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Observability and measurement allocation analysis for networked systems

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Abstract: This paper deals with the strong state observability analysis for Networked Systems which are composed of interconnected subsystems that exchange data over communication networks. More precisely, we interest to determine which data or measurements must arrive via the network to a subsystem in order to ensure that a local observer can estimate its state vector. The proposed approach is based on a graph-theoretic approach and assumes only the knowledge of the system's structure. It is suited to study large scale distributed systems because based on classical and well-known graph theory algorithms, which have polynomial complexity orders.

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Keywords: Networked Systems, strong state observability, measurements allocation, graph theory.

1. INTRODUCTION

Networked Control systems (NCS) are in general composed of a large number of interconnected devices or subsystems that exchange data through communication networks. Examples include industrial automation, building supervision, automotive control, ... NCSs provide many advantages such as modular and flexible system design, fast implementation, distribution. The observability of the internal state of each subsystem is one of the main properties which is strongly linked to the configuration of the distributed subsystems and to the data they exchange. Indeed, even if the global system is observable, when we subdivide it into several subsystems, the latter may be not structurally observable and in this case, it may be impossible to achieve the state estimation with a local observer. This can be quite awkward since the state estimate is useful in the control law synthesis, supervision, fault detection and so on. Thus, an analysis of the observability of the distributed system, in function of the informations exchanged on the network, is important for the observer design and so in the general conception of the system. Many studies use unknown input observers to deal with the observation of decentralized systems even if it is not in the context of Networked Control Systems. In this way, in the early 70's, Sanders et al. [1974] propose, under some decoupling assumptions, the design of a filter for interconnected dynamical systems in which the information pattern is decentralized. In Saif and Guan [1992], the authors propose a method for the design of decentralized reduced state estimator for large scale systems composed by interconnected systems using unknown input observers under some "matching condition". Also on the basis of unknown input observers, in Hou and Müller [1994], decentralized state function observer are designed for large scale interconnected systems. In the latter papers as in most others, the proposed approaches to address the observability or on decentralized systems deal with algebraic and geometric tools. The use of such tools requires the exact knowledge of the state space matrices characterizing the system's model. However, in many modeling problems, only zero entries of these matrices, which are determined by the physical laws, are fixed while

the remaining entries are not precisely known. To study the properties of these systems in spite of poor knowledge we have on them, the idea is that we only keep the zero/non-zero entries in the state space matrices. Thus, we consider models where the fixed zeros are conserved while the non-zero entries are replaced by free parameters. The analysis of such systems, called structured systems, requires a low computational burden which allows one to deal with large scale systems. Many studies on structured systems are related to the graph-theoretic approach to analyse some system properties such as controllability, observability or the solvability of several classical control problems including disturbance rejection, input-output decoupling, ... These works are reviewed in the survey (Dion et al. [2003]) from which it results that the graph-theoretic approach provides simple, efficient and elegant solutions.

In this context, the state and input observability conditions provided in Boukhobza et al. [2007] for a linear systems with unknown inputs are a good point of departure to enhance the observability of Networked Control Systems. Besides, we address the problem of measurement allocation over the network. The paper is organised as follows: after Section 2, which is devoted to the problem formulation, a digraph representation of Networked Control systems is given in Section 3. The main result is enounced in Section 4. Finally, a conclusion ends the paper.

2. PROBLEM STATEMENT

In this paper, we consider networked systems having the numerically non-specified following model:

$$\Sigma_{\Lambda} : \begin{cases} \dot{x} = A^{\lambda}x + B^{\lambda}u \\ y = C^{\lambda}x + D^{\lambda}u \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^q$ and $y \in \mathbb{R}^p$ are respectively the state vector, the unknown input vector and the output vector. A^{λ} , B^{λ} , C^{λ} and D^{λ} represent matrices which elements are either fixed to zero or assumed to be nonzero free parameters noted λ_i . These parameters forms a vector $\Lambda = (\lambda_1, \dots, \lambda_h)^T \in \mathbb{R}^h$.

If all parameters λ_i are numerically fixed, we obtain a so-called admissible realization of structured system Σ_Λ . We say that a property is true generically if it is true for almost all the realizations of structured system Σ_Λ . Here, "for almost all the realizations" is to be understood as "for all parameter values ($\Lambda \in \mathbb{R}^h$) except for those in some proper algebraic variety in the parameter space". The proper algebraic variety for which the property is not true is the zero set of some nontrivial polynomial with real coefficients in the h system parameters $\lambda_1, \lambda_2, \dots, \lambda_h$.

Consider that Σ_Λ is a networked distributed system constituted of several subsystems Σ_i^R , $i = 1, \dots, N$. Each subsystem satisfies to a model of the form:

$$\Sigma_i^R \begin{cases} \dot{x}_i = A_i^\lambda x_i + B_i^\lambda u_i + \sum_{j=1, j \neq i}^N (A_{i,j}^\lambda x_j + B_{i,j}^\lambda u_j) \\ y_i = C_i^\lambda x_i + D_i^\lambda u_i \end{cases} \quad (2)$$

where for $i = 1, \dots, N$, $x_i \in \mathbb{R}^{n_i}$, $y_i \in \mathbb{R}^{m_i}$ and $u_i \in \mathbb{R}^{p_i}$ represent respectively the state vector, the local measurement vector and the local input vector of subsystem Σ_i^R . Obviously, the whole state vector of system Σ_Λ is constituted by the aggregation of the state vectors of subsystems Σ_i^R , $i = 1, \dots, N$ i.e. $x = (x_1^T, \dots, x_N^T)^T$. Similarly, the whole input vector is $u = (u_1^T, \dots, u_N^T)^T$ and the whole output vector is $y = (y_1^T, \dots, y_N^T)^T$. If a state or an input component of subsystem Σ_j^R acts on the dynamics of another subsystem Σ_i^R ($i \neq j$), we consider this component "as an unknown input" for subsystem Σ_i^R . Even if its dynamics has a known expression in the global model, it won't be implemented in the local observers used to reconstruct the state vector of each subsystems. In fact, this consideration is more pertinent and realistic from a practical point of view because implementing the whole system's model in each local observer can be, on the one hand, tedious and unpracticable and on the other hand does not allow to benefit from advantages of the subdivision of a large scale system into several smaller subsystems. Moreover, as we are interested in the observability problem, by the term "output" we mean here measurements. Otherwise, without loss of generality and for the sake of homogeneity, all the inputs are assumed to be unknown. Concerning the control input components, which are known, they are considered to be measured.

In the framework of this paper, the subsystems can exchange some informations through a network. We consider without loss of generality that the measurements arriving to each subsystem Σ_i^R are related to other subsystems Σ_j^R ($i \neq j$) and can be modeled in the most general form as:

$$\tilde{y}_i = \sum_{j=1, j \neq i}^N \tilde{C}_{i,j}^\lambda x_j(t) + \tilde{D}_{i,j}^\lambda u_j(t)$$

The aim is to study the generic strong state observability (Trentelman et al. [2001]) of structured subsystems constituting Σ_Λ . This property is necessary and sufficient to ensure that the state components can be deduced from only the knowledge of the available measurements. First, we recall the definition of the generic strong state observability in the case of a structured linear system:

Definition 1. Structured linear system Σ_Λ is generically strongly observable if, for almost all its realizations, $y(t) = 0$ for $t \geq 0$ implies $x(t) = 0$ for $t \geq 0$ whatever the unknown inputs $u(t)$.

Roughly speaking, the generic strong observability means that it is possible to express the state of Σ_Λ in function of the

available measurements and their time derivatives.

In the framework of networked systems, we are interested in the generic strong observability of the state of each subsystem Σ_i^R , $i = 1, \dots, N$, and we consider two cases. In the first one, we assume that there are no external measurements arrive through the network to Σ_i^R . So, subsystem Σ_i^R can only use its own measurement vector y_i to estimate or reconstruct its state components. We call this case the decentralized autonomous observation scheme. In the second case, we assume that subsystem Σ_i^R is linked to the network and can use the measurement vectors y^i and \tilde{y}^i to reconstruct its state components. We call this case the decentralized interconnected observation scheme. In order to handle these notions of local strong observability in the two previously described cases, we associate to each subsystem Σ_i^R , the following system:

$$\Sigma_i^{RA} \begin{cases} \dot{x}_i = A_i^\lambda x_i + B_i^\lambda u_i + \sum_{j=1, j \neq i}^N \tilde{B}_{i,j}^\lambda \tilde{u}_j \\ y_i = C_i^\lambda x_i + D_i^\lambda u_i \end{cases} \quad (3)$$

where for $i, j = 1, \dots, N$, $j \neq i$, unknown input $\tilde{u}_j = (x_j^T, u_j^T)$ represents the interactions due to other subsystems and so matrix $\tilde{B}_{i,j}^\lambda = (A_{i,j}^\lambda \ B_{i,j}^\lambda)$. System Σ_i^{RA} is equivalent to system Σ_i^R by considering that vectors x_j and u_j , $j \neq i$ are unknown inputs for subsystem Σ_i^R . We define now the state strong observability, relatively to these considered observation schemes, as follows:

Definition 2. Consider linear structured system Σ_Λ constituted of several subsystems Σ_i^R , $i = 1, \dots, N$.

We say that that state vector $x_i(t)$ is generically locally strongly observable in a decentralized autonomous observation scheme if for system Σ_i^{RA} , having $y_i(t) = 0$ for $t \geq 0$ implies $x_i(t) = 0$, $\forall t \geq 0$ whatever the unknown inputs $u_i(t)$ and $\tilde{u}_j(t)$, $j \neq i$.

Similarly, state $x_i(t)$ is generically strongly observable in a decentralized interconnected observation scheme if for system Σ_i^{RA} , having $y_i(t) = 0$ and $\tilde{y}^i(t) = 0$ for $t \geq 0$ implies that $x_i(t) = 0$, $\forall t \geq 0$ whatever the unknown inputs $u_i(t)$ and $\tilde{u}_j(t)$, $j \neq i$.

The aim of this paper is firstly to characterize graphically the strong observability of the each subsystems Σ_i^R for both the considered observation schemes. Then, assuming that each subsystem is strongly observable if it is not submitted to external physical actions from other subsystems, we study the problem of data distribution over the network.

3. GRAPH REPRESENTATION OF STRUCTURED LINEAR SYSTEMS

3.1 Digraph definition for structured linear system

To structured system Σ_Λ constituted by subsystems Σ_i^R , $i = 1, \dots, N$, we associate a digraph noted $\mathcal{G}(\Sigma_\Lambda)$ which is constituted by a vertex set \mathcal{V} and an edge set \mathcal{E} . More

precisely, $\mathcal{V} = \bigcup_{i=1}^N (\mathbf{X}_i \cup \mathbf{U}_i \cup \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)$, where $\mathbf{X}_i =$

$\{\mathbf{x}_{i,1}, \dots, \mathbf{x}_{i,n_i}\}$ is the set of state vertices for subsystem i , $\mathbf{U}_i = \{\mathbf{u}_{i,1}, \dots, \mathbf{u}_{i,q_i}\}$ is the set of input vertices for subsystem i , $\mathbf{Y}_i = \{\mathbf{y}_{i,1}, \dots, \mathbf{y}_{i,p_i}\}$ is the set of output vertices for subsystem i , $\tilde{\mathbf{Y}}_i = \{\tilde{\mathbf{y}}_{i,1}, \dots, \tilde{\mathbf{y}}_{i,\tilde{p}_i}\}$ is the set of output vertices associated to the measurements arriving through the network to

subsystem i . The edge set is

$$\mathcal{E} = \bigcup_{i=1}^N \left(A_i\text{-edges} \cup B_i\text{-edges} \cup C_i\text{-edges} \cup D_i\text{-edges} \right.$$

$$\left. \cup_{j=1, j \neq i}^N \left(A_{i,j}\text{-edges} \cup B_{i,j}\text{-edges} \cup \tilde{C}_{i,j}\text{-edges} \cup \tilde{D}_{i,j}\text{-edges} \right) \right),$$

where

$$A_i\text{-edges} = \{(\mathbf{x}_{i,j}, \mathbf{x}_{i,k}) \mid A_i^\lambda(k, j) \neq 0\},$$

$$B_i\text{-edges} = \{(\mathbf{u}_{i,h}, \mathbf{x}_{i,\ell}) \mid B_i^\lambda(\ell, h) \neq 0\},$$

$$\text{for } j \neq i, A_{i,j}\text{-edges} = \{(\mathbf{x}_{j,\ell}, \mathbf{x}_{i,h}) \mid A_{i,j}^\lambda(h, \ell) \neq 0\},$$

$$B_{i,j}\text{-edges} = \{(\mathbf{u}_{j,\ell}, \mathbf{x}_{i,h}) \mid B_{i,j}^\lambda(h, \ell) \neq 0\},$$

$$C_i\text{-edges} = \{(\mathbf{x}_{i,h}, \mathbf{y}_{i,\ell}) \mid C_i^\lambda(\ell, h) \neq 0\},$$

$$D_i\text{-edges} = \{(\mathbf{u}_{i,h}, \mathbf{y}_{i,\ell}) \mid D_i^\lambda(\ell, h) \neq 0\},$$

$$\text{for } j \neq i, \tilde{C}_{i,j}\text{-edges} = \{(\mathbf{x}_{j,\ell}, \tilde{\mathbf{y}}_{i,h}) \mid \tilde{C}_{i,j}^\lambda(h, \ell) \neq 0\}$$

$$\text{and } \tilde{D}_{i,j}\text{-edges} = \{(\mathbf{u}_{j,\ell}, \tilde{\mathbf{y}}_{i,h}) \mid \tilde{D}_{i,j}^\lambda(h, \ell) \neq 0\}.$$

Here, $M(i, j)$ is the (i, j) th element of matrix M and $(\mathbf{v}_1, \mathbf{v}_2)$ denotes a directed edge from vertex $\mathbf{v}_1 \in \mathcal{V}$ to vertex $\mathbf{v}_2 \in \mathcal{V}$.

Example 3. Consider the following structured distributed system constituted of three subsystems:

Subsystem 1:

$$A_1^\lambda = \begin{pmatrix} 0 & \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 \\ 0 & 0 & 0 & 0 & \lambda_4 & 0 \\ 0 & \lambda_5 & 0 & 0 & 0 & 0 \end{pmatrix}, A_{1,2}^\lambda = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, A_{1,3}^\lambda = \begin{pmatrix} 0 & 0 \\ 0 & \lambda_7 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$B_1^\lambda = \begin{pmatrix} 0 \\ \lambda_8 \\ 0 \\ 0 \\ 0 \end{pmatrix}, B_{1,2}^\lambda = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \lambda_9 \end{pmatrix}, C_1^\lambda = \begin{pmatrix} \lambda_{10} & 0 & \lambda_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{12} & \lambda_{13} & 0 \\ 0 & 0 & \lambda_{14} & 0 & 0 & 0 \end{pmatrix},$$

$$B_{1,3}^\lambda = 0 \text{ and } D_1^\lambda = 0.$$

Subsystem 2:

$$A_2^\lambda = \begin{pmatrix} 0 & \lambda_{15} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda_{16} \end{pmatrix}, A_{2,1}^\lambda = A_{2,3}^\lambda = 0, B_2^\lambda = \begin{pmatrix} 0 \\ \lambda_{17} \\ \lambda_{18} \end{pmatrix}, B_{2,1}^\lambda = B_{2,3}^\lambda = 0, C_2^\lambda = \begin{pmatrix} \lambda_{19} & 0 & 0 \\ 0 & 0 & \lambda_{20} \end{pmatrix} \text{ and } D_2^\lambda = \begin{pmatrix} 0 \\ \lambda_{21} \end{pmatrix}.$$

Subsystem 3:

$$A_3^\lambda = \begin{pmatrix} 0 & \lambda_{22} \\ 0 & 0 \end{pmatrix}, A_{3,1}^\lambda = 0, A_{3,2}^\lambda = \begin{pmatrix} 0 & 0 & \lambda_{23} \\ 0 & 0 & \lambda_{24} \end{pmatrix}, B_3^\lambda = \begin{pmatrix} 0 \\ \lambda_{25} \end{pmatrix}, B_{3,1}^\lambda = B_{3,2}^\lambda = 0, C_3^\lambda = \begin{pmatrix} \lambda_{26} & 0 \\ 0 & 0 \end{pmatrix} \text{ and } D_3^\lambda = \begin{pmatrix} 0 \\ \lambda_{27} \end{pmatrix}.$$

To such a model, we associate the digraph of figure 1. For a sake of simplicity in this example, the set of output vertices associated to the measurements \tilde{Y} arriving through the network to each subsystem are not represented.

3.2 Notations and definitions

- Two edges $e_1 = (\mathbf{v}_1, \mathbf{v}'_1)$ and $e_2 = (\mathbf{v}_2, \mathbf{v}'_2)$ are v -disjoint if $\mathbf{v}_1 \neq \mathbf{v}_2$ and $\mathbf{v}'_1 \neq \mathbf{v}'_2$. Some edges are v -disjoint if they are mutually v -disjoint.

- Path P containing vertices $\mathbf{v}_{r_0}, \dots, \mathbf{v}_{r_i}$ is denoted $P = \mathbf{v}_{r_0} \rightarrow \mathbf{v}_{r_1} \rightarrow \dots \rightarrow \mathbf{v}_{r_i}$, where $(\mathbf{v}_{r_j}, \mathbf{v}_{r_{j+1}}) \in \mathcal{E}$ for $j = 0, 1, \dots, i-1$. We say that P covers $\mathbf{v}_{r_0}, \mathbf{v}_{r_1}, \dots, \mathbf{v}_{r_i}$.

- A path is simple when every vertex occurs only once in this path. Some paths are disjoint if they have no common vertex. Path P is a \mathbf{Y} -topped path if its end vertex belongs to \mathbf{Y} .

Let \mathcal{V}_1 and \mathcal{V}_2 denote two subsets of \mathcal{V} .

- The cardinality of \mathcal{V}_1 is noted $\text{card}(\mathcal{V}_1)$.

- $\theta(\mathcal{V}_1, \mathcal{V}_2)$ is the maximal number of v -disjoint edges from \mathcal{V}_1 to \mathcal{V}_2 .

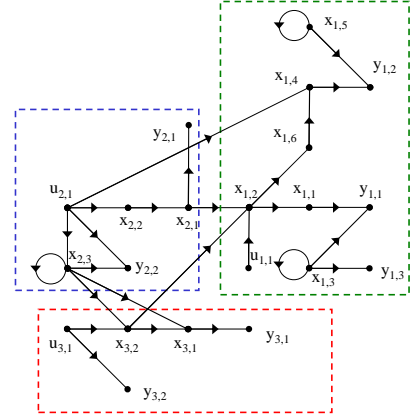


Figure 1. Digraph associated to Example 3

- A path P is said a \mathcal{V}_1 - \mathcal{V}_2 path if its begin vertex belongs to \mathcal{V}_1 and its end vertex belongs to \mathcal{V}_2 . If the only vertices of P belonging to $\mathcal{V}_1 \cup \mathcal{V}_2$ are its begin and its end vertices, P is said a direct \mathcal{V}_1 - \mathcal{V}_2 path.

- A set of ℓ disjoint \mathcal{V}_1 - \mathcal{V}_2 paths is called a \mathcal{V}_1 - \mathcal{V}_2 linking of size ℓ . The linkings, which consist of a maximal number of disjoint \mathcal{V}_1 - \mathcal{V}_2 paths, are called maximum \mathcal{V}_1 - \mathcal{V}_2 linkings. We define by $\rho(\mathcal{V}_1, \mathcal{V}_2)$ the size of these maximum \mathcal{V}_1 - \mathcal{V}_2 linkings.

- $V_{ess}(\mathcal{V}_1, \mathcal{V}_2) \stackrel{def}{=} \{\mathbf{v} \in \mathcal{V} \mid \mathbf{v} \text{ is covered by every maximum } \mathcal{V}_1\text{-}\mathcal{V}_2 \text{ linking}\}$.

- $\mathbf{S} \subseteq \mathcal{V}$ is a separator between sets \mathcal{V}_1 and \mathcal{V}_2 if every path from \mathcal{V}_1 to \mathcal{V}_2 contains at least one vertex in \mathbf{S} . We call minimum separators between \mathcal{V}_1 and \mathcal{V}_2 any separators having the smallest size. According to Menger's Theorem, the latter is equal to $\rho(\mathcal{V}_1, \mathcal{V}_2)$.

- There exist two uniquely determined minimum separators between \mathcal{V}_1 and \mathcal{V}_2 noted $\mathbf{S}^o(\mathcal{V}_1, \mathcal{V}_2)$ such that:

- $\mathbf{S}^o(\mathcal{V}_1, \mathcal{V}_2)$ is the set of begin vertices of all direct $V_{ess}(\mathcal{V}_1, \mathcal{V}_2)$ - \mathcal{V}_2 paths, where $V_{ess}(\mathcal{V}_1, \mathcal{V}_2) \cap \mathcal{V}_2$ is considered, in the present definition, as input vertices. Vertex subset $\mathbf{S}^o(\mathcal{V}_1, \mathcal{V}_2)$ is called the minimum output separator.

- $\mathbf{S}^i(\mathcal{V}_1, \mathcal{V}_2)$ is the set of begin vertices of all direct \mathcal{V}_1 - $V_{ess}(\mathcal{V}_1, \mathcal{V}_2)$ paths, where $V_{ess}(\mathcal{V}_1, \mathcal{V}_2) \cap \mathcal{V}_2$ is considered, in the present definition, as input vertices. Vertex subset $\mathbf{S}^i(\mathcal{V}_1, \mathcal{V}_2)$ is called the minimum input separator.

4. MAIN RESULTS

We first recall hereafter the graphic conditions which ensure the generic strong observability of a classical structured linear systems Boukhobza et al. [2007]:

Proposition 1. Structured linear system Σ_Λ is generically strongly observable iff in its associated digraph $\mathcal{G}(\Sigma_\Lambda)$

Cond1. $\theta(\mathbf{X} \cup \mathbf{U}, \mathbf{X} \cup \mathbf{Y}) = n + \theta(\mathbf{U}, \mathbf{X} \cup \mathbf{Y})$;

Cond2. every state vertex is the begin vertex of a \mathbf{Y} -topped path;

Cond3. $\{\mathbf{x}_i \mid \rho(\mathbf{U} \cup \{\mathbf{x}_i\}, \mathbf{Y}) = \rho(\mathbf{U}, \mathbf{Y})\} \subseteq V_{ess}(\mathbf{U}, \mathbf{Y} \cup \mathbf{X})$.

4.1 Strong observability conditions for networked systems

Let us now consider that the structured linear system Σ_Λ is a distributed system and constituted of several subsystems Σ_i^R , $i = 1, \dots, N$. We apply the previous result in order to obtain the strong observability conditions of a given subsystem Σ_i^R for both decentralized autonomous and decentralized interconnected observation schemes. We can consider for subsystem

Σ_i^R that the unknown input set is constituted by its own unknown input set plus all the state and unknown components related to the other subsystems Σ_j^R , $j = 1, \dots, i-1, i+1, \dots, N$ which act on Σ_i^R . Let us denote by $\tilde{\mathbf{U}}_i = \{\mathbf{v} \in \bigcup_{j=1, j \neq i}^N (\mathbf{U}_j \cup \mathbf{X}_j) \text{ such that } \theta(\{\mathbf{v}\}, \mathbf{X}_i) \neq 0\}$ and $\bar{\mathbf{U}}_i = \tilde{\mathbf{U}}_i \cup \mathbf{U}_i$. Thus, when we study subsystem Σ_i^R , we consider in $\mathcal{G}(\Sigma_\Lambda)$ that all the edges ending by an element of $\tilde{\mathbf{U}}_i$ are removed. This digraph is similar to the one of Σ_i^{RA} .

When we deal with the autonomous scheme, only the measurements done on Σ_i^R are available for the local observer dedicated to Σ_i^R . Thus, we deduce from Proposition 1:

Proposition 2. Consider structured system Σ_Λ represented by digraph $\mathcal{G}(\Sigma_\Lambda)$ and constituted by subsystems Σ_i^R , $i = 1, \dots, N$. Subsystem Σ_i^R is generically strongly observable in an autonomous scheme iff in its associated digraph \mathcal{G}_i

Cond1. $\theta(\mathbf{X}_i \cup \bar{\mathbf{U}}_i, \mathbf{X}_i \cup \mathbf{Y}_i) = n_i + \theta(\bar{\mathbf{U}}_i \cup \mathbf{X}_i, \mathbf{X}_i \cup \mathbf{Y}_i)$ (maximal matching condition).

Cond2. every state vertex in \mathbf{X}_i is the begin vertex of a \mathbf{Y}_i -topped path (output connectivity condition);

Cond3. $\{\mathbf{x}_\ell \in \mathbf{X}_i \mid \rho(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\}, \mathbf{Y}_i) = \rho(\bar{\mathbf{U}}_i, \mathbf{Y}_i)\} \subseteq V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i)$ (length condition).

Concerning the subsystems of Example 3, we have:

Subsystem 1 has 6 state components ($n_1 = 6$). It is submitted to one internal unknown input: $\mathbf{u}_{1,1}$ and three external inputs: $\mathbf{x}_{2,1}$, $\mathbf{u}_{2,1}$ from Subsystem 2 and $\mathbf{x}_{3,2}$ from Subsystem 3 and so $\bar{\mathbf{U}}_1 = \{\mathbf{u}_{1,1}, \mathbf{x}_{2,1}, \mathbf{u}_{2,1}, \mathbf{x}_{3,2}\}$, $\mathbf{X}_1 = \{\mathbf{x}_{1,1}, \mathbf{x}_{1,2}, \mathbf{x}_{1,3}, \mathbf{x}_{1,4}, \mathbf{x}_{1,5}, \mathbf{x}_{1,6}\}$, $\mathbf{Y}_1 = \{\mathbf{y}_{1,1}, \mathbf{y}_{1,2}, \mathbf{y}_{1,3}\}$. We have that $\theta(\bar{\mathbf{U}}_1, \mathbf{X}_1 \cup \mathbf{Y}_1) = 2$ and $\theta(\bar{\mathbf{U}}_1 \cup \mathbf{X}_1, \mathbf{X}_1 \cup \mathbf{Y}_1) = 7 \neq n_1 + \theta(\bar{\mathbf{U}}_1, \mathbf{X}_1 \cup \mathbf{Y}_1) = 8$. Thus, the maximal matching condition is not satisfied and Subsystem 1 is not generically strongly observable in an autonomous observation scheme. Note that the output connectivity condition is satisfied. Finally, we have that $\mathbf{u}_{2,1}$, $\mathbf{x}_{1,1}$, $\mathbf{x}_{1,2}$, $\mathbf{x}_{1,4}$, $\mathbf{y}_{1,1}$ and $\mathbf{y}_{1,2}$ constitute $V_{ess}(\bar{\mathbf{U}}_1, \mathbf{Y}_1)$. On the other hand, $\rho(\bar{\mathbf{U}}_1 \cup \{\mathbf{x}_{1,5}\}, \mathbf{Y}_1) = \rho(\bar{\mathbf{U}}_1 \cup \{\mathbf{x}_{1,6}\}, \mathbf{Y}_1) = \rho(\bar{\mathbf{U}}_1, \mathbf{Y}_1) = 2$. Yet, neither $\mathbf{x}_{1,5}$ nor $\mathbf{x}_{1,6}$ are essential. Thus, the length condition is also not satisfied.

Subsystem 2 is not submitted to inputs from other subsystems. For such subsystem, $\mathbf{X}_2 = \{\mathbf{x}_{2,1}, \mathbf{x}_{2,2}, \mathbf{x}_{2,3}\}$, $\mathbf{Y}_2 = \{\mathbf{y}_{2,1}, \mathbf{y}_{2,2}\}$ and $\bar{\mathbf{U}}_2 = \mathbf{U}_2 = \{\mathbf{u}_{2,1}\}$. We have that $\theta(\bar{\mathbf{U}}_2, \mathbf{X}_2 \cup \mathbf{Y}_2) = 2$ and $\theta(\bar{\mathbf{U}}_2 \cup \mathbf{X}_2, \mathbf{X}_2 \cup \mathbf{Y}_2) = 4 = n_2 + \theta(\bar{\mathbf{U}}_2, \mathbf{X}_2 \cup \mathbf{Y}_2)$. Thus, the maximal matching condition is satisfied. Furthermore, the output connectivity condition is also satisfied. Finally, $\rho(\mathbf{U}_2, \mathbf{Y}_2) = 1$ and $V_{ess}(\bar{\mathbf{U}}_2, \mathbf{Y}_2) = \{\mathbf{u}_{2,1}\}$ but $\rho(\mathbf{U}_2 \cup \{\mathbf{x}_{2,1}\}, \mathbf{Y}_2) = \rho(\mathbf{U}_2 \cup \{\mathbf{x}_{2,2}\}, \mathbf{Y}_2) = 2$, which implies that the length condition is also satisfied. Therefore Subsystem 2 is generically strongly observable in an autonomous scheme.

Subsystem 3 is submitted to one internal input: $\mathbf{u}_{3,1}$ and one external input from Subsystem 2: $\mathbf{x}_{2,3}$. $\mathbf{X}_3 = \{\mathbf{x}_{3,1}, \mathbf{x}_{3,2}\}$, $\mathbf{Y}_3 = \{\mathbf{y}_{3,1}, \mathbf{y}_{3,2}\}$, $\bar{\mathbf{U}}_3 = \{\mathbf{u}_{3,1}, \mathbf{x}_{2,3}\}$. On the one hand, $\theta(\bar{\mathbf{U}}_3, \mathbf{X}_3 \cup \mathbf{Y}_3) = 2$ and $\theta(\bar{\mathbf{U}}_3 \cup \mathbf{X}_3, \mathbf{X}_3 \cup \mathbf{Y}_3) = 4 = n_3 + \theta(\bar{\mathbf{U}}_3, \mathbf{X}_3 \cup \mathbf{Y}_3)$. Thus, the maximal matching condition is satisfied. Furthermore, the output connectivity condition is also satisfied. Moreover, $\rho(\bar{\mathbf{U}}_3, \mathbf{Y}_3) = 2$ and $V_{ess}(\bar{\mathbf{U}}_3, \mathbf{Y}_3) = \{\mathbf{u}_{3,1}, \mathbf{x}_{2,3}, \mathbf{x}_{3,1}, \mathbf{y}_{3,1}, \mathbf{y}_{3,2}\}$. Yet, $\rho(\bar{\mathbf{U}}_3 \cup \{\mathbf{x}_{3,2}\}, \mathbf{Y}_3) = 2$

and $\mathbf{x}_{3,2} \notin V_{ess}(\bar{\mathbf{U}}_3, \mathbf{Y}_3)$. Thus, the length condition is not satisfied and Subsystem 3.

In the next part of the paper we will interest in determining what external measurements must be transmitted via the network to a subsystem in order to make it strongly observable in a distributed interconnected observation scheme. At this aim, we first give the conditions of strong observability for this scheme:

Proposition 3. Consider structured system Σ_Λ represented by digraph $\mathcal{G}(\Sigma_\Lambda)$ and constituted by subsystems Σ_i^R , $i = 1, \dots, N$. Subsystem Σ_i^R is generically strongly observable in an interconnected (I.C.) scheme iff in its associated digraph $\mathcal{G}_i(\Sigma_\Lambda)$

Cond1. $\theta(\mathbf{X}_i \cup \bar{\mathbf{U}}_i, \mathbf{X}_i \cup \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) = n_i + \theta(\bar{\mathbf{U}}_i, \mathbf{X}_i \cup \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)$ (I.C. maximal matching condition).

Cond2. every state vertex in \mathbf{X}_i is the begin vertex of a \mathbf{Y}_i -topped path (output connectivity condition);

Cond3. $\{\mathbf{x}_\ell \in \mathbf{X}_i \mid \rho(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\}, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) = \rho(\bar{\mathbf{U}}_i, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)\} \subseteq V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)$ (I.C. length condition).

Consider Example3, and assume that the measurement $\tilde{y}_{3,1} = \mathbf{x}_{2,3}$ is transmitted to Subsystem 3. In this case, $V_{ess}(\bar{\mathbf{U}}_3, \mathbf{Y}_3 \cup \tilde{\mathbf{Y}}_3) = \{\mathbf{u}_{3,1}, \mathbf{x}_{2,3}, \mathbf{y}_{3,2}\}$ and so the I.C. length condition is satisfied. Hence, Subsystem 3 is generically strongly observable in an interconnected scheme when it receives via the network the measurement $\tilde{y}_{3,1} = \mathbf{x}_{2,3}$.

4.2 Necessary external measurements transmitted via network for the strong observability

Let us assume that each subsystem is strongly locally observable or in other words that subsystems

$$\begin{cases} \dot{x}_i(t) = A_i^\lambda x_i(t) + B_i^\lambda u_i(t) \\ y_i(t) = C_i^\lambda x_i(t) + D_i^\lambda u_i(t) \end{cases} \quad i = 1, \dots, N$$

are strongly observable. If it is not the case, we must add some local measurements. This aspect has been already treated (Boukhobza and Hamelin [2009]) and have no major relevance for the present study because is not related to networked systems. In this case, the following conditions are true:

• $\theta(\mathbf{X}_i \cup \mathbf{U}_i, \mathbf{X}_i \cup \mathbf{Y}_i) = n_i + \theta(\mathbf{U}_i, \mathbf{X}_i \cup \mathbf{Y}_i)$ (local maximal matching condition).

• every state vertex in \mathbf{X}_i is the begin vertex of a \mathbf{Y}_i -topped path (output connectivity condition);

• $\{\mathbf{x}_\ell \in \mathbf{X}_i \mid \rho(\mathbf{U}_i \cup \{\mathbf{x}_\ell\}, \mathbf{Y}_i) = \rho(\mathbf{U}_i, \mathbf{Y}_i)\} \subseteq V_{ess}(\mathbf{U}_i, \mathbf{Y}_i)$ (local length condition).

Note that all subsystems of Example 3 satisfy the conditions above and are strongly observable if they are not submitted to external inputs.

The aim of this subsection is to study what are the measurements which must be transmitted to subsystem Σ_i^R in order to recover the strong observability in the distributed interconnected observation scheme. More precisely, we start from a subsystem which has enough local sensors in order to guarantee that it is strongly observable if it is not submitted to the physical interactions from other subsystems. Our objective is to provide, for each subsystem, the set of external measurements represented by $\tilde{\mathbf{Y}}_i$ which allow to recover the conditions of Proposition 3, knowing that the output connectivity condition is already satisfied since each subsystem is strongly locally observable. To answer to this question, our proposed procedure consists of two steps which correspond one by one to conditions **Cond1.** and **Cond3.** of Proposition3.

4.3 Transmitted external measurements for the I.C. maximal matching condition

The first objective of the measurement $\tilde{\mathbf{Y}}_i$ arriving to subsystem Σ_i^R is to obtain $\theta(\mathbf{X}_i \cup \bar{\mathbf{U}}_i, \mathbf{X}_i \cup \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) = n_i + \theta(\bar{\mathbf{U}}_i, \mathbf{X}_i \cup \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)$. This maximal matching condition is also equivalent to have $\forall \mathbf{V}_1 \subseteq \mathbf{X}_i \cup \bar{\mathbf{U}}_i$

$$\theta(\mathbf{V}_1, \mathbf{X}_i \cup \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) = \text{card}(\mathbf{V}_1 \cap \bar{\mathbf{X}}_i) + \theta(\mathbf{V}_1 \cap \bar{\mathbf{U}}_i, \mathbf{X}_i \cup \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) \quad (4)$$

When the latter condition is not satisfied for some vertex subset \mathbf{V}_1 , we say that there is dilation in the digraph of the system. The aim of transmitted measures at this first stage is to eliminate all these dilations. As in Commault et al. [2008], we also use the Dulmage-Mendelsohn decomposition. Hence, we define a bipartite graph in order to localize the dilations occurring in the digraph of subsystem Σ_i^R . This bipartite graph is noted $B_i(\Sigma_\Lambda) = (V^+, V^-, W)$, where V^+ and V^- are two disjoint vertex subsets and W is the edge set. More precisely, $V^+ = \mathbf{X}_i^+ \cup \bar{\mathbf{U}}_i^+$ and $V^- = \mathbf{Y}_i^- \cup \mathbf{X}_i^-$, with $\mathbf{X}_i^+ = \{\mathbf{x}_{i,1}^+, \mathbf{x}_{i,2}^+, \dots, \mathbf{x}_{n_i,i}^+\}$, $\bar{\mathbf{U}}_i^+ = \{\bar{\mathbf{u}}_{i,1}^+, \bar{\mathbf{u}}_{i,2}^+, \dots, \bar{\mathbf{u}}_{q_i,i}^+\}$, $\mathbf{X}_i^- = \{\mathbf{x}_{i,1}^-, \mathbf{x}_{i,2}^-, \dots, \mathbf{x}_{i,n}^-\}$, $\mathbf{Y}_i^- = \{\mathbf{y}_{i,1}^-, \mathbf{y}_{i,2}^-, \dots, \mathbf{y}_{i,p}^-\}$. Edge set W is defined such that $(\mathbf{v}_i^+, \mathbf{v}_j^-) \in W$ iff there exists an edge $(\mathbf{v}_i, \mathbf{v}_j)$ in the associated digraph $\mathcal{G}(\Sigma_\Lambda)$. A matching in a bipartite graph $B_i(\Sigma_\Lambda) = (V^+, V^-, W)$ is an edge set $M \subseteq W$ such that all the edges of M are disjoint. A matching is maximal if it has a maximal cardinality which is equal to $\theta(V^+, V^-)$ which is equal by construction of the digraph to $\theta(\mathbf{X}_i \cup \bar{\mathbf{U}}_i, \mathbf{X}_i \cup \mathbf{Y}_i)$. Then, the fact that condition **Cond1.** of Proposition 3 is not satisfied *i.e.* $\theta(V^+, V^-) < \text{card}(\mathbf{X}_i^+) + \theta(\bar{\mathbf{U}}_i^+, V^-)$, implies that some additional measurement vertices $\tilde{\mathbf{Y}}_i$ are needed to complete V^- . This number is less or equal to $\text{card}(V^+) - \theta(V^+, V^-)$. In addition to determining the exact number of required transmitted measurements, another problem is to precise as finely as possible because obviously transmitting any data does not allow to recover the maximal matching condition. At this aim, let us associate to each maximal matching M , a non bipartite digraph noted $B_M(\Sigma_\Lambda) = (V^+, V^-, \bar{W})$ where $(\mathbf{v}_1, \mathbf{v}_2) \in \bar{W} \Leftrightarrow (\mathbf{v}_1, \mathbf{v}_2) \in W$ or $(\mathbf{v}_2, \mathbf{v}_1) \in M$.

We denote by $\partial^+ M$ (resp. $\partial^- M$) the set of vertices in V^+ (resp. in V^-) covered by the edges of M and we note $\mathbf{S}_0^+ = V^+ \setminus \partial^+ M$. Then, we use the following algorithm:

- ↪ Find a maximal matching M in $B(\Sigma_\Lambda)$,
- ↪ $\bar{\mathbf{V}}_0^+ = \mathbf{S}_0^+ \cup \{\mathbf{v} \in V^+, \exists \text{ a path in } B_M(\Sigma_\Lambda) \text{ from } \mathbf{S}_0^+ \text{ to } \mathbf{v}\}$
- ↪ $\mathbf{V}_0^+ = \bar{\mathbf{V}}_0^+ \setminus \{\mathbf{v} \in \bar{\mathbf{V}}_0^+ \cap \bar{\mathbf{U}}_i^+, \nexists \text{ a path in } B_M(\Sigma_\Lambda) \text{ from } \mathbf{S}_0^+ \text{ to } \mathbf{v} \text{ or from } \mathbf{v} \text{ to } \mathbf{S}_0^+ \cap \mathbf{X}_i^+\}$
- ↪ $\mathbf{V}_0^- = \{\mathbf{v} \in V^-, \exists \text{ a path in } B_M(\Sigma_\Lambda) \text{ from } \mathbf{V}_0^+ \text{ to } \mathbf{v}\}$.

It is important to note that the obtained subsets \mathbf{V}_0^+ and \mathbf{V}_0^- are the same whatever the choice of the maximal matching M (Dulmage and Mendelsohn [1958], Murota [1987]). Using the previous algorithm, we have:

Proposition 4. Consider structured system Σ_Λ represented by digraph $\mathcal{G}(\Sigma_\Lambda)$ and constituted by subsystems Σ_i^R , $i = 1, \dots, N$. To recover the maximal matching condition, for subsystem Σ_i^R the minimal number of additional measures transmitted by the network, noted $\tilde{\mathbf{Y}}_i$, is equal to $\gamma = \text{card}(\mathbf{V}_0^+) - \text{card}(\mathbf{V}_0^-)$. These additional measurements γ concern unknown inputs in $\mathbf{V}_0^+ \cap \bar{\mathbf{U}}_i^+$ such that we obtain a maximal matching of size $\text{card}(\mathbf{V}_0^+)$ between \mathbf{V}_0^+ and $\mathbf{V}_0^- \cup \tilde{\mathbf{Y}}_i$.

Proof: the proof is similar to the one given in Commault et al. [2008], Boukhobza and Hamelin [2009]. \triangle

Let us now illustrate the previous settings by considering Subsystem 1 of Example 3. The bipartite graph associated to such system is presented in Figure 2.

↪ Select a maximal matching M in this bipartite graph, which

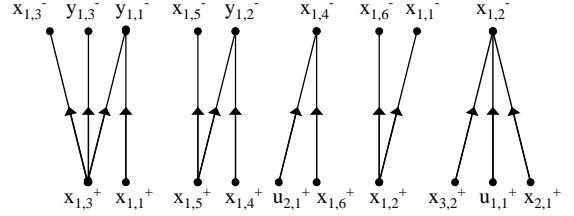


Figure 2. Bipartite graph associated to Subsystem 1 of Example 3

can be: $M = \{(\mathbf{x}_{1,3}^+, \mathbf{x}_{1,3}^-), (\mathbf{x}_{1,1}^+, \mathbf{y}_{1,1}^-), (\mathbf{x}_{1,5}^+, \mathbf{x}_{1,5}^-), (\mathbf{x}_{1,4}^+, \mathbf{y}_{1,2}^-), (\mathbf{u}_{2,1}^+, \mathbf{x}_{1,4}^-), (\mathbf{x}_{1,6}^+, \mathbf{x}_{1,6}^-), (\mathbf{x}_{3,2}^+, \mathbf{x}_{1,2}^-)\}$. In this case,

$$\mathbf{S}_0^+ = \{\mathbf{x}_{1,6}^+, \mathbf{u}_{1,1}^+, \mathbf{x}_{2,1}^+\}.$$

$$\Leftrightarrow \bar{\mathbf{V}}_0^+ = \{\mathbf{u}_{2,1}^+, \mathbf{x}_{1,6}^+, \mathbf{x}_{3,2}^+, \mathbf{u}_{1,1}^+, \mathbf{x}_{2,1}^+\}$$

$$\Leftrightarrow \mathbf{V}_0^+ = \{\mathbf{u}_{2,1}^+, \mathbf{x}_{1,6}^+\}.$$

$$\Leftrightarrow \mathbf{V}_0^- = \{\mathbf{x}_{1,4}^-\}.$$

Since $\text{card}(\mathbf{V}_0^+) - \text{card}(\mathbf{V}_0^-) = 1$, only one transmitted measurement is sufficient to guarantee the maximal matching condition. Moreover, the only possibility is to transmit $\mathbf{u}_{2,1}$ as it is the only external variable to Subsystem 1 in \mathbf{V}_0^+ . Note that in this case, the length condition has been also recovered by the transmission of $\mathbf{u}_{2,1}$ even if this subsystem is submitted to other external inputs.

It can be useful to specify more precisely the additional measurements transmitted to the considered subsystem to recover the I.C. maximal matching condition. At this aim, let us define by $\bar{B}(\Sigma_\Lambda)$ the non-directed graph corresponding to $B(\Sigma_\Lambda)$ and $\mathbf{V}_0 \stackrel{\text{def}}{=} \mathbf{V}_0^+ \cup \mathbf{V}_0^-$. We call \mathbf{V}_0 -path every simple path of $\bar{B}(\Sigma_\Lambda)$ which covers only vertices of \mathbf{V}_0 . For each \mathbf{V}_0 -path P , we define a vertex subset $\vartheta(P)$ such that $\vartheta(P) = \{\mathbf{v} \in \mathbf{V}_0^+, \text{ such that } P \text{ covers } \mathbf{v}\}$. Finally, we say that a \mathbf{V}_0 -path P is maximal, if there does not exist a \mathbf{V}_0 -path P' such that $\vartheta(P) \subset \vartheta(P')$. To recover the maximal matching condition, it is necessary to have:

$$\text{for each maximal } \mathbf{V}_0\text{-path } P, \theta(\vartheta(P), \tilde{\mathbf{Y}}_i^-) \neq 0 \quad (5)$$

Moreover to recover the maximal matching condition, it is necessary to have, for each maximal \mathbf{V}_0 -path P :

$$\theta(\mathbf{V}_0^+, \tilde{\mathbf{Y}}_i^- \cup \mathbf{V}_0^-) - \theta(\mathbf{V}_0^+ \setminus \vartheta(P), \tilde{\mathbf{Y}}_i^- \cup \mathbf{V}_0^-) > 0 \quad (6)$$

Conditions (5) and (6) ensure that there is at least one measurement in each subset $\vartheta(P) \cap \bar{\mathbf{U}}_i$. Indeed, $\vartheta(P)$ satisfy, by construction $\theta(\vartheta(P), \mathbf{V}_0^-) = \text{card}(\vartheta(P)) - 1$ and so necessitates the transmission of one measurement to recover the maximal matching condition.

4.4 Transmitted external measurements for the I.C. length condition

Assume that condition **Cond1.** of Proposition 3 is now verified. Hence, we may have transmit some measurements to subsystem Σ_i^R to the system in order to satisfy I.C. maximal matching condition. In this case, we can consider in the sequel that all the vertices associated to the measured external inputs of Σ_i^R are removed from $\bar{\mathbf{U}}_i$ and from the digraph as well as all the

edges beginning by these vertices.

Let us denote by $\Delta_i \stackrel{def}{=} \left\{ \mathbf{x}_\ell \in \mathbf{X}_i \mid \rho(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\}, \mathbf{Y}_i) = \rho(\bar{\mathbf{U}}_i, \mathbf{Y}_i) \right\}$. If Condition **Cond1.** of Proposition 3 is satisfied then it is easy to see that $\rho(\bar{\mathbf{U}}_i, \mathbf{Y}_i) = \theta(\bar{\mathbf{U}}_i, \mathbf{Y}_i)$. We can deduce that, whatever the transmitted measurements represented by vertex subset $\tilde{\mathbf{Y}}_i$, we have $V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) \cap \mathbf{X}_i \subseteq V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i) \cap \mathbf{X}_i$. Consequently, if an element \mathbf{x}_ℓ is in Δ_i but not in $V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i)$, then adding measures $\tilde{\mathbf{Y}}_i$ anywhere on $\bar{\mathbf{U}}_i$, we cannot obtain that $\mathbf{x}_\ell \in V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)$. Hence, the only way to ensure condition **Cond3.** of Proposition 3 is to remove from Δ_i all the elements which do not belong to $V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i)$.

According to this fact, to recover condition **Cond3.** of Proposition 3, it is necessary and sufficient to transmit some measurements $\tilde{\mathbf{Y}}_i$, such that, for each $\mathbf{x}_\ell \in \Delta_i \setminus V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i)$, we have $\rho(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\}, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) > \rho(\bar{\mathbf{U}}_i, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)$. Considering virtually \mathbf{x}_ℓ as an input vertex, a necessary and sufficient condition to guarantee inequality $\rho(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\}, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) > \rho(\bar{\mathbf{U}}_i, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)$ is that the added measurements $\tilde{\mathbf{Y}}_i$ concern only input vertices covered by any direct $\bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\}$ - $\mathbf{S}^i(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_i\}, \mathbf{Y}_i)$ path which has a nonzero length (Commaut and Dion [2007]). But there can exist some edges arriving to \mathbf{x}_ℓ since the latter is not really an input vertex. Thus, the only other measurements which can guarantee inequality $\rho(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\}, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) > \rho(\bar{\mathbf{U}}_i, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)$ are also the ones done on any direct $\bar{\mathbf{U}}_i \cup \{\mathbf{x}_k\}$ - $\mathbf{S}^i(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_i\}, \mathbf{Y}_i)$ where \mathbf{x}_k is such that $\rho(\{\mathbf{x}_k\}, \{\mathbf{x}_\ell\}) = \rho(\{\mathbf{x}_\ell\}, \{\mathbf{x}_k\}) = 1$. Finally, since $\tilde{\mathbf{Y}}_i$ represent measurements transmitted via the network concerning other subsystems, they concern only the unknown input vertices $\bar{\mathbf{U}}_i$. Thus, considering \mathbf{x}_ℓ or all the \mathbf{x}_k such that $\rho(\{\mathbf{x}_k\}, \{\mathbf{x}_\ell\}) = \rho(\{\mathbf{x}_\ell\}, \{\mathbf{x}_k\}) = 1$, the elements of $\bar{\mathbf{U}}_i$ which covered by any direct $\bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\}$ - $\mathbf{S}^i(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_i\}, \mathbf{Y}_i)$ path are the same than the ones covered by any direct $\bar{\mathbf{U}}_i \cup \{\mathbf{x}_k\}$ - $\mathbf{S}^i(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_i\}, \mathbf{Y}_i)$ path. Thus, it is necessary and sufficient to consider virtually all the vertices belonging to $\Delta_i \setminus V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i)$ as input vertices.

Let us define, for each $\mathbf{x}_\ell \in \Delta_i \setminus V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i)$, vertex subset $\delta_\ell \stackrel{def}{=} \left\{ \mathbf{u}_j \in \bar{\mathbf{U}}_i, \mathbf{u}_j \text{ is the begin vertex of some direct } \bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\} - \mathbf{S}^i(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_i\}, \mathbf{Y}_i) \text{ path, such that } \rho(\bar{\mathbf{U}}_i \cup \{\mathbf{x}_\ell\}, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i) > \rho(\bar{\mathbf{U}}_i \setminus \{\mathbf{u}_j\}, \mathbf{Y}_i) \right\}$. We can state:

Proposition 5. Consider structured system Σ_Λ represented by digraph $\mathcal{G}(\Sigma_\Lambda)$ and constituted by subsystems Σ_i^R , $i = 1, \dots, N$. Assume that each subsystem is strongly locally observable. To recover the I.C. length condition of Proposition 3, for subsystem Σ_i^R , the measurements transmitted by the network, noted $\tilde{\mathbf{Y}}_i$ must concern all the external components in $\mathbf{V}_U \stackrel{def}{=} \bigcup_{\mathbf{x}_\ell \in \Delta_i \setminus V_{ess}(\bar{\mathbf{U}}_i, \mathbf{Y}_i)} \delta_\ell$ such that $\mathbf{V}_U \subseteq \mathbf{S}^o(\bar{\mathbf{U}}_i, \mathbf{Y}_i \cup \tilde{\mathbf{Y}}_i)$.

Proof: The proof is immediate from the previous settings. \triangle

5. CONCLUSION

An important problem that must be considered when dealing with control over network, is the validity of some properties as the observability. For network distributed systems, an alternative to the centralized observation scheme, which can be

quite complicated to realize when we deal with a large scale system, is to consider a decentralized distributed observation scheme or a completely autonomous observation scheme. The first scheme corresponds to the case when the subsystem is connected to the network and receive some informations from the other subsystems. The second scheme is related to the case when the subsystem have only its own measurements to reconstruct a part of the state and input components as in a network cut for example. In this paper, we propose first an analysis tool to study the generic strong observability of network distributed structured linear systems in both distributed interconnected and distributed autonomous schemes. In a second part of the paper, we give a procedure of measurement distribution through the network. More precisely, for any given subsystem Σ_i^R , we indicate quite precisely, which external measurement must be transmitted through the network from the other subsystem in order to ensure the strong observability of Σ_i^R in a decentralized interconnected observation scheme.

Our approach is based on a graph-theoretic approach, and needs few information about the system. The provide results are very easy to check by means of well-known combinatorial techniques and simply by hand for small systems. Indeed, from a computational point of view, our proposed approach needs few information about the system and is quite easy to check by means of well-known combinatorial techniques or simply by hand for small systems. Indeed, it uses classical programming techniques like Ford-Fulkerson algorithm to compute the input separators in a digraph and Dulmage-Mendelsohn decomposition of a bipartite graph. These algorithms are free from numerical difficulties and lead polynomial complexity order implementations.

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