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# Space-time structure of long ocean swell fields

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**Abstract.** The space-time structure of long period ocean swell fields is investigated, with particular attention given to features in the direction orthogonal to the propagation direction. This study combines space-borne Synthetic Aperture Radar (SAR) data with numerical model hindcasts and time series recorded by in situ instruments. In each dataset the swell field is defined by a common storm source. The correlation of swell height time series is very high along a single great circle path with a time shift given by the deep water dispersion relation of the dominant swells. This correlation is also high for locations situated on different great circles in entire ocean basins. Given the Earth radius  $R$ , we define the distance from the source  $R\alpha$  and the transversal angle  $\beta$  so that  $\alpha$  and  $\beta$  would be equal the the colatitude and longitude if the pole was set at the storm center. Outside of land influence, the swell height field at time  $t$ ,  $H_{ss}(\alpha, \beta, t)$  is well approximated by a function  $H_{ss,0}(t - R\alpha/C_g)/\sqrt{(\alpha \sin(\alpha))}$  times another function  $r_2(\beta)$ , where  $C_g$  is a representative group speed.  $r_2(\beta)$  derived from SAR data is very broad, with a width at half the maximum that is larger  $70^\circ$ , and varies significantly from storm to storm. Land shadows introduce further modifications so that in general  $r_2$  is a function of  $\beta$  and  $\alpha$ . This separation of variables and the smoothness of the  $H_{ss}$  field, allows the estimation of the full field of  $H_{ss}$  from sparse measurements, such as wave mode SAR data, combined with one time series, such as provided by a single buoy.

## 1. Introduction

Sea states in large ocean basins are the result of the superposition of wave trains that propagate at different frequencies and directions. A wave train can be a locally-generated wind sea and/or one swell field radiated from a distant storm [e.g. *Gain*, 1918; *Barber and Ursell*, 1948]. Long swells are generated by severe storms. During such events, the strongest winds are found in a well defined region, and for a limited time, typically of the order of 12 to 24 hours. We call "swell field" the ensemble of swell trains that were all generated by this same meteorological event. This swell field may eventually cover a full ocean basin or more, and has a lifetime that can extend over a few weeks due to the swell propagation across ocean basins, sometimes more than 20 000 km from their source [*Munk et al.*, 1963].

The first thorough investigation of swell was motivated by forecasting needs on the coast of Morocco during the colonial war of 1907–1912 [*Gain*, 1918], because the absence of harbors was making the transportation of troops very difficult with unpredictable delays in ship offloading due to heavy swells, a concern soon shared by commercial shipping. Similar reasons, namely the amphibious landing of 1943, again on the coast of Morocco (before many further landings elsewhere), also produced a revival of interest in swell prediction in the 1940s [*Sverdrup and Munk*, 1947; *Ursell*, 1999]. The observed dispersive nature of swells also led to the implementation of the first spectral wave models, designed and implemented by *Gelci et al.* [1957] after a long experience

with other methods of swell forecasting in Morocco [*Gelci and Cazalé*, 1953].

Other investigations have complemented these North Atlantic studies, culminating with the work of *Snodgrass et al.* [1966] which essentially focused on the evolution of the swell energy along the propagation direction. That analysis of swell height was later refined by [*Gjevik et al.*, 1988] who investigated the effects of the motion of the source storm, and [*Ardhuin et al.*, 2009a] who provided accurate estimations of the dissipation rates of swell energy. All these studies essentially followed swells along a great circle and showed that it was possible to forecast swell heights at great distances. This method was pioneered by *Montagne* [1922] who used observations in the Azores to forecast swell heights in Morocco.

The development of buoy networks and the very important contribution of swell observation from satellite later revealed the consistency of swell parameters also in the transversal direction, on the scale of entire ocean basins as revealed by *Hasselmann et al.* [1996], *Voorrips et al.* [1997] or more recently by *Collard et al.* [2009].

However, whereas the structure of frequency and direction in swell fields has been widely studied, the characteristics of the two-point space-time covariance of swell heights are essentially unexplored. In particular, the evolution of the energy distribution in the direction orthogonal to the propagation direction is not known. The correlation scales of the total significant wave height  $H_s$  has been investigated by *Tournadre* [1993], and *Greenslade and Young* [2005]. The latter study reveals an strong anisotropy of  $H_s$  in swell-dominated regions, with a maximum correlation scale in the direction perpendicular to dominant swell propagation directions. New assimilation techniques, that use swell partition information [e.g. *Voorrips et al.*, 1997; *Aouf et al.*,

2006a, b], would benefit from similar studies of swell partition parameters.

The goal of the present paper is thus to explore the correlation scales and structures of swell fields, in isolation from the rest of the sea state, with a view to provide information on the generating storms, and useful parameterization for the assimilation of swell-related measurements in numerical wave models. In particular we shall investigate the correlation in space and time, an effort dictated by the propagative nature of swells.

First, in section 2, a simple and robust method for assembling swell fields from ensembles of swell partitions automatically generated from wave spectra is presented. The tracking is both applied to numerical wave model output and to measurements of wave buoys. Using this tracking method, a field can be isolated from the other fields and studied separately. The swell significant wave height ( $H_{ss}$ ) of such fields is then studied. The present work widely uses model results to reveal features of the structure of the  $H_{ss}$  field, which are subsequently validated by confrontation with synthetic aperture radar (SAR) observations. The general properties of these features are presented in section 3 and used to generate similar  $H_{ss}$  fields using sparse SAR data, which provided comparable features as discussed in section 4. Finally, in section 5, a parameterization of the  $H_{ss}$  field is proposed and confronted to SAR observations.

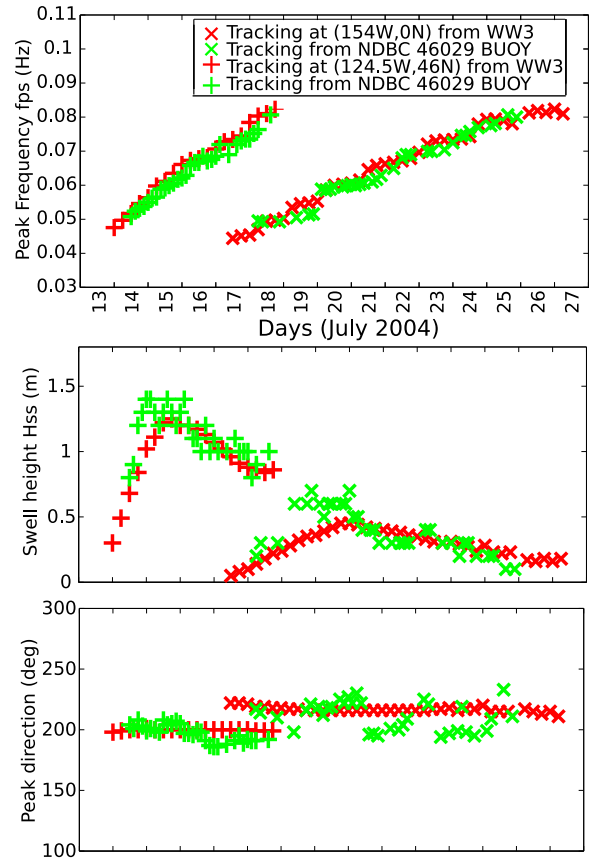
## 2. Space-Time Tracking of Swell Fields

### 2.1. Space-Time Tracking Method

To study the structure of a swell field, it is necessary to identify and isolate its contribution to the sea state. Spectral partitioning is the first step of this task. At a given observation point, a set of partitions of the ocean wave spectrum is defined. Each partition corresponds to the energetic contribution of a wave system [Gerling, 1992]. In order to associate swell partitions at different times and places to a common source, thus producing a field, Voorrips *et al.* [1997] proposed a cross-assignment tracking method based on the hypothesis of strong correlation between bulk parameters of two partitions related to the same swell field. To decide if two partitions are related to the same field or not, they used a set of empiric thresholds limiting the variability of the energy and the (mean or peak) frequency and direction in neighboring points in space and time. With a different approach, Aarnes and Krogstad [2001] estimated a meteorological source event time and location from each partitions group. A group is then validated or refined by verifying that it corresponds to a unique generating event.

In the present work, we exclusively use the uniqueness of the source to assemble the swell field. In our procedure, the time and location of the meteorological source are first estimated and this information is used to filter the swell partition dataset. This procedure is well suited to our goal of defining clean or ideal swell fields, but it will naturally lead to gaps and thus is not applicable as such for a compre-

hensive assembly of the swell field. This approach may also select a specific type of swell field.



**Figure 1.** Parameters of a swell field, generated by a storm centered at (177°W, 55°S) on 9 July 2004. Results are shown with green plus signs for NDBC buoy data 51028 (central equatorial Pacific) and green crosses for 46029 (Southern California Bight). Model results for the same locations are processed in the same way and shown with red color.

Practically, a swell partition is accepted in the field if its peak frequency  $f_{ps}$  and its mean direction at the peak wave frequency  $\theta_{ps}$  are close enough to those given by linear wave theory applied to a point source, denoted ( $f_{ps,lin}, \theta_{ps,lin}$ ). This follows the linear model for deep water swells at great distances from their generation areas, as given by Barber and Ursell [1948]. In this model the swell energy travels along the great circle of direction  $\theta_{ps}$  at the observation point, with a group speed, prescribed by deep water linear wave theory, namely  $C_g(f) = g/(4\pi f)$ . The space-time correspondence of  $f_{ps,lin}$  and  $\theta_{ps,lin}$  are given by

$$f_{ps,lin} = \frac{g(t - t_s)}{4\pi R\alpha}, \quad (1)$$

$$\theta_{ps,lin} = \arccos\left(\frac{\sin \phi_s - \sin \phi \cos \alpha}{\sin \phi \sin \alpha}\right) \quad (2)$$

and

$$\alpha = \arccos(\cos \phi \cos \phi_s \cos(\lambda - \lambda_s) + \sin \phi \sin \phi_s) \quad (3)$$

where  $\alpha$  is the angular distance between the point-source of coordinates  $(\lambda_s, \phi_s)$  and the observation point of coordinates  $(\lambda, \phi)$ ,  $t_s$  denotes the time of the source event and  $t$

the observation time,  $R$  denotes the Earth radius and  $g$  the acceleration of gravity. The linear time dependence of  $f_{ps,lin}$  at a given point is consistent with the ridge-like pattern in the time-frequency diagrams of the wave energy, conspicuous in any measurement of remote swells [e.g. *Munk et al.*, 1963]. The investigations of *Snodgrass et al.* [1966] and *Hasselmann et al.* [1996], among others, confirmed the ability of the model (1)–(3) to estimate the space-time structure of directions and frequencies when cutting across a swell field along a great circle.

It is decided that a partition belongs to a swell field if and only if the relative spectral distance between observed and expected values of  $(f_{ps}, \theta_{ps})$

$$D(f_{ps}, \theta_{ps}) = \frac{1}{f_{ps}} \left[ (f_{ps} \cos \theta_{ps} - f_{ps,lin} \cos \theta_{ps,lin})^2 + (f_{ps} \sin \theta_{ps} - f_{ps,lin} \sin \theta_{ps,lin})^2 \right]^{\frac{1}{2}} \quad (4)$$

is below a fixed threshold of 30%. This threshold has been calibrated to be relatively strict, as we would rather be sure that only contributions to the field of interest are retained rather than trying to keep more contributions which present strong differences with deep linear propagation from a point-source. As a result, this criterion is almost never met close to the source, and our fields usually have gaps near the source. Filling the hole near the source could be done by relaxing the constraint on  $D$  and using the continuity of  $f_{ps}$  and  $\theta_{ps}$  fields, as done by e.g. *Gerling* [1992] or *Hanson and Phillips* [2001].

At a given point the partition which minimizes the distance  $D$  is further selected, so that at any location only a single partition is part of the field. This requires that the input spectra are smooth enough so that noise is not mistaken for multiple peaks [e.g. *Portilla et al.*, 2009]. The present work uses model spectra that are smooth enough and SAR data that is smoothed before partitioning as described by *Collard et al.* [2009]. When using buoy data, the spectra are first averaged over 3 hours and interpolated on the model frequency grid, making the spectra smooth enough for a simple partitioning.

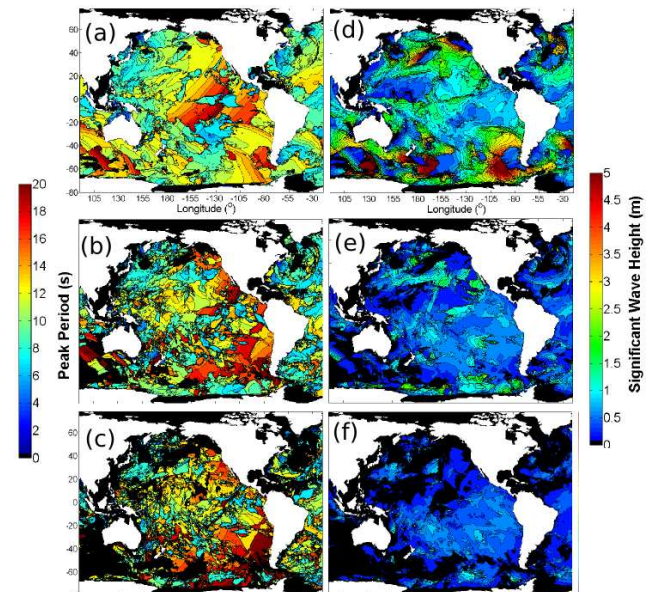
In order to be consistent with the propagation model (1)–(3), only long swells with peak frequency lower than 0.08 Hz are considered below.

## 2.2. Structure of swell fields

Two examples of swell fields are presented. The storms investigated in this paper are part of the database further analyzed by *Ardhuin et al.* [2009a] for which the storm location was also verified against Quikscat satellite wind fields obtained from CERSAT. In Figure 1, the tracking method is applied to partitioned spectra (using the method of *Gerling* [1992] for the partitioning) derived from the National Data Buoy Center (NDBC) platforms 51028 and 46029. Both wave measurement systems are pitch-and-roll 3 m diameter disc buoys, located respectively on the Equator, off Xmas Island (Kiritimati), Kiribati (153.913°W, 0°N) and in the South California Bight (124.510°W, 46.144°N). The tracked swell system has been generated by a storm which occurred

in the south-west Pacific Ocean on 09 July 2004 and was centered at (177°E, 55°S). This particular July 09 2004 storm is also illustrated by figures 2–3 in *Collard et al.* [2009].

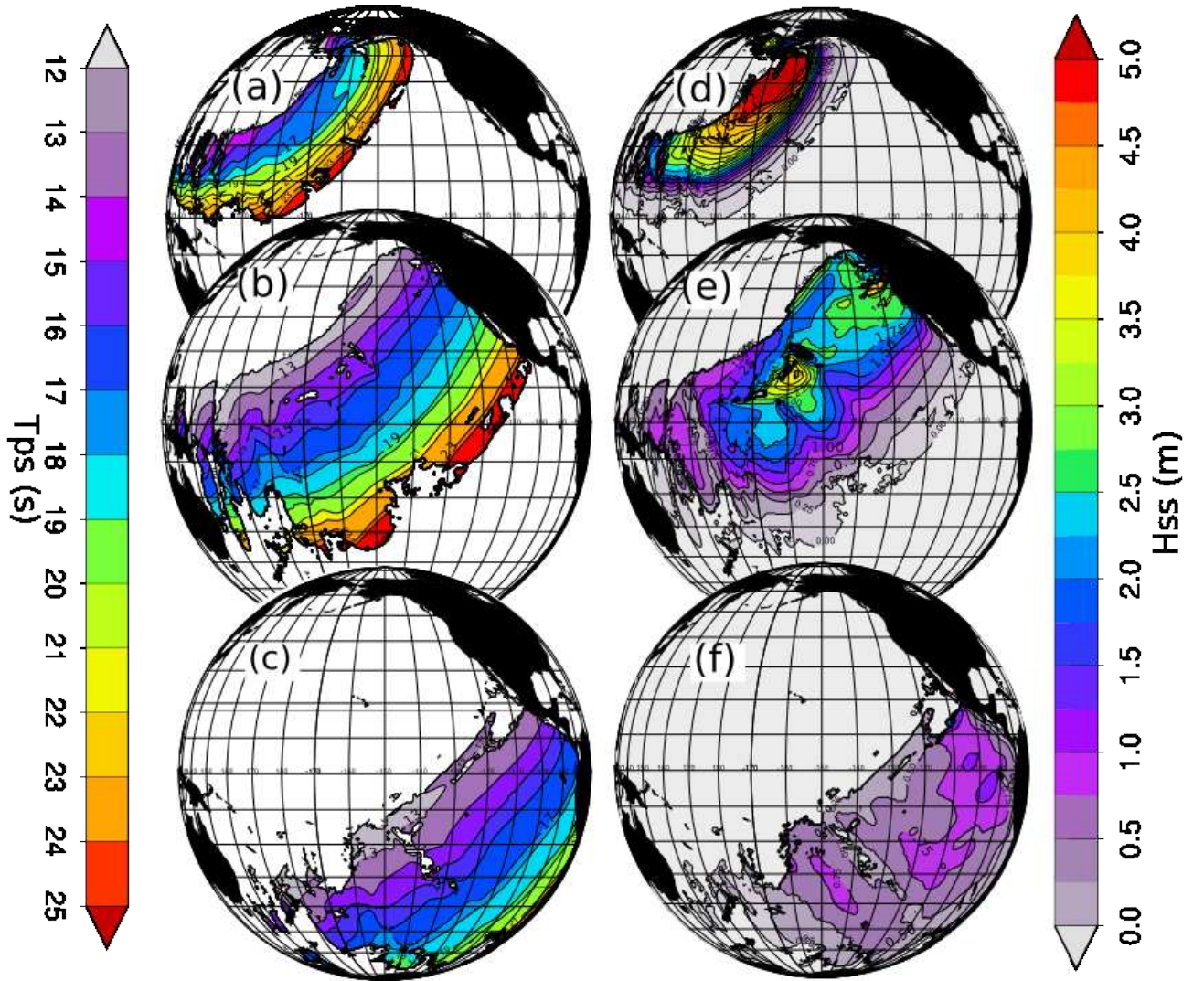
Swell parameters are compared with the tracking result obtained from the spectra computed by the wave model WAVEWATCH III<sup>TM</sup> (hereinafter WWATCH) using the version 3.14 [Tolman, 2008, 2009] with physical parameterization modified according to *Ardhuin et al.* [2009b] and corresponding to the TEST441 set-up described by [Ardhuin et al., 2009c]. This parameterization includes, in particular, an explicit swell dissipation consistent with the observations of *Ardhuin et al.* [2009a]. It should be noted that the sub-grid island blocking scheme of *Tolman* [2003] is used, as modified by *Tolman* [2007], together with the third-order Ultimate Quickest propagation scheme including the correction for spurious effects of spectral discretization (the garden sprinkler effect) as proposed by *Tolman* [2002].



**Figure 2.** Modelled peak periods (left) and significant wave heights (right) for the 3 most energetic swell partitions on June 03 2004, at 00h UTC. The top, middle and bottom panels show, respectively, the locally first, second and third partition by decreasing order of energy. In black are the areas where no swell partition is detected.

The model was ran on a 0.5° by 0.5° grid covering the entire ocean, with local two-way nested zooms covering Tahiti and the Tuamotus at 10 times finer resolution and in other areas that have no impact on the present result. Computation outputs are given every 6 hours. Output wave spectra are discretized over 32 frequencies exponentially spaced from 0.038 Hz to 0.72 Hz so that the bandwidth between two successive frequencies  $f_i$  and  $f_{i+1}$  is  $0.10f_i$ , and 24 directions with a constant 15° directional resolution. All model output fields are available at the URL <http://tinyurl.com/yesofy>.





**Figure 3.** Space-time tracking of a modelled swell system illustrated by (a,b,c) the peak periods  $T_{ps}$  and (d,e,f) the swell heights  $H_{ss}$ . The tracked system has been generated by the storm which occurred on 24 February 2004 at (160°E, 42°N). The system is shown on 27/02 (top) 01/03 (middle) and 06/03 (bottom) at 00h UTC. The central longitude is 150°W.

As often reported, the peak frequency and direction time series are in excellent agreement with the model given by (1)–(3) and thus with the numerical model WWATCH, which is based on the same geometrical optics principle with limited numerical error [Tolman, 2002]. The peak direction  $\theta_{ps}$  varies weakly around the source direction 200° for 51028 and 220° for 46029. The normalized root mean square difference (NRMSD) between the WWATCH model and the observations is re-

spectively of 3.9% and 2.8% for these two time series, where the NRMSD between time series of model results  $(X(t_i))_{i=1..n}$  and observations  $(Y(t_i))_{i=1..n}$  is defined by

$$NRMSD = \frac{1}{RMS} \left( \frac{1}{n} \sum_{i=1}^n (X(t_i) - Y(t_i))^2 \right)^{\frac{1}{2}}, \quad (5)$$

with  $RMS$  the root mean square of  $(Y(t_i))_{i=1..n}$ . It has been further verified on other storms (not shown) that the observed peak periods and those obtained with the numerical wave model coincide very closely. Compared

to both observations and model results, the point-source model gives arrival times that are on average 6 h too early for 17 s swells and up to 18 h too early for 13 s swells, with a gradual increase for intermediate periods, for all propagation distances. This effect will not be corrected in the following processing. It suggests that the afterglow of the storm, when the winds are decreasing, may be the time where most energy is generated for the shorter swells. That property could be used to refine the point-source model.

Swell dispersion also induces a progressive separation of the lowest and highest frequency components, resulting in larger durations of the swell events as the distance from the source increases. Here the swell is significant for 5 days at buoy 51028, located 6800 km from the source, and 9 days at 46029, located 10900 km from the source.

The modelled  $H_{ss}$  are clearly less accurate than frequencies or directions, with NRMSDs of 14.1% and 27.5% respectively at 51028 and 46029. These high NRMSD values are typical of such long swells whereas significant wave height of the total sea state  $H_s$  is generally better predicted by this state-of-the-art model, in the range 5–15% for the open ocean, in particular on the eastern part of ocean basins [Ardhuin *et al.*, 2009b, c].

From these comparisons of model and buoy observations it appears that periods and directions are well described by eqs. (1)–(3), but there is a need to improve the predictions of  $H_{ss}$ , which requires a further understanding of the structure of the  $H_{ss}$  field and possibly the assimilation of observations based on such structures.

However, qualitative features of the time series seem to be well reproduced by the model: a rapid increase of  $H_{ss}$  precedes an energetic peak with frequencies close to 0.06 Hz (periods close to 17 s) and directions around 200° for 51028 and 220° for 46029, followed by a slower decrease of  $H_{ss}$  with the gradual arrival of higher frequency components of the system.

Figures 2 and 3 illustrate the application of the swell systems tracking method to the model. Figure 2 represents the peak period ( $T_{ps}$ ) and significant wave height fields on 03 June 2004 at 00h UTC for the 3 most energetic partitions obtained by partitioning WWATCH spectra during run-time using the method of *Hanson and Phillips* [2001] as described in *Tolman* [2009]. Although buoy spectra are partitioned using the (possibly) different method by *Gerling* [1992], the two methods have been checked on model output and give equivalent results for our applications. In Figure 2, the parameters  $H_{ss}$  and  $T_{ps}$  are shown for partitions with decreasing energy. From one point to the next of the ocean surface, this local ordering can make the swell system jump from one storm source to another. As a consequence, the structure of the fields from separate sources is difficult to understand from the peak periods (Figure

2, left). The significant wave height field by itself is even more difficult to interpret.

Figure 3 represents the results of the tracking method applied to the same partitions. The tracked swell system has been generated by a source event which occurred in the north-west Pacific Ocean on 24 February 2004 and was centered at (160°E, 42°N). Results are given at 3-day intervals on 27 February, 01 March 2004, and 06 March 2004, all at 00h UTC. A remarkable coherence of the peak period field is now observed. This coherence seems to be very little affected during the propagation and is still observed at very long distance from the source (on the last panel, the system is located at more than 9000km from its source). The peak period field exhibits a conservation of a structure that is in good agreement with the model (1)–(3). The dispersion-induced spreading in the propagation direction is also clearly visible from panel 1 to panel 3.

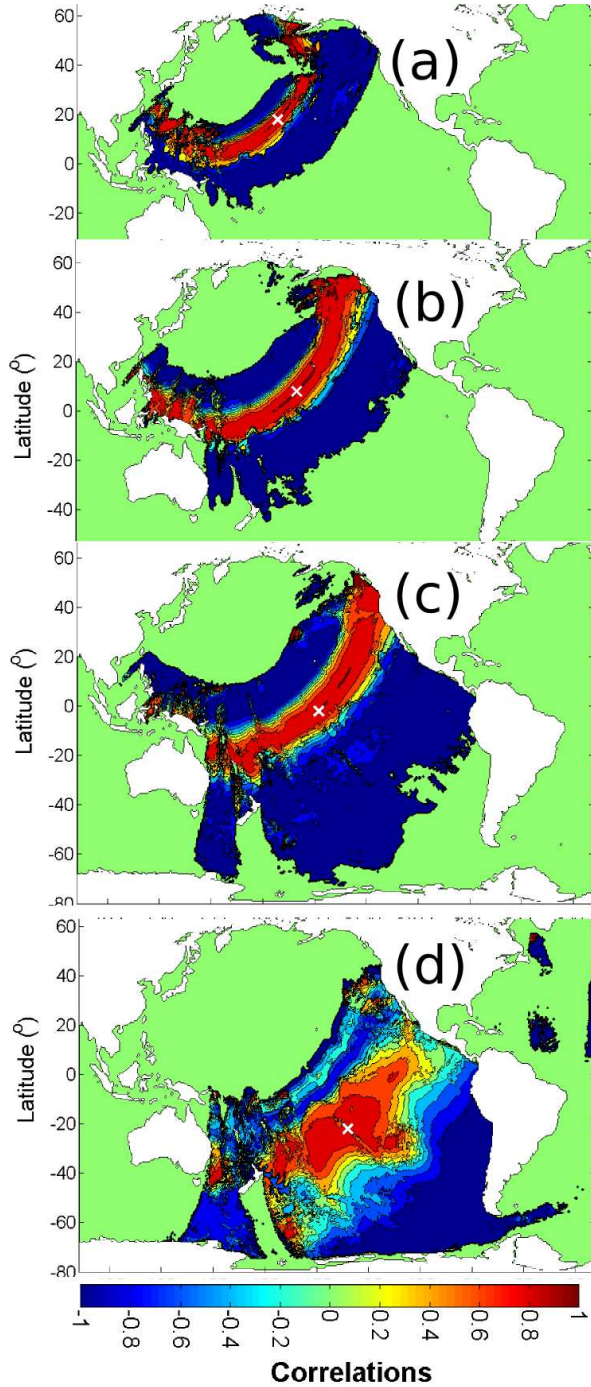
The structure of the  $H_{ss}$  field appears more complex. On the first panel, the distribution of  $H_{ss}$  for different outgoing directions results from the wave generation in the storm. Farther from the source, a general decrease of  $H_{ss}$  with the distance from the source is observed. Outside of the areas sheltered by islands, this decrease of  $H_{ss}$  is mainly due to dispersion and angular spreading, with a secondary effect of dissipation [Collard *et al.*, 2009; Ardhuin *et al.*, 2009a].

We will now further analyze the space-time structure of the  $H_{ss}$  field of long swell systems. Because the model results provide a continuous coverage in space and time which makes it easier to study, the modelled  $H_{ss}$  fields are first analyzed. The fair qualitative agreement of model predictions with observations suggests that some features found here are probably also present in the observations, what will be verified in section 4.

### 3. Patterns of Modelled Swell Heights

#### 3.1. Significant Wave Heights Correlations

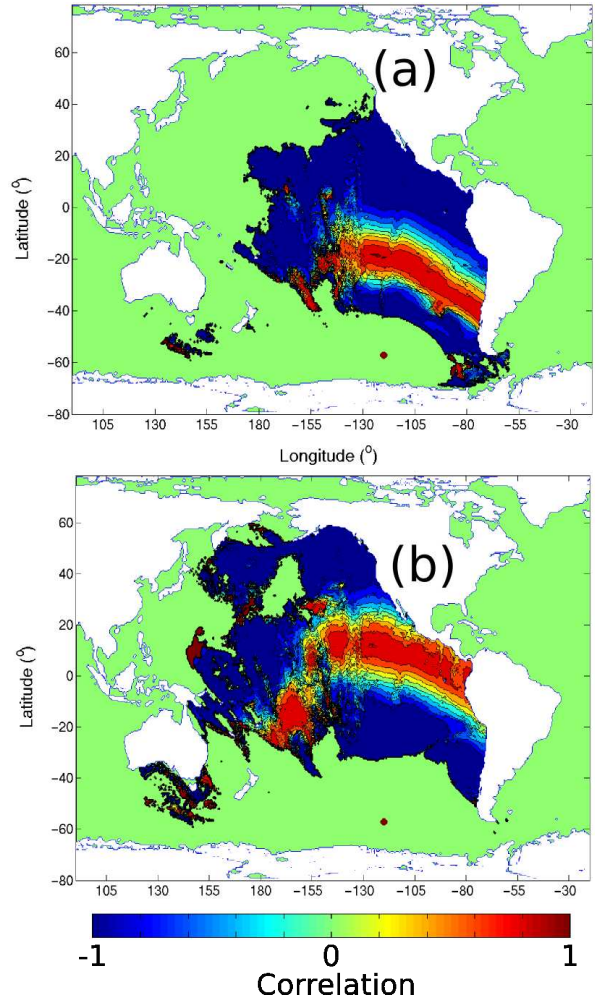
The spatial structures of the total  $H_s$  fields, and not  $H_{ss}$  as studied here, have been investigated by [e.g. *Greenlade and Young*, 2005]. These structures were studied in the context of sequential assimilation systems and the time correlation has been given relatively less attention [Pinto *et al.*, 2005]. Yet, in the very early swell investigations, correlations of swell height time series have been examined. When confronted with the task of practically improving the swell forecasting method of *Gain* [1918], based on the analysis of storm paths in the North Atlantic, *Montagne* [1922] soon realized that swell heights observed in the Azores corresponded closely, for many storms, with the observations of his own services in Morocco, with a time shift due to the time of propagation. Similar conclusions were also drawn on the relation of microseismic activity in Europe compared to swells in Morocco [Bernard, 1937].



**Figure 4.** Significant wave height time series correlations between a reference point  $P_0$  and the other points. (a)  $P_0=(173^\circ\text{W},20^\circ\text{N})$ , (b)  $P_0=(163^\circ\text{W},10^\circ\text{N})$ , (c)  $P_0=(153^\circ\text{W},0^\circ\text{N})$ , (d)  $P_0=(130^\circ\text{W},20^\circ\text{S})$ .

Following that earlier effort, we shall thus investigate correlations between  $H_{ss}$  time series of a given swell system at two different points. These correlations are obviously expected to change from event to event, with possibly some common features. After choosing a reference point  $P_0$ , the correlation between the time series

at  $P_0$ ,  $H_{ss,0}(t_i)$  for  $i$  in  $\{1..n\}$ , and at any other point  $P_j$ ,  $H_{ss,j}(t_i)$ , is investigated.



**Figure 5.** Same as Figure 4 for the August 9 2004 storm, with  $P_0$  at (a)  $(124^\circ\text{W},17^\circ\text{S})$  or (b)  $(125^\circ\text{W},12^\circ\text{N})$ , correlated with times series at other locations. The red dot indicates the source location.

Pearson's linear correlation coefficient  $C_j$  is used, allowing for a time shift  $\tau_j$ , so that  $C_j$  is given by

$$C_j = \frac{\sum_{i=1}^n (H_{ss,0}(t_i - \tau_j) - \overline{H_{ss,0}}) (H_{ss,j}(t_i) - \overline{H_{ss,j}})}{(n-1)\sigma_0\sigma_j} \quad (6)$$

where  $\overline{H_{ss,0}}$  and  $\sigma_0$  are given by

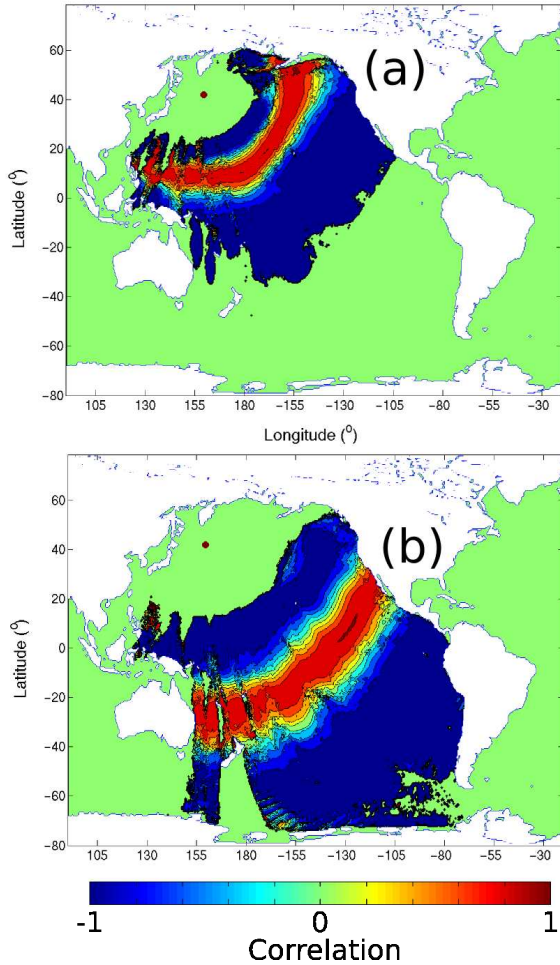
$$\overline{H_{ss,0}} = \frac{1}{n} \sum_{i=1}^n H_{ss,0}(t_i), \quad (7)$$

and

$$\sigma_0 = \left( \frac{1}{n-1} \sum_{i=1}^n (H_{ss,0}(t_i) - \overline{H_{ss,0}})^2 \right)^{\frac{1}{2}}, \quad (8)$$

with similar definitions for  $\overline{H_{ss,j}}$  and  $\sigma_j$ .





**Figure 6.** Same as Figure 4 for the February 24 2004 storm, with  $P_0$  at (a)  $(170^\circ\text{W}, 20^\circ\text{N})$  or (b)  $(125^\circ\text{W}, 10^\circ\text{N})$ . The red dot indicates the source location.

Figure 4 represents the result without time shift ( $\tau_j = 0$ ) obtained for the system generated by the source-event which occurred on 16 February 2004 at 00h UTC and was centered at  $(160^\circ\text{E}, 37^\circ\text{N})$ . The value of  $C_j$  for this swell system is given at  $P_j$  by the color-scale, if the system is simultaneously detected in  $P_0$  and  $P_j$  during at least 24h. The four panels correspond respectively to  $P_0=(173^\circ\text{W}, 20^\circ\text{N})$ ,  $P_0=(163^\circ\text{W}, 10^\circ\text{N})$ ,  $P_0=(153^\circ\text{W}, 0^\circ\text{N})$  and  $P_0=(130^\circ\text{W}, 20^\circ\text{S})$ . On each panel, the location of  $P_0$  is represented by a white cross.

These results show a remarkable  $H_{ss}$  correlations structure. On the first three panels, strong correlations ( $C_j > 0.9$ ) are observed for all the points located on the arc of a circle, at the same distance from the source as  $P_0$ . These strong correlations are observed for all the propagation directions. This is not too surprising since the basic time series of rising and falling  $H_{ss}$  is likely to be a common feature of any localized source, but the high correlation level indicates a degree of similarity that may be used to reconstruct a field from sparse data, even for locations not aligned with other observations. These remarkable features are observed for all the different swell systems studied here. Other exam-

ples are presented in Figure 5 and 6. In Figure 5, the correlation pattern appears less isotropic than in Figure 4, probably due to the translation speed and rotation of the generating storm. The blob of high correlation at  $10^\circ\text{S}$  and  $160^\circ\text{W}$  is located at a distance closer to the source than the east part of the high correlations front, probably because the associated waves were generated at a different stage of the storm evolution.

However, for all the systems, the area with strong correlations is less regular and symmetric at very long distance from the source (more than 7000km on the fourth panel of Figure 4). The principal cause of this evolution appears to be island shadowing. Indeed, the most important correlations decreases are located in the shadow areas of Pacific islands. In the example presented in Figure 4, the decrease in correlation is mostly to be found in the south west region of the Pacific basin, where the swell field propagated between many islands. Obviously the stretching of the time series due to dispersion (Figure 1) is another source of decorrelation that could be easily corrected for.

### 3.2. Structure of Swell Height Fields in the Transverse Direction

The strong correlations observed in the direction orthogonal to the propagation direction suggest a quasi-linear relation between the swell system  $H_{ss}$  observed at points located at the same distance from the source. This leads us to examine the coefficient  $r(\alpha, \beta)$  that gives the best correlation

$$H_{ss}(\alpha, \beta, t) \simeq r(\alpha, \beta)H_{ss,0}(t), \quad (9)$$

where  $\alpha$  and  $\beta$  are the co-latitude and longitude when setting the North Pole on the storm center. Formally we take

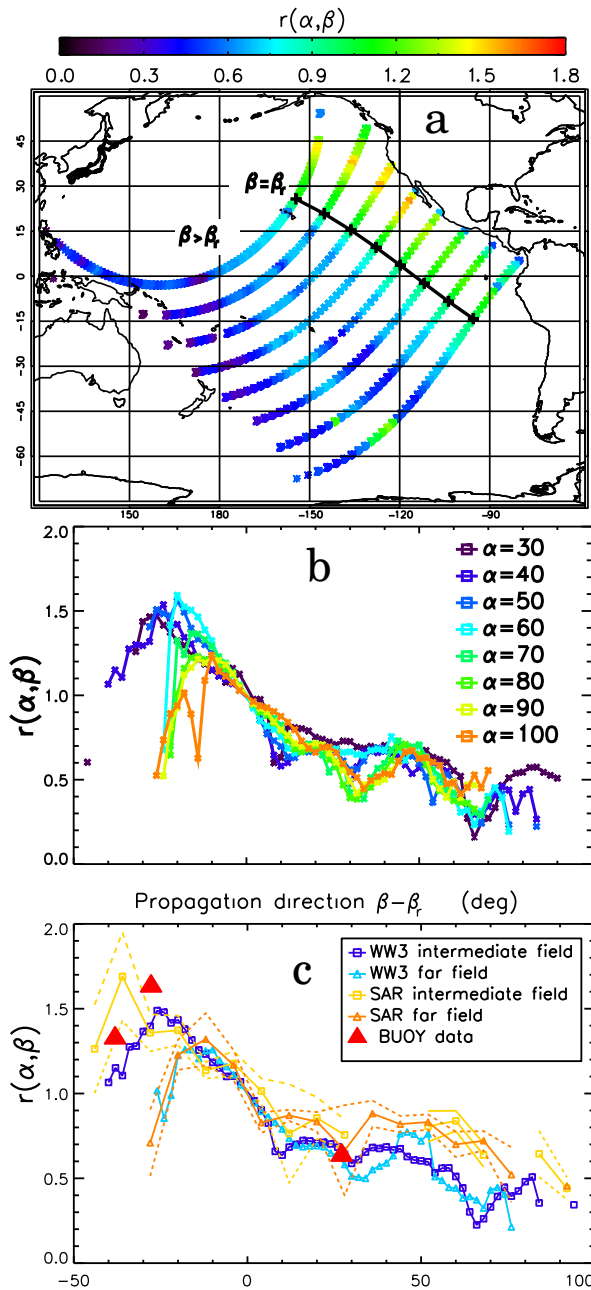
$$r(\alpha, \beta) = \sqrt{\frac{1}{n} \sum_{i=1}^n \frac{H_{ss}(\alpha, \beta, t_i)^2}{H_{ss}(\alpha, \beta_r, t_i)^2}}, \quad (10)$$

where  $\beta_r$  is a reference direction. Figure 7.a and 7.b give two representations of  $r$  for the system generated on 16 February 2004 at  $(160^\circ\text{E}, 37^\circ\text{N})$  for distances  $\alpha$  from  $30^\circ$  to  $100^\circ$ , with a  $10^\circ$  step. Defining  $\beta_r$  as the closest direction from the system mean direction outside the source in which waves do not cross island groups,  $\beta - \beta_r$  takes values in the range  $[-180^\circ, 180^\circ]$  and we compute  $r(\alpha, \beta)$  every  $8^\circ$ .

Figure 7.a gives the location of the points where the ratio  $r$  is computed, with the value of  $r$  indicated by the color-scale. Figure 7.b gives transverse profiles of  $r(\alpha, \beta)$  for the different values of  $\alpha$  and this time the color-scale indicates the value of  $\alpha$ .

Figure 7.b reveals that the transverse structure is essentially the same for different values of  $\alpha$ , except for islands shadow regions. For example the shadow of the Tuamotus (French Polynesia,  $20^\circ\text{S}$ ,  $140^\circ\text{W}$ ) explains the change of  $r$  for  $\beta = \beta_r + 40^\circ$  as  $\alpha$  increases from  $60^\circ$  to  $70^\circ$ .



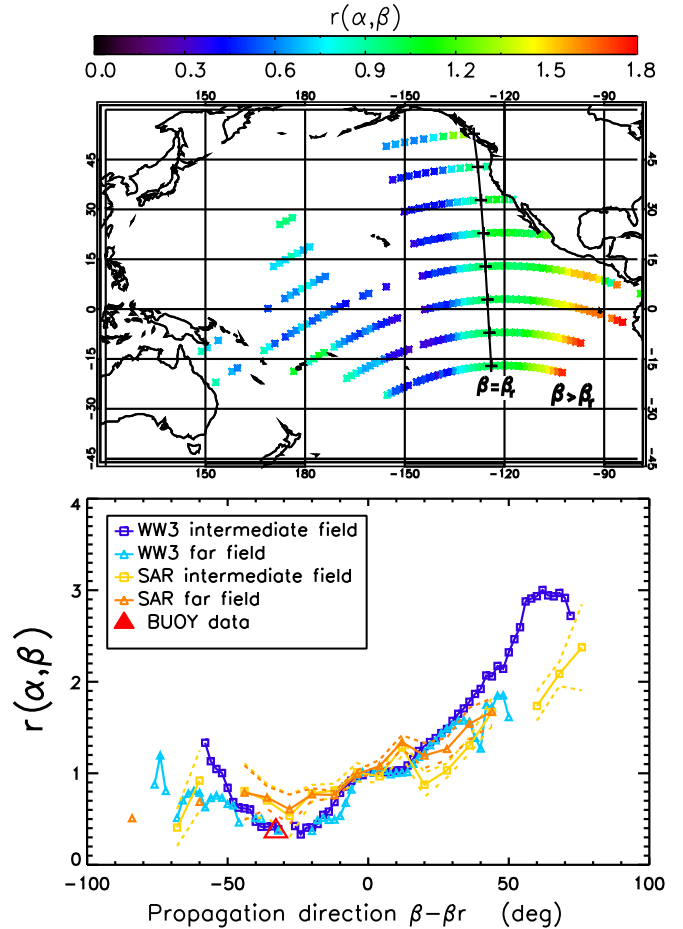


**Figure 7.** Transverse structure of the system generated on 16 February 2004, at (160°E, 37°N). (a) spatial distribution of  $r(\alpha, \beta)$ , inferred from the model, with the reference direction  $\beta_r$  in black. (b) transverse profiles of  $r(\alpha, \beta - \beta_r)$  for the model only. (c) transverse profiles of  $r(\alpha, \beta - \beta_r)$  for intermediate ( $\alpha < 60^\circ$ ) and far fields ( $\alpha > 60^\circ$ ) inferred from model, SAR and buoy data. The dashed lines correspond to + or - one standard deviation of the expected SAR error given the scatter index of SAR observations compared to buoys [Collard *et al.*, 2009], divided by  $\sqrt{N}$  where  $N$  is the number of SAR observations for one given estimate of  $r$ .

The variability imposed by islands appears to be significant for large islands groups, such as the Hawaii chain (20°N, 155°W) for  $\alpha > 40^\circ$  and  $\beta - \beta_r \sim 10^\circ$  or the Tuamotus. Outside of these main island shadow

zones,  $r(\alpha, \beta)$  generally varies very little with  $\alpha$ . This low variability is a common feature of all the different systems studied here (see Figure 8 and 9 for other examples).

#### 4. Estimation of Transverse Structures in SAR Data

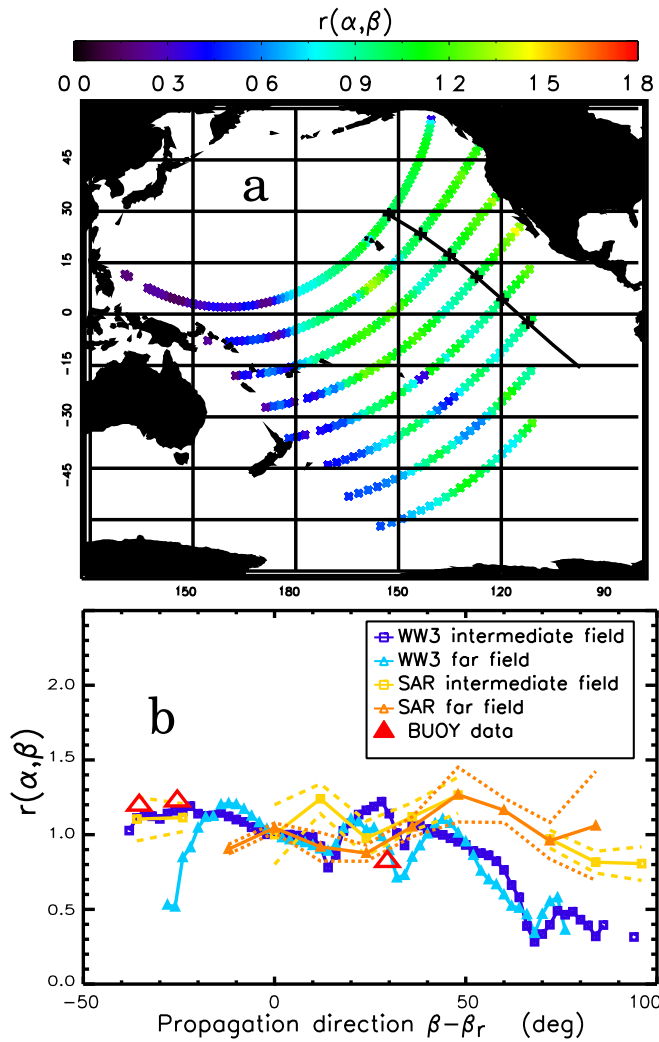


**Figure 8.** Same as figure 7, for the storm which occurred on 9 August 2004 at 00h UTC, located at (120°W, 57°S).

The structure of the modelled  $H_{ss}$  fields is now verified, still expecting a similar smooth variation of  $H_{ss}$  with  $\alpha$  to build an estimate of  $r(\alpha, \beta)$  using SAR data. We define the structure function  $r(\alpha_0, \beta)$  by propagating SAR observations located at distances  $\alpha$  in the range  $[\alpha_1, \alpha_2]$  to the distance  $\alpha_0$ . This propagation is similar to the generation of "fireworks" described by [Collard *et al.*, 2009], but the value of  $H_{ss}(\alpha, \beta)$  given by the SAR observation is now rescaled by a factor

$$\frac{r(\alpha_0)}{r(\alpha)} = \sqrt{\frac{\alpha \sin \alpha}{\alpha_0 \sin \alpha_0}}, \quad (11)$$

which corresponds to a propagation without dissipation away from a point source [e.g. Collard *et al.*, 2009].



**Figure 9.** Same as figure 7, for the February 24 2004 storm.

Because of the limited number of SAR observations, the resolution is limited over  $\alpha$  or  $\beta$ . Since we wish to examine details in the  $\beta$  dimension, a very coarse resolution in the  $\alpha$  dimension is taken by dividing up the oceans in an intermediate field, with  $40^\circ \leq \alpha \leq 60^\circ$  and a far field with  $\alpha > 60^\circ$ . Observations for  $\alpha < 40^\circ$  are not retained because eq. (11) is not generally valid in the near field.

Figure 7.c represents the result obtained for the 16 February 2004-(160°E,37°N) storm. The SAR data used here are level 2 products provided by the European Space Agency (ESA) and collected during ENVISAT mission. These products are unambiguous wave directional spectra processed from SAR scenes using the method of *Chapron et al.* [2001], as updated by *Johnsen and Collard* [2004], with further bias corrections given by *Collard et al.* [2009]. On this figure, the intermediate field and far field structures estimated from the model are also superposed.

A good agreement of the different structures appears in the east part of the Pacific basin, for  $\beta - \beta_r < 0^\circ$ ,

where few islands are in the propagation path. On the contrary, for  $\beta - \beta_r > 0^\circ$ , modelled  $H_{ss}$  are underestimated. This low bias also exists in the close field structure. Outside of island shadows, the model and SAR observations generally show little variability (Figure 5 to 9) with some exceptions.

For a given value of  $\alpha$ ,  $r$  generally exhibits a peak with a very slow decrease with  $\beta$ . The half width of  $r$  is typically larger than  $60^\circ$ . In the case of the 24 February 2004 storm,  $r$  is almost isotropic with  $\beta$ . Model results are typically less isotropic than SAR observations, possibly due to an incorrect dependence of the swell damping rate on the wind-wave angle in the parameterization by *Ardhuin et al.* [2009b], or similar defects in the forcing wind fields or wind-wave generation parameterization.

Analysis of modelled spectra thus suggests a large scale coherence of the space-time  $H_{ss}$  field structure, with a transverse structure imprinted by the source storm and islands shadows.

## 5. Modelling $H_s$ Structure of a Swell System: Synthetic Field

Given the features of the  $H_{ss}$  field, there may be a use for parameterized synthetic fields that could represent the correlations of  $H_{ss}$ , for example in an assimilation system for wave hindcasting or forecasting. For simplicity we propose such a parameterization of  $H_{ss}$  outside island shadows.

Given the generally good approximation of  $T_{ps}$  and  $\theta_{ps}$  with  $T_{ps,lin}$  and  $\theta_{ps,lin}$ , periods and directions space-time structures are represented by the deep water linear propagation from a point-source (1)–(2). The temporal shift  $\tau$  which maximizes the correlation of  $H_{ss}$  time series observed at two points  $P_0$  and  $P$  located respectively at distances  $\alpha_0$  and  $\alpha$  from the storm is replaced by

$$\tau(\alpha, t) = \frac{4\pi R(\alpha - \alpha_0)f_{ps,lin}}{g} \quad (12)$$

where  $f_{ps,lin}$  is given by (1) at the point  $P$  at time  $t$ .

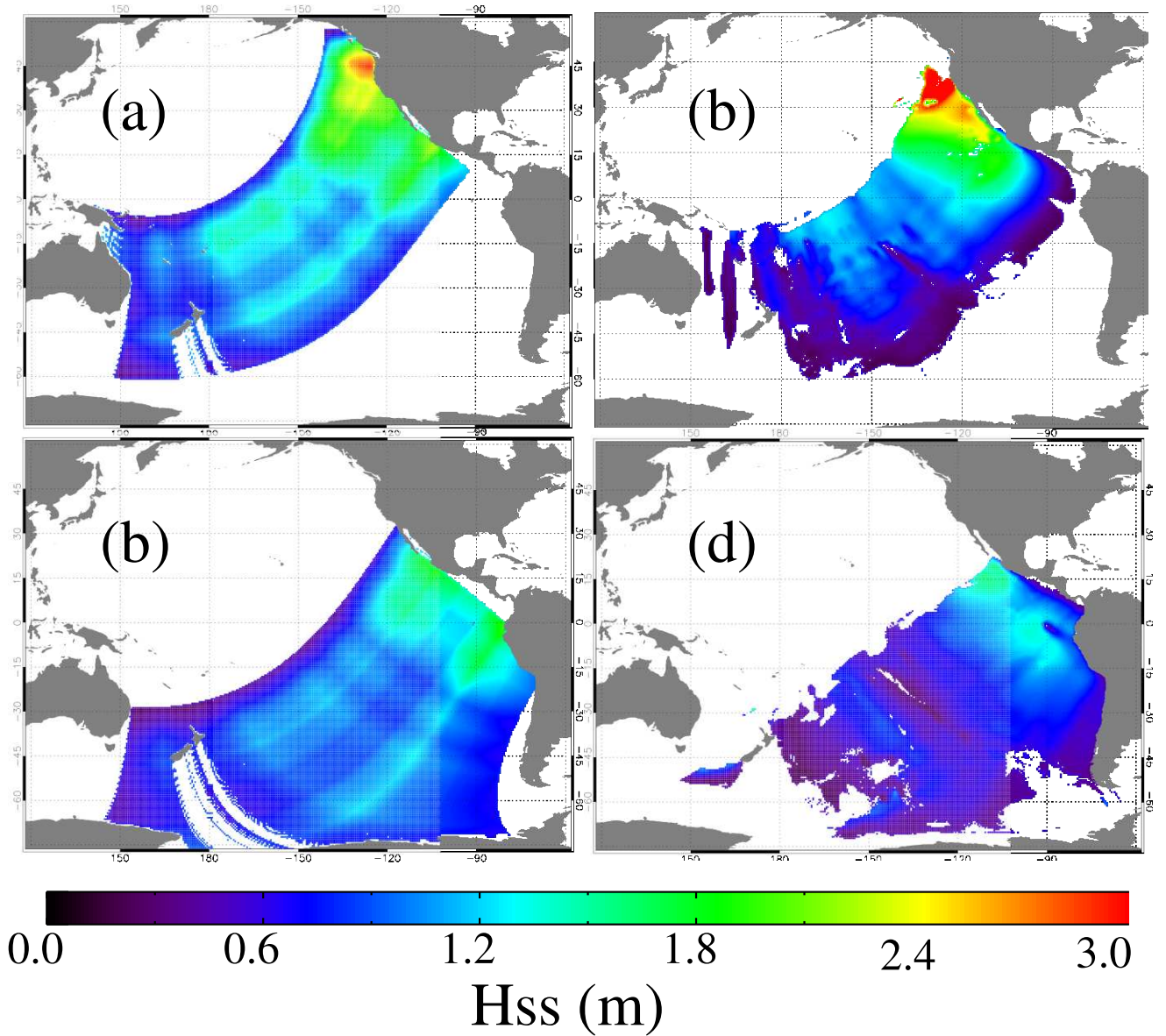
Secondly, we seek a parameterization for  $r$  by separating the variables, with a form  $r(\alpha, \beta) = r_1(\alpha)r_2(\beta)$ . It is further assumed that  $r_1$  is given only by the principal causes of  $H_{ss}$  decrease, namely dispersion and angular spreading, which is represented by the asymptotic form (11).

This gives the parameterized field

$$\hat{H}_{ss}(\alpha, \beta, t) = \sqrt{\frac{\alpha_0 \sin \alpha_0}{\alpha \sin \alpha}} H_{ss,0}(t - \tau(\alpha, t)) r(\beta) \quad (13)$$

where  $H_{ss,0}(t)$  is a known reference  $H_{ss}$  time series at the point  $(\alpha_0, \beta_0)$ , given by an in situ sensor or inferred from a broadband sismograph [e.g. *Bromirski et al.*, 1999].

The field (13) is now compared to observations. Using the storm of 16 February 2004 as an example, we use for  $H_{ss,0}(t)$  the  $H_{ss}$  time series given by the Xmas



**Figure 10.** (a and c), synthetic field of  $H_{ss}$  on 25 and 28 February 2004. (b and d) modelled field at the same time (right).

Island buoy (number 51028). Here  $\tau$  is defined using eq. (12) in which  $f_{ps,lin}$  is replaced by the time-dependent peak period time series at the buoy. This very simple approximation could be refined by taking the spectral content given by the buoy. We use for  $r_2(\beta)$  the intermediate field structure estimated from SAR observations and thus reconstruct a synthetic field  $\hat{H}_{ss}$  using (13), which is compared with SAR observations in the far field.

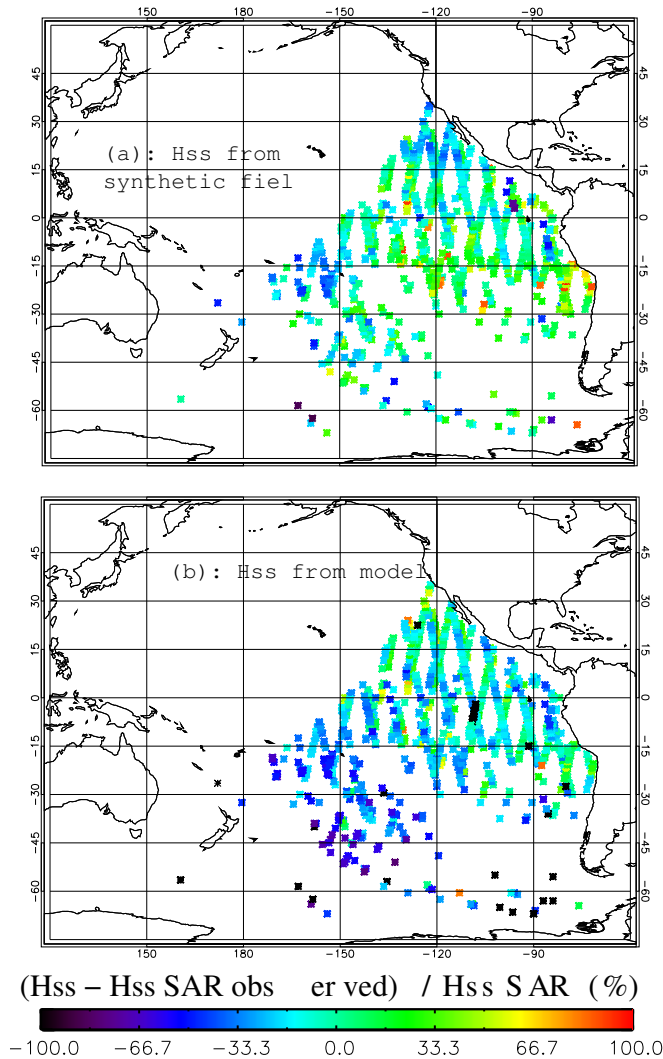
The two left panels of Figure 10 represent the synthetic  $\hat{H}_{ss}$  field obtained at two different times: on 25 February 2004, when it is mostly in the intermediate field, and on 28 February 2004, when it is mostly in the far field, both at 00h UTC.

The two right panels represent the modelled field extracted from the model computation at the same times. Figure 11 represents the relative error between  $\hat{H}_{ss}$  and SAR observations (top panel) and between modelled  $H_{ss}$  and SAR observations (bottom panel) in the far field region. As the synthetic  $\hat{H}_{ss}$  field represents the  $H_{ss}$  field outside islands shadow, only SAR observations which are located outside this shadow are considered here. In practice, an observation is not retained if the great circle path joining its location to the source point is less than 20 km from an important island.

Clearly the synthetic swell field is very smooth due to the many simplifying assumptions, in particular for the decay along the propagation path. Yet, the modelled and synthetic fields present a global NRMSD of 35.4% and 33.6% respectively, suggesting that our very crude first



parameterization does capture the essential features of the swell field, up to a point that it may already be useful as a source of correction for today's best numerical models.



**Figure 11.** Difference  $H_{ss} - H_{ss,SAR}$  for  $\alpha > 60^\circ$ , for (a) the synthetic wave field and (b) the model results.  $H_{ss,SAR}$  is derived from the local SAR observations.

Although the variation of  $H_{ss}$  with  $\alpha$  in Figure 10 does not look very realistic, the key element here is certainly the variation of  $H_{ss}$  with  $\beta$ , in particular for this example with a relatively poor model prediction in the area East of New Zealand (Figure 6). The lower NRMSD for the systems components whose propagation path crosses few islands confirms the good agreement of (13) with the observations. It should be noted that these differences are not much larger than the expected error of SAR-derived  $H_{ss}$  which is about 24%. A further quantitative validation should thus be performed with more accurate buoy measurements, or by aggregating SAR observations to reduce the errors.

## 6. Conclusions and perspectives

Several studies from [Montagne, 1922] and [Barber and Ursell, 1948] to [Collard et al., 2009] have shown that long swell systems often follow closely wave linear theory for a point-source, and swells may travel with high space-time correlations over distances and times as large as 10 000 km and 10 days. However, because the variations of swell wave height  $H_{ss}$  were not known as a function of the direction  $\beta$  transversal to the propagation, previous investigations of swells for forecasting or geophysical applications had to rely on measurements aligned along propagation great circles [e.g. Snodgrass et al., 1966]. Here we have performed the first investigation of  $H_{ss}$  as a function of  $\beta$ . Although the analysis of many more swell events will be needed to accumulate confidence in these results, two important results are obtained.

First, the distribution of  $H_{ss}$  is very broad, broader than  $\cos \beta$ , so that a swell field generally covers a very large part of an ocean basin, with some occasional propagation towards other basins, as suggested by the modelling study of Alves [2006]. As a result, the correlations of geophysical processes with swell can be very difficult to interpret, and non-local swell effects may be mistaken as local effects. For example, while both mid-ocean and coastal sources of microseisms probably co-exist, it is hard to distinguish microseismic signals generated at a coast [e.g. Bromirski et al., 1999] from other microseismic signals thought to be generated in deep water, but not so far from another coast [e.g. Kedar et al., 2008].

Second, the variations of  $H_{ss}$  are largely invariant as a function of distance from the storm  $\alpha$ , except for island shadow effects. As a result, the swell field can be parameterized by separating the variables  $\beta$  and  $\alpha$ , with the time closely related to  $\alpha$  due to the propagation. In one example, a first crude parameterization of  $H_{ss}$  based on this principle was shown to agree already slightly better than the best forecasting model available today [Bidlot, 2008]. There is thus a clear potential for improving swell hindcasts and forecasts by developing time-dependent covariance models based on these observations, or by directly assimilated gridded (level 3) synthetic swell parameters. Clearly, the parameterization of islands in the synthetic swell fields is an obvious next step, and this could easily use the subgrid masking algorithms employed in wave models [Tolman, 2003]. This was not done here in order to keep the procedure as simple as possible and also because no detailed local validation of these algorithm has yet been performed. The simple decay law for  $H_{ss}$  with distance  $\alpha$  could also be refined to represent explicitly the spectral contents of the wave field and include a period-dependent dissipation rate, based on, for example, [Ardhuin et al., 2009a]. Further methodological improvements may come from a comprehensive study of swell fields, that may show how and when  $H_{ss}$  may be directly parameterized based on the storm structure, translation and rotation. It is quite



possible that our procedure for assembling swell fields has biased our view of swells towards fields that do conform well to the point source model, and we will further examine how representative these fields are.

Swells are today the most poorly predicted part of the sea state [Ardhuin *et al.*, 2008, 2009c], with detrimental impact on delicate marine operations and, unfortunately, accidents due to heavy swells on the coast. Several assimilation method have been imagined to improve wave forecasts [Voorrips *et al.*, 1997; Pinto *et al.*, 2005; Aouf *et al.*, 2006a] with, so far, very limited practical benefits [Bidlot *et al.*, 2007; Bidlot, 2008]. Exploiting the very large correlation scales of the swell field revealed in the present study should help pushing further the time horizon of the assimilation impact.

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