318. no. 5850, 629 - 632, DOI: 10.1126.
- 347 Soden, B. J., and Held, I. M. (2006), An Assessment of Climate Feedbacks in Coupled
348 Ocean-Atmosphere Models, *J. Climate*. 19, 3354.
- 349 Van der Vaart, A. W. (2000), *Asymptotic Statistics*. Cambridge University Press.

350 **Figure 1** – In all charts, f is truncated gaussian $\mathcal{N}(M_f, \sigma_f)$ as in RB07. Upper left panel (a):
 351 pdf of ΔT with $M_f = 0.65$ and $\sigma_f = 0.20, 0.15, 0.10$. Arrows represent the decreasing sensitivity
 352 spread $S_{\Delta T}$ obtained for decreasing values of σ_f . Upper right panel (b): climate sensitivity
 353 spread $S_{\Delta T}$ as a function of feedback spread S_f , for $M_f = 0.60, 0.65, 0.70$. Feedback spread S_f is
 354 measured by standard deviation ($\simeq \sigma_f$) but climate sensitivity spread $S_{\Delta T}$ is measured by IQR
 355 (see appendix 1 for explanation). Lower left panel (c): cdf of ΔT . Arrows represent the stable
 356 probability $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$ obtained for decreasing values of $\sigma_f = 0.20, 0.15, 0.10$. Lower
 357 right panel (d): probability $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$ as a function of feedback spread S_f , spread
 358 measured with IQR.

359 **Figure 2** – X is centered gaussian with standard deviation S_X . Right panel: probability for
 360 X to exceed respectively 1 and 3, as functions of S_X . Left panel: probability for X to fall within
 361 interval $[1, 3]$ as a function of S_X .

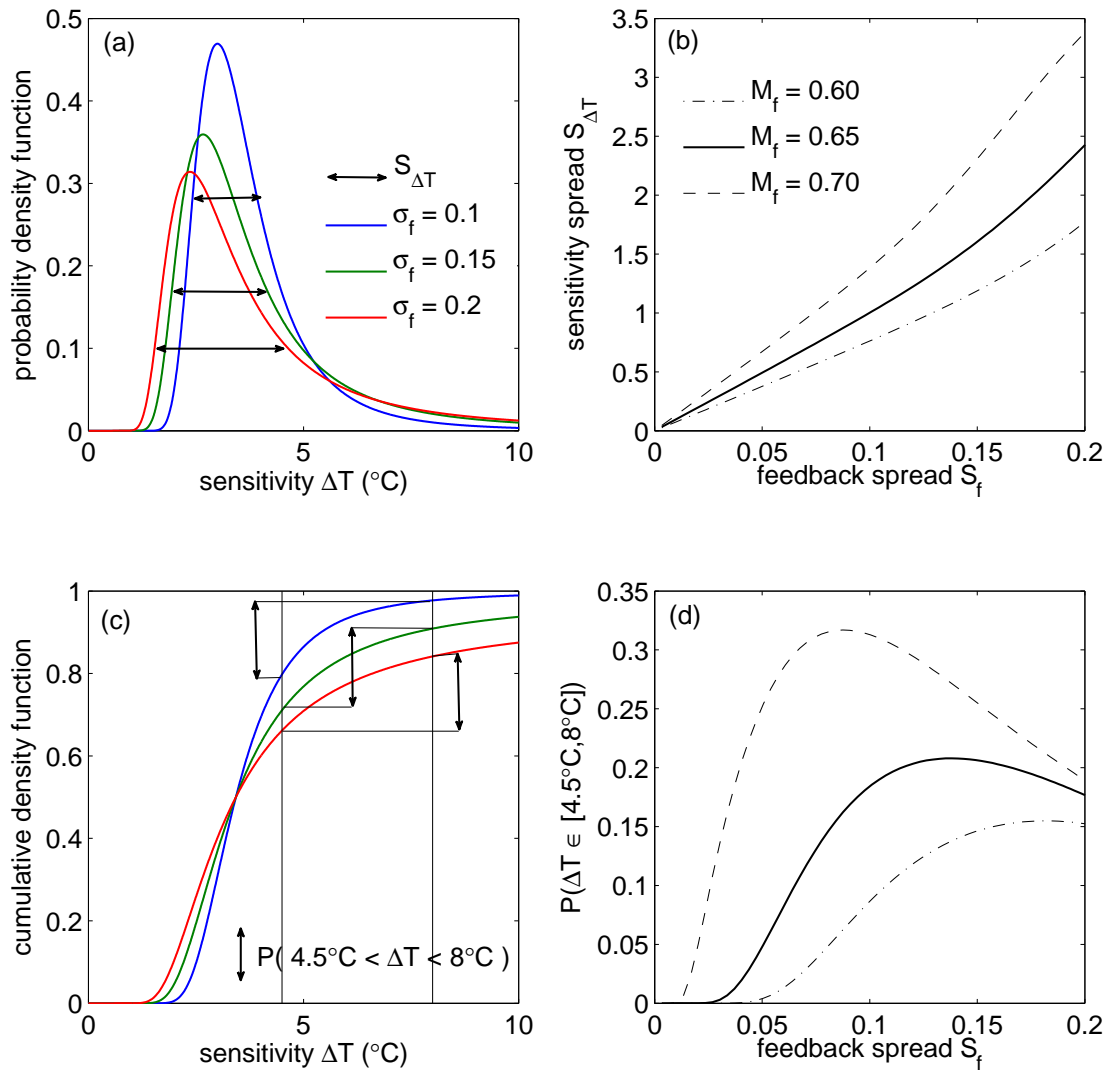


Figure 1. In all charts, f is truncated gaussian $\mathcal{N}(M_f, \sigma_f)$ as in RB07. Upper left panel (a): pdf of ΔT with $M_f = 0.65$ and $\sigma_f = 0.20, 0.15, 0.10$. Arrows represent the decreasing sensitivity spread $S_{\Delta T}$ obtained for decreasing values of σ_f . Upper right panel (b): climate sensitivity spread $S_{\Delta T}$ as a function of feedback spread S_f , for $M_f = 0.60, 0.65, 0.70$. Feedback spread S_f is measured by standard deviation ($\simeq \sigma_f$) but climate sensitivity spread $S_{\Delta T}$ is measured by IQR (see appendix 1 for explanation). Lower left panel (c): cdf of ΔT . Arrows represent the stable probability $\mathbb{P}(\Delta T \in [4.5^\circ\text{C}, 8^\circ\text{C}])$ obtained for decreasing values of $\sigma_f = 0.20, 0.15, 0.10$. Lower right panel (d): probability $\mathbb{P}(\Delta T \in [4.5^\circ\text{C}, 8^\circ\text{C}])$ as a function of feedback spread S_f , spread measured with IQR.

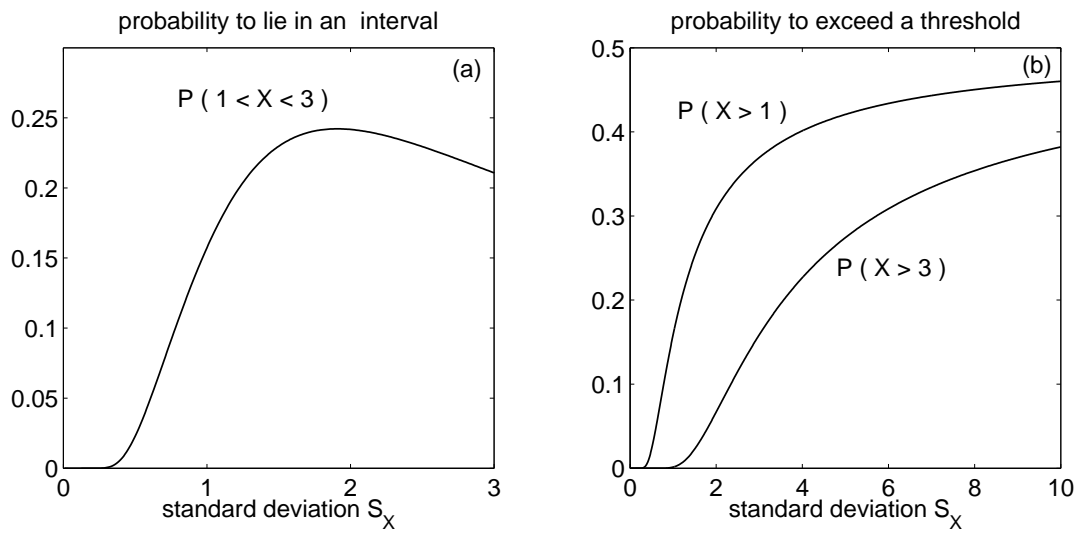


Figure 2. X is centered gaussian with standard deviation S_X . Right panel: probability for X to exceed respectively 1 and 3, as functions of S_X . Left panel: probability for X to fall within interval $[1, 3]$ as a function of S_X .