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# <sup>1</sup> Why climate sensitivity may not be so unpredictable?

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7 Different explanations have been proposed as to why the range of climate  
8 sensitivity predicted by GCMs have not lessened substantially in the last decades,  
9 and subsequently if it can be reduced. One such study (*Why is climate sen-*  
10 *sitivity so unpredictable?*, Roe and Baker, 2007 [①]) addressed these questions  
11 using rather simple theoretical considerations and reached the conclusion that  
12 reducing uncertainties on climate feedbacks and underlying climate processes  
13 will not yield a large reduction in the envelope of climate sensitivity. In this  
14 letter, we revisit the premises of this conclusion. We show that it results from  
15 a mathematical artefact caused by a peculiar definition of uncertainty used  
16 by these authors. Applying standard concepts and definitions of descriptive  
17 statistics to the exact same framework of analysis as Roe and Baker, we show  
18 that within this simple framework, reducing inter-model spread on feedbacks  
19 does in fact induce a reduction of uncertainty on climate sensitivity, almost  
20 proportionally. Therefore, following Roe and Baker assumptions, climate sen-  
21 sitivity is actually not so unpredictable.

## 1. Introduction

22     Uncertainties in projections of future climate change described in the last Assessment  
23     Report of the IPCC (IPCC, 2007 [2007]) are high, as illustrated by the broad range of  
24     climate sensitivity – defined as the global mean temperature increase for a doubling  
25     of CO<sub>2</sub> – simulated by general circulation models (GCMs). Attempts to explain this  
26     fact have focused mainly on uncertainties in our understanding of the individual physical  
27     feedback processes (especially associated to clouds), difficulties to represent them faithfully  
28     in GCMs, nonlinearity of some processes and complex interactions among them giving rise  
29     to a chaotic behaviour of the climate system (Randall et al. [2007<sup>a</sup>]). A review of these  
30     explanations can be found in Bony et al., 2006 [2006]. Nevertheless, in this letter, we leave  
31     aside all these considerations to focus our interest solely on the explanation proposed by  
32     Roe and Baker, 2007 [2007] (RB07) which somewhat differ from the above-mentioned. This  
33     study uses the framework of feedback analysis, which has often been used to describe the  
34     relationship between physical processes involved in global warming and climate sensitivity  
35     (see for instance Lu and Cai, 2008 [2008], Dufresne and Bony, 2008 [2008], Soden and Held,  
36     2006 [2006]). The feedback analysis framework assumes a linear approximation of radiative  
37     feedbacks, resulting in a simple relationship between a global feedback gain  $f$  and climate  
38     sensitivity  $\Delta T$ . In this classic setting, the main originality of RB07 approach consists  
39     in analyzing explicitly the way uncertainties on  $f$ , due to a limited understanding of  
40     their underlying physical processes, propagates into uncertainties on  $\Delta T$ : assuming  $f$   
41     is a random variable with mean  $\bar{f}$  and standard deviation  $\sigma_f$ , RB07 uses this simple  
42     probabilistic model to highlight several fundamental properties of uncertainty propagation

43 from feedbacks to climate sensitivity. The most prominent conclusion of this analysis is  
44 that reducing uncertainties on  $f$  does not yield a large reduction in the uncertainty of  
45  $\Delta T$ , and thus that improvements in the understanding of physical processes will not yield  
46 large reductions in the envelope of future climate projections. This conclusion, if true,  
47 would clearly have crucial implications for climate research and policy.

48 In section 2, we revisit the premises of RB07 conclusion. We highlight that it is the  
49 result of a peculiar way of defining uncertainty. Moreover, we show in section 5 that  
50 this conclusion is a pure mathematical artefact with no connection whatsoever to climate.  
51 Since the basic question of uncertainty definition appears to be at stake, section 3 briefly  
52 recalls widely used definitions and elementary results on uncertainty and its propagation  
53 as they can be found in Descriptive Statistics textbooks. In section 4, we apply these  
54 standard concepts and definitions to the exact same framework of analysis as RB07. We  
55 show that within this simple framework, reducing inter-model spread on feedbacks does  
56 in fact induce a reduction of uncertainty on climate sensitivity, almost proportionally.  
57 Finally, section 6 concludes.

## 2. Overview of RB07 approach

58 RB07 uses the feedback analysis framework. Denoting  $\Delta T_0$  the Planck temperature  
59 response to the radiative perturbation and  $f$  the feedback gain (referred to as feedback  
60 factor in RB07), they obtain:

$$61 \quad \Delta T = \frac{\Delta T_0}{1 - f} \quad (1)$$

62 RB07 then assumes uncertainty on Planck response to be neglectible so that the entire  
63 spread on  $\Delta T$  results from the uncertainty on the global feedback gain  $f$ . To model

64 this uncertainty, RB07 assumes that  $f$  follows a gaussian distribution with mean  $\bar{f}$ , stan-  
65 dard deviation  $\sigma_f$  and implicit truncation for  $f > 1$  (implications of this truncation are  
66 discussed in appendix 1). Then, they derive an exact mathematical expression of the  
67 distribution of  $\Delta T$  through equation (1). This simple probabilistic climatic model is then  
68 used by RB07 to analyze the way uncertainties on  $f$ , due to a limited understanding of  
69 their underlying physical processes, propagates into uncertainties on  $\Delta T$ . Their analysis  
70 highlights two fundamental properties of uncertainty propagation:

- 71 • Amplification: The term in  $\frac{1}{1-f}$  in equation (1) amplifies uncertainty on feedbacks, all  
72 the more intensely as  $\bar{f}$  is close to (though lower than) one. Small uncertainties on feed-  
73 backs are thus converted in large uncertainties on the rise of temperature.
- 74 • Insensitivity: Quoting RB07, “*reducing uncertainty on  $f$  has little effect in reducing*  
75 *uncertainty on  $\Delta T$ ”, also stated as “*the breadth of the distribution of  $\Delta T$  is relatively*  
76 *insensitive to decreases in  $\sigma_f$ .*”*

77 We fully subscribe to the first property and elaborate further on it in section 4. However,  
78 we are puzzled by the second property, that is, the claimed insensitivity of uncertainty  
79 on  $\Delta T$  to uncertainty on feedbacks. The reason why one may find this second assertion a  
80 priori puzzling, is that it intuitively seems to be at a contradiction with the first property  
81 highlighted. Indeed, if small uncertainties on  $f$  are amplified into large uncertainties  
82 on  $\Delta T$ , it suggests that a strong dependency exists between both uncertainties, rather  
83 than no or little dependency. We therefore dig into the details of RB07 argumentation  
84 regarding this assertion. To get to that conclusion, it appears that RB07 actually focus  
85 on the probability  $\mathbb{P}(\Delta T \in [4.5^\circ\text{C}, 8^\circ\text{C}])$  that  $\Delta T$  lies in the interval  $[4.5^\circ\text{C}, 8^\circ\text{C}]$  in

86 response to a sustained doubling of the  $CO_2$  concentration. This interval is defined as  
87 immediately above the range obtained with the CMIP3/AR4 GCMs (IPCC, 2007 [a]).  
88 They study graphically how this probability fluctuates with the level of uncertainty on  
89 feedbacks, by plotting for several values of  $\sigma_f$  the obtained cumulative distribution of  
90  $\Delta T$ . Doing this graphical analysis, they observe that the probability of large temperature  
91 increase  $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$  is insensitive to  $\sigma_f$ . This observation is easily verifiable:  
92 we replicated RB07 cumulative distribution chart in figure 1c, and we computed several  
93 values of  $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$  for  $\bar{f} = 0.65$  and  $\sigma_f$  ranging from 0.10 to 0.20, finding  
94 it to fluctuate between 0.18 and 0.20. Therefore, in agreement with RB07, it is fair to  
95 say that the probability of large temperature increase (i.e.  $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$ ) is quite  
96 insensitive to  $\sigma_f$  in this domain. However, concluding from this observation that “*the*  
97 *breadth of the distribution of  $\Delta T$  is relatively insensitive to decreases in  $\sigma_f$ ” and that  
98 “*reducing uncertainty on  $f$  has little effect in reducing uncertainty on  $\Delta T$ ” implicitly  
99 assumes two very different definitions of uncertainty: while on the side of feedback the  
100 uncertainty is measured by standard deviation  $\sigma_f$ , on the side of sensitivity the probability  
101  $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$  is used as a metric of uncertainty. As will be developed in section 3,  
102 standard deviation is a standard, consensual uncertainty metric but the probability to lie  
103 in a fixed interval is not. While under this peculiar double definition of uncertainty RB07  
104 conclusion holds, it is fair to ask whether it would still hold with a different uncertainty  
105 metric for  $\Delta T$ ; second, whether the probability to lie in a fixed interval can be considered  
106 an acceptable measure of distribution breadth; and third, what are the implications of**

107 using such an asymmetric definition of uncertainty. The following sections attempt to  
108 answer these questions.

### 3. Standard measurement and propagation of distribution spread

109 To investigate the first question, which relates to the basic issue of uncertainty definition,  
110 we briefly recall a few standard definitions and concepts, as they can be found almost  
111 identically in most Descriptive Statistics textbooks. For details, the reader can refer for  
112 instance to Barlow, 1989 [ @ ], Van der Vaart, 2000 [ @ ], Reinard, 2006 [ @ ], James and  
113 Eadie, 2006 [ @ ] to mention but a few such textbooks.

114 Descriptive Statistics primary purpose is to provide metrics summarizing a sample of  
115 observations and similarly, in probabilistic terms, metrics summarizing the probability  
116 density function (pdf) underlying them. Technically, the correspondance between both  
117 is simply that a sample summary is an *estimator* (a function of the data) which esti-  
118 mates a distribution summary *estimand* (a parameter). In the present case, we study  
119 continuous random variables thus we are rather concerned about pdf metrics than sam-  
120 ple metrics, even though these pdfs actually aim at fitting a sample of observations, in  
121 that case CMIP3/AR4 GCMs simulations (Meehl et al. [2007]). Descriptive Statistics  
122 usually group metrics under three categories: location, scale and shape parameters. The  
123 so-called location parameters are meant to identify the center of a distribution. Most  
124 common location measures are mean, mode and median. The so-called scale parameters,  
125 also referred to as dispersion, variability, variation, scatter or spread measures, describe  
126 how far from the above-defined center possible values covered by the distribution tend  
127 to be. This second group of metrics is the one we are interested in for our discussion,



128 as it is concerned with the measurement of distribution spread. Most common measures  
 129 are standard deviation, interquartile range (IQR), range or median absolute deviation  
 130 (MAD), more rarely full width at half maximum (FWHM). Variance and coefficient of  
 131 dispersion should also be mentioned though they are not expressed in the same unit as the  
 132 variable. Above mentioned references give complete mathematical expressions, properties,  
 133 strengths and limitations of these. We underline a property of particular interest to our  
 134 discussion: above mentioned measures of spread are invariant in location and linear in  
 135 scale. In other words, denoting  $S$  any particular measure of spread amongst those listed  
 136 above,  $X$  a random variable and  $Y = aX + b$  then:

$$137 \quad S_Y = |a| \cdot S_X \quad (2)$$

138 Further, in the general case of a dependency of the type  $Y = \phi(X)$ :

$$139 \quad S_Y \simeq |\phi'(M_X)| \cdot S_X \quad (3)$$

140 where  $\phi'$  represents the first derivative of  $\phi$  and  $M$  is a location parameter. This linear  
 141 approximation is commonly used to combine errors on measurements, though generally  
 142 in its multivariate formulation, and is thus sometimes referred to as the error propagation  
 143 framework. It may also be used to study the way uncertainty on some input variable(s)  
 144 propagates into uncertainty on an output obtained from a determinist function, as in  
 145 section 4.

#### 4. Standard uncertainty propagation in RB07 feedback model

146 We now analyse the dependency between uncertainty on feedbacks and uncertainty on  
 147 climate sensitivity in RB07 model. Denoting  $S_{\Delta T}$  a measure of climate sensitivity spread,

148  $S_f$  a measure of feedback spread and  $M_f$  a measure of feedback location, the uncertainty  
 149 propagation recalled in equation (3) can be applied straightforward to equation (1), lead-  
 150 ing to:

$$151 \quad S_{\Delta T} \simeq \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \quad (4)$$

152 Note that Equation (4) holds for any choice of pdf for feedback factor  $f$  and thus applies  
 153 more generally than in the particular case of a truncated gaussian pdf chosen by RB07.  
 154 Equation (4) also provides a simple relationship between  $S_{\Delta T}$ ,  $S_f$  and  $M_f$  which translates  
 155 into the following two properties:

156 • Amplification: In agreement with RB07 first above recalled result, for a fixed level  
 157 of feedback uncertainty  $S_f$ , the level of sensitivity uncertainty  $S_{\Delta T}$  is amplified when  
 158 feedback  $M_f$  approaches one. Since estimates of feedback parameters in CMIP3/AR4  
 159 models ( Soden and Held, 2006 [ @ ], Randall et al. [2007<sup>a</sup>]) suggest  $M_f$  is close enough  
 160 to one ( $M_f \simeq 0.65$ ) and hence yields substantial amplification, it seems that “*the climate*  
 161 *system is operating in a regime in which small uncertainties in feedbacks are amplified in*  
 162 *the resulting climate sensitivity uncertainty*”, to quote RB07.

163 • Proportionality: In disagreement with RB07 second above recalled result, for a fixed level  
 164 of average feedback  $M_f$ , the level of climate sensitivity uncertainty  $S_{\Delta T}$  is proportional  
 165 to the level of feedback uncertainty  $S_f$  ( $S_{\Delta T} \simeq 9.8 S_f$  for  $M_f \simeq 0.65$ ). This simple  
 166 relationship between both uncertainties is intuitive. Indeed, when  $S_f = 0$ , feedbacks are  
 167 determinists and  $\Delta T$  also is, considering no other source of uncertainty in the climate  
 168 system, hence  $S_{\Delta T} = 0$ . As values of  $f$  get increasingly scattered, resulting values of  
 169 climate sensitivity also get more scattered proportionally (figure 1a and 1b).

170 This proportionality has general validity in the sense that it holds for any above-recalled  
 171 standard spread measure and for any distribution of  $f$ . However, it is an approximation for  
 172 small values of  $S_f$ . We therefore find it relevant to investigate how this linear dependency  
 173 is affected when  $S_f$  increases. To perform this analysis, we exhibit more precise results  
 174 on uncertainty propagation in RB07 model. First, when spread is measured by IQR, an  
 175 exact relationship holds for any value of  $S_f$  and any distribution of  $f$  (appendix 2):

$$176 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} S_f \cdot \left\{ 1 - \frac{w_f}{1-M_f} S_f - \frac{1-w_f^2}{4(1-M_f)^2} S_f^2 \right\}^{-1} \quad (5)$$

177 where  $w_f$  measures the asymmetry of  $f$  distribution. Hence, when  $S \equiv \text{IQR}$ , the dependency  
 178 between  $S_{\Delta T}$  and  $S_f$  is always overlinear when  $w_f \geq 0$ , eg when  $f$  has a symmetric or right  
 179 skewed distribution. When it is left skewed, the dependency is sublinear for small values  
 180 of  $S_f$  but eventually becomes overlinear when  $S_f$  is large enough. Second, when spread  
 181 is measured by standard deviation, a second order Taylor expansion of equation (1) leads  
 182 to a more accurate approximation (appendix 3):

$$183 \quad S_{\Delta T} \simeq \frac{\Delta T_0}{(1-M_f)^2} S_f \cdot \left\{ 1 + \frac{2w_f}{1-M_f} S_f + \frac{k_f-1}{(1-M_f)^2} S_f^2 \right\}^{\frac{1}{2}} \quad (6)$$

184 Again, overlinearity prevails when  $w_f \geq 0$  or  $S_f$  large enough, which is connected to the  
 185 convexity of the dependency between  $\Delta T$  and  $f$ . Third, when  $S$  is standard deviation  
 186 and  $f$  distribution is log-normal, an exact formula holds for any  $S_f$ :

$$187 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \cdot \left\{ 1 + \left[ \frac{S_f}{1-M_f} \right]^2 \right\} \quad (7)$$

188 and is again overlinear. Finally, overlinear relationships can also be derived when the  
 189 distribution of  $f$  is assumed to be gamma or beta (equations (12) and (14) in appendix  
 190 4).

191 To summarize the above discussion, its main outcome is rather intuitive and has actually  
 192 few to do with climate: if the spread of feedback factor values decreases, the resulting  
 193 spread of climate sensitivity values also decreases. Secondly, the dependency is as follows:  
 194 it is linear for small feedback spreads and tends to get overlinear for larger values. Last,  
 195 the proportionality coefficient in the dependency sharply increases as feedback intensifies.

## 5. Properties of the probability to lie in a fixed interval

196 We now focus on whether the probability to lie in a fixed interval can be considered  
 197 an acceptable measure of distribution breadth, as implicitly done by RB07 to reach their  
 198 main conclusion. We approach this question very generally: let  $X$  be a continuous random  
 199 variable with location  $M_X$ , spread  $S_X$  and pdf  $p_X$ . Let  $[a, b]$  be a fixed interval near but  
 200 above the center ( $M_X < a < b$ ). Then, when  $S_X \rightarrow 0$  the variable becomes determinist  
 201 ( $X = M_X$ ) and it results that  $\mathbb{P}(X \in [a, b])$  equals to zero since  $M_X \notin [a, b]$ . When  
 202  $S_X \rightarrow +\infty$  the distribution covers such a wide range of values that the probability to  
 203 exceed any given threshold slowly increases towards 0.5 (figure 2b). In particular  $\mathbb{P}(X >$   
 204  $a) \rightarrow 0.5$  and  $\mathbb{P}(X > b) \rightarrow 0.5$ , hence  $\mathbb{P}(X \in [a, b]) = \mathbb{P}(X > a) - \mathbb{P}(X > b) \rightarrow 0$   
 205 (appendix 5). Hence the dependency between  $\mathbb{P}(X \in [a, b])$  and  $S_X$  is characterized by  
 206 a non monotonous function that increases, flattens and then decreases to zero (figure  
 207 2a). In light of this non monotonous dependency, it is difficult to hold  $\mathbb{P}(X \in [a, b])$   
 208 as a valid measure for the width of  $X$  distribution. Further, the observed insensitivity  
 209 of  $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$  to feedback spread  $S_f$ , which lead authors to their conclusion,  
 210 happens to proceed directly from the above described dependency: this flattening of the

211 dependency is a pure mathematical artefact which systematically manifests under these  
212 definitions, and has nothing to do with climate.

213 Finally, if one still wants to stick to this peculiar, asymmetric definition of uncertainty, it  
214 has to be noted that in RB07 model, even though the dependency is flat in the domain  
215  $S_f \in [0.1, 0.2]$ , the dependency is strong for  $S_f < 0.1$  when  $M_f \approx 0.65$  and subsequently  
216 leads to a steep decrease of  $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$  to zero (figure 1d). In fact, since  
217 feedback current estimates suggest  $S_f \simeq 0.09$  and  $M_f \approx 0.65$  (Soden and Held, 2006 [②],  
218 Randall et al. [2007<sup>a</sup>]), the domain of strong dependency may actually already be reached  
219 to date.

## 6. Conclusion

220 Developments in section 5 suggest that, while the probability  $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$   
221 may be of interest practically, this metric is irrelevant to describe “*the breadth of the dis-*  
222 *tribution of climate sensitivity*” which was RB07 explicit intent. To address this question,  
223 any measure of distribution spread chosen amongst those classically used in Descriptive  
224 Statistics and recalled in section 3, appear to us more appropriate. With such measures of  
225 spread, we showed in section 4 that in RB07 framework, when the spread of feedback pa-  
226 rameter  $S_f$  decreases, the resulting spread of climate sensitivity  $S_{\Delta T}$  values also decreases.  
227 Further, we also highlighted that in this framework, the decrease is approximately linear  
228 for  $S_f$  small and tends to be overlinear (i.e. to be steeper) for larger values of  $S_f$  owing  
229 to the convexity of the dependency between  $\Delta T$  and  $f$ .

230 Other than the definition issue discussed here, the relevance of RB07 simplified model to  
231 describe the dependency between climate sensitivity and feedbacks may also be discussed

232 but this was beyond the scope of this article. In any case, if one holds this model to be  
 233 accurate, a decrease of the spread on feedback will lead to a decrease of the uncertainty  
 234 on climate sensitivity and a narrowing of the envelope of future climate projections. If  
 235 enough studies are undertaken to better understand and assess the physical processes  
 236 involved in the different feedbacks, neither are doomed to remain at their current level.

## Appendix

### 1 – Implications of the truncation

237  
 238 Since the linear feedback model of RB07 implicitly assumes  $f \leq 1$ , the gaussian distribu-  
 239 tion  $\mathcal{N}(\bar{f}, \sigma_f)$  proposed by RB07 is implicitly truncated for  $f > 1$  – otherwise equation  
 240 (1) would produce negative values of  $\Delta T$ . This truncation has several implications. First,  
 241  $\sigma_f$  (resp.  $\bar{f}$ ) does not exactly match standard deviation (resp. mean) of the truncated  
 242 distribution. For instance, when  $(\bar{f}, \sigma_f) = (0.75, 0.25)$  the standard deviation of  $f$  equals  
 243 0.18 and its mean equals 0.67. Second, it introduces some negative skewness in the dis-  
 244 tribution of  $f$  ( $-0.39$  in the same example) which becomes more and more asymmetric as  
 245  $\sigma_f$  and  $\bar{f}$  increases. Finally, since the truncated gaussian pdf is finite and non zero in the  
 246 vicinity of  $f = 1$ , the obtained pdf of climate sensitivity behave as a Pareto distribution  
 247 in  $\mathcal{O}(\Delta T^{-2})$  for high values, and hence does not have a finite mean, nor a finite variance.  
 248 Hence, the truncated gaussian model of RB07 forbids the use of standard deviation as  
 249 a measure of climate sensitivity spread, which explains the use of IQR in figure 1. For  
 250 the purpose of RB07 which is to study climate sensitivity spread, assuming a parametric  
 251 distribution of  $f$  – such as log-normal, gamma or beta – which leads to finite mean and  
 252 deviation for sensitivity and exact mathematical expressions of the dependency between

the deviation of  $\Delta T$  and the deviation of  $f$  (appendix 3), would be in our view more convenient. However, the results on the dependency between  $S_{\Delta T}$  and  $S_f$  presented in section 4 are general and also hold under RB07 gaussian assumption. Therefore, RB07 truncated gaussian is in our view mathematically inconvenient, but it does not affect uncertainty propagation: for a gaussian distribution just as for any other, the spread dependency is approximately linear for small spreads and overlinear otherwise, as equation (4) and (5) demonstrate and as figure 1b illustrates.

## 2 – Exact uncertainty propagation equation for IQR

If  $X$  is a continuous random variable  $X$ , we denote  $X_\alpha$  its  $\alpha$ -quantile,  $S_X = X_{0.75} - X_{0.25}$  its interquantile range,  $M_X = X_{0.50}$  its median and  $w_X = \frac{X_{0.75} + X_{0.25} - 2X_{0.50}}{X_{0.75} - X_{0.25}}$  a dimensionless, quantile-based metric of asymetry. We thus have  $X_{0.75} = M_X + \frac{1}{2}S_X(1 + w_X)$  and  $X_{0.25} = M_X - \frac{1}{2}S_X(1 - w_X)$ . Since when  $\Phi$  is a diffeomorphism, we also have  $[\Phi(X)]_\alpha = \Phi(X_\alpha)$ , hence from (1):

$$\begin{aligned} S_{\Delta T} &= \Delta T_{0.75} - \Delta T_{0.25} = \frac{\Delta T_0}{(1-f_{0.75})} - \frac{\Delta T_0}{(1-f_{0.25})} = \frac{\Delta T_0}{(1-f_{0.75})(1-f_{0.25})} S_f \\ &= \frac{\Delta T_0}{(1-M_f)^2} S_f \cdot \left\{ 1 - \frac{w_f}{1-M_f} S_f - \frac{1-w_f^2}{4(1-M_f)^2} S_f^2 \right\}^{-1} \end{aligned}$$

## 3 – Second order term in uncertainty propagation equation

Assuming  $Y = \phi(X)$ , we analyse the way the approximation of the relationship between both spread measures  $S_Y$  and  $S_X$  is modified when a second order term is introduced in the Taylor development of  $\phi$  about  $M_X$ :

$$Y \simeq \phi(M_X) + \phi'(M_X)(X - M_X) + \frac{1}{2}\phi''(M_X)(X - M_X)^2 \quad (8)$$

272 When the chosen spread measure  $S$  is standard deviation, calculations can be performed  
 273 explicitly:

$$274 \quad S_Y \simeq | \phi'(M_X) | \cdot S_X \cdot \left\{ 1 + \left[ \frac{\phi''(M_X)}{\phi'(M_X)} w_X \right] S_X + \left[ \frac{\phi''(M_X)^2}{4\phi'(M_X)^2} (k_X - 1) \right] S_X^2 \right\}^{\frac{1}{2}} \quad (9)$$

275 Equation (9) shows that non linear terms in the resulting relationship between  $S_Y$  and  $S_X$   
 276 depends on the shape of the distribution  $p(x)$  through its skewness  $w_X$  (a dimensionless  
 277 measure of assymetry) and kurtosis  $k_X$  (a dimensionless measure of peakedness), and on  
 278 the shape of function  $\phi$  through the curvature factor  $\frac{\phi''(M_X)}{\phi'(M_X)}$  (the rate of increase of the  
 279 slope in  $M_X$ ). A remarkable consequence of equation (9) is that when  $X$  distribution is  
 280 symmetric ( $w_X = 0$ ) and since kurtosis always exceeds one (Jensen inequality) hence the  
 281 dependency of  $S_Y$  to  $S_X$  is always over linear. Actually, sublinearity would require quite  
 282 special conditions: a distribution  $p(x)$  with low kurtosis and high skewness, simultaneously  
 283 with a function  $\phi$  characterized by strong curvature with sign opposite to skewness.

284 Applying equation (9) to model (1), it follows:

$$285 \quad S_{\Delta T} \simeq \frac{\Delta T_0}{(1-M_f)^2} S_f \cdot \left\{ 1 + \frac{2w_f}{1-M_f} S_f + \frac{k_f-1}{(1-M_f)^2} S_f^2 \right\}^{\frac{1}{2}} \quad (10)$$

286 **4 – Exact uncertainty propagation equations for standard deviation**

287 Since the domain of value of  $f$  in RB07 model is  $] - \infty, 1]$ , we assume single tailed  
 288 distributions defined on this support to avoid a truncation and make mathematical  
 289 developments more convenient. For several usual distributions, the relationship be-  
 290 tween  $S_{\Delta T}$  and  $S_f$  can thus be explicited. Assuming a log-normal distribution with pdf  
 291  $\frac{1}{(1-f)\sigma\sqrt{2\pi}} \exp \left[ -\frac{(\ln(1-f)-\mu)^2}{2\sigma^2} \right]$ , mean  $M_f = 1 - e^{\mu+\frac{\sigma^2}{2}}$  and variance  $S_f^2 = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$  we



292 obtain  $S_{\Delta T}^2 = \Delta T_0^2 \cdot e^{-2\mu + \sigma^2} (e^{\sigma^2} - 1)$ . Recombining :

$$293 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \cdot \left\{ 1 + \left[ \frac{S_f}{1-M_f} \right]^2 \right\} \quad (11)$$

294 Assuming a gamma distribution with pdf  $(1-f)^{k-1} \frac{\exp(-(1-f)/\theta)}{\Gamma(k)\theta^k}$ , mean  $M_f = 1 - \theta k$  and

295 variance  $S_f^2 = \theta^2 k$ , we obtain  $S_{\Delta T}^2 = \Delta T_0^2 \cdot [\theta^2(k-1)(k-2)]^{-1}$ . Recombining :

$$296 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \cdot \left\{ 1 - \left[ \frac{S_f}{1-M_f} \right]^2 \right\}^{-1} \cdot \left\{ 1 + \left[ \frac{S_f}{1-M_f} \right]^2 \right\}^{-\frac{1}{2}} \quad (12)$$

297 Assuming a beta distribution with pdf  $\frac{\Gamma(2k)}{\theta\Gamma(k)^2} (1 - \frac{1-f}{\theta})^{k-1} (\frac{1-f}{\theta})^{k-1}$  on  $[1 - \theta, 1]$ , mean

298  $M_f = 1 - \frac{\theta}{2}$  and variance  $S_f^2 = \theta^2[8k + 4]^{-1}$ , we obtain  $S_{\Delta T}^2 = \Delta T_0^2 \cdot [k(2k-1)] \cdot [\theta^2(k -$

299  $1)^2(k-2)]^{-1}$ . Recombining :

$$300 \quad S_{\Delta T} = \frac{\Delta T_0}{(1-M_f)^2} \cdot S_f \cdot \left\{ 1 - \left[ \frac{S_f}{1-M_f} \right]^2 \right\}^{\frac{1}{2}} \left\{ 1 - 2 \left[ \frac{S_f}{1-M_f} \right]^2 \right\}^{\frac{1}{2}} \left\{ 1 - 3 \left[ \frac{S_f}{1-M_f} \right]^2 \right\}^{-1} \left\{ 1 - 5 \left[ \frac{S_f}{1-M_f} \right]^2 \right\}^{-\frac{1}{2}} \quad (13)$$

### 301 5 – Dependency between spread and probability weight of an interval

302 Assume  $X_1$  is a random real variable with pdf  $p_1(x)$ , cdf  $P_1(x)$ , center  $M_1$  and spread

303  $S_1 > 0$ . Let  $[a, b]$  be a fixed interval near but above the center (eg  $M_1 < a$ ). For  $\lambda > 0$ ,

304 we introduce  $X_\lambda = \lambda(X_1 - M_1) + M_1$ , which has pdf  $\frac{1}{\lambda} p(\frac{x-M_1}{\lambda} + M_1)$ , cdf  $P(\frac{x-M_1}{\lambda} + M_1)$ ,

305 center  $M_1$  and spread  $\lambda S_1$ . To analyse the dependency between the probability of a real

306 variable to fall in  $[a, b]$  and the spread of its underlying distribution, we study  $F(\lambda; a, b) =$

307  $\mathbb{P}(X_\lambda \in [a, b])$ .  $F$  can be expressed using the cdf of  $X_\lambda$ :

$$308 \quad \begin{aligned} F(\lambda; a, b) &= P(\frac{b-M_1}{\lambda} + M_1) - P(\frac{a-M_1}{\lambda} + M_1) \\ F(0; a, b) &= P(-\infty) - P(-\infty) = 0 \quad \text{since } M_1 < a < b \\ F(+\infty; a, b) &= P(M_1) - P(M_1) = 0 \end{aligned} \quad (14)$$

309 Since  $F(0; a, b) = F(+\infty; a, b) = 0$ , and  $F \geq 0$ , then  $F$  reaches a maximum, and it

310 has the general pattern mentioned in the text. It is also straightforward to obtain that

311  $F(\lambda; a, b) \sim \frac{(b-a)p_1(M_1)}{\lambda^2}$  for large  $\lambda$ .

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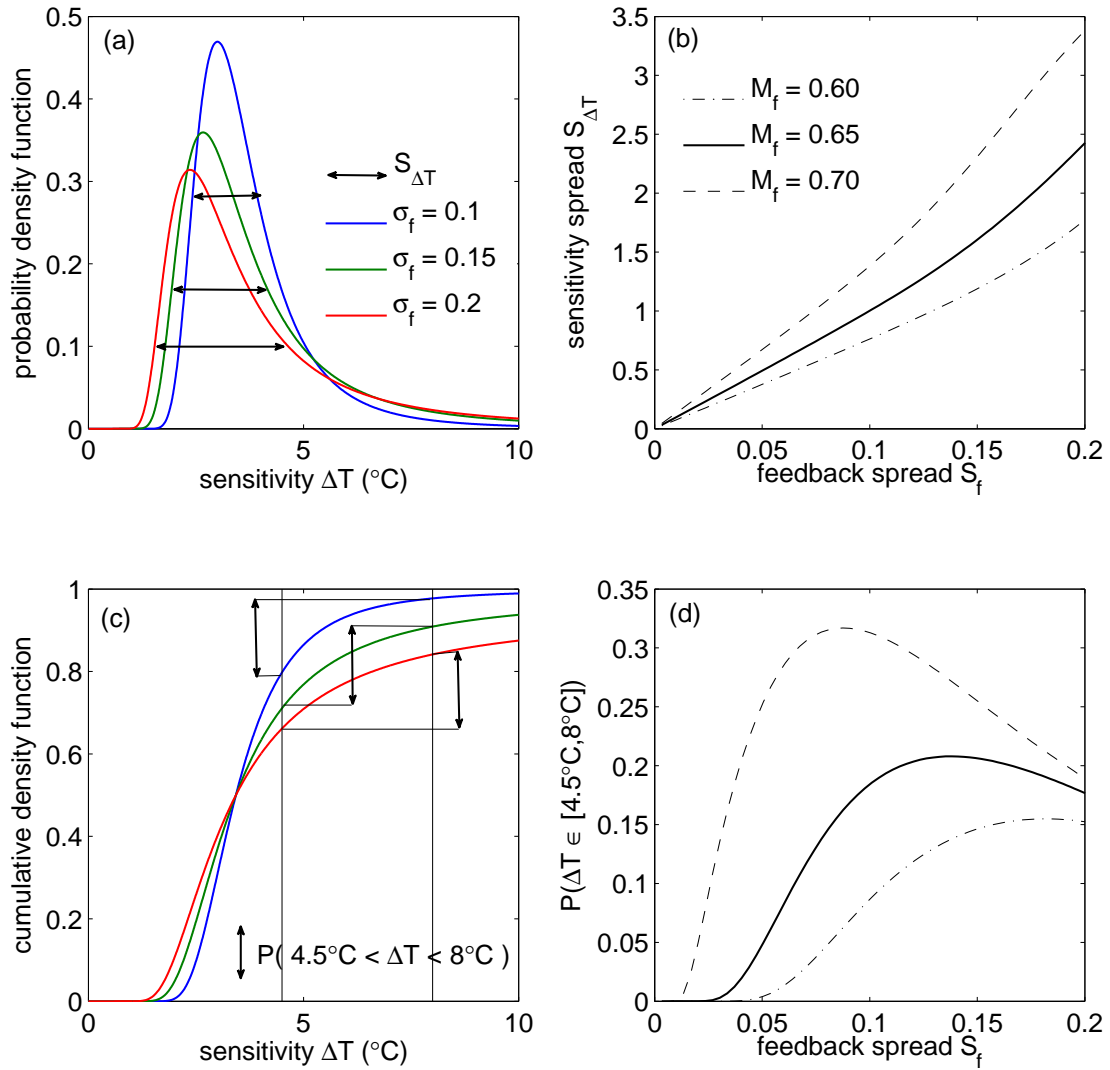
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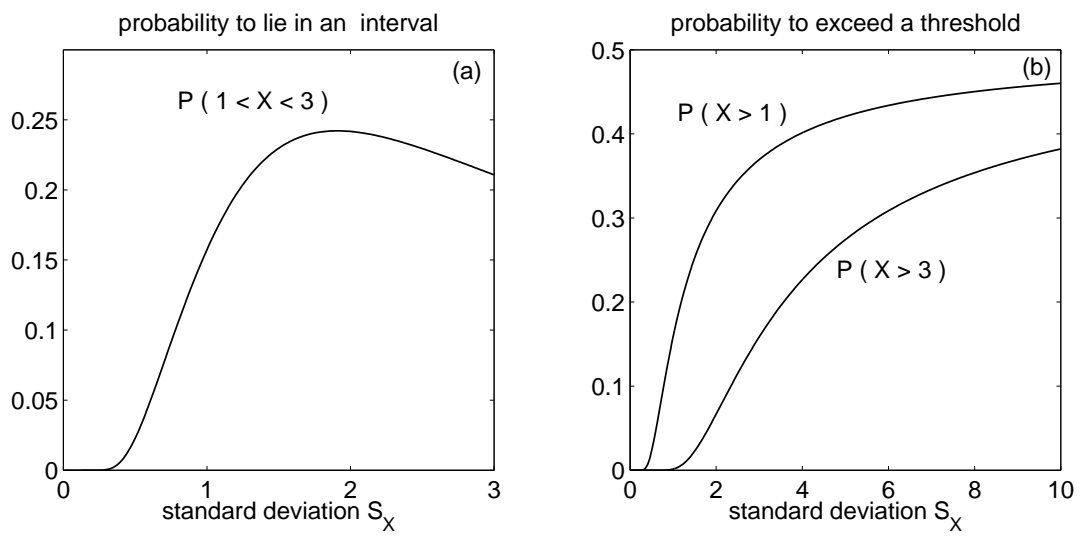
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350 **Figure 1** – In all charts,  $f$  is truncated gaussian  $\mathcal{N}(M_f, \sigma_f)$  as in RB07. Upper left panel (a):  
 351 pdf of  $\Delta T$  with  $M_f = 0.65$  and  $\sigma_f = 0.20, 0.15, 0.10$ . Arrows represent the decreasing sensitivity  
 352 spread  $S_{\Delta T}$  obtained for decreasing values of  $\sigma_f$ . Upper right panel (b): climate sensitivity  
 353 spread  $S_{\Delta T}$  as a function of feedback spread  $S_f$ , for  $M_f = 0.60, 0.65, 0.70$ . Feedback spread  $S_f$  is  
 354 measured by standard deviation ( $\simeq \sigma_f$ ) but climate sensitivity spread  $S_{\Delta T}$  is measured by IQR  
 355 (see appendix 1 for explanation). Lower left panel (c): cdf of  $\Delta T$ . Arrows represent the stable  
 356 probability  $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$  obtained for decreasing values of  $\sigma_f = 0.20, 0.15, 0.10$ . Lower  
 357 right panel (d): probability  $\mathbb{P}(\Delta T \in [4.5^\circ C, 8^\circ C])$  as a function of feedback spread  $S_f$ , spread  
 358 measured with IQR.

359 **Figure 2** –  $X$  is centered gaussian with standard deviation  $S_X$ . Right panel: probability for  
 360  $X$  to exceed respectively 1 and 3, as functions of  $S_X$ . Left panel: probability for  $X$  to fall within  
 361 interval  $[1, 3]$  as a function of  $S_X$ .



**Figure 1.** In all charts,  $f$  is truncated gaussian  $\mathcal{N}(M_f, \sigma_f)$  as in RB07. Upper left panel (a): pdf of  $\Delta T$  with  $M_f = 0.65$  and  $\sigma_f = 0.20, 0.15, 0.10$ . Arrows represent the decreasing sensitivity spread  $S_{\Delta T}$  obtained for decreasing values of  $\sigma_f$ . Upper right panel (b): climate sensitivity spread  $S_{\Delta T}$  as a function of feedback spread  $S_f$ , for  $M_f = 0.60, 0.65, 0.70$ . Feedback spread  $S_f$  is measured by standard deviation ( $\simeq \sigma_f$ ) but climate sensitivity spread  $S_{\Delta T}$  is measured by IQR (see appendix 1 for explanation). Lower left panel (c): cdf of  $\Delta T$ . Arrows represent the stable probability  $\mathbb{P}(\Delta T \in [4.5^\circ\text{C}, 8^\circ\text{C}])$  obtained for decreasing values of  $\sigma_f = 0.20, 0.15, 0.10$ . Lower right panel (d): probability  $\mathbb{P}(\Delta T \in [4.5^\circ\text{C}, 8^\circ\text{C}])$  as a function of feedback spread  $S_f$ , spread measured with IQR.



**Figure 2.**  $X$  is centered gaussian with standard deviation  $S_X$ . Right panel: probability for  $X$  to exceed respectively 1 and 3, as functions of  $S_X$ . Left panel: probability for  $X$  to fall within interval  $[1, 3]$  as a function of  $S_X$ .