Bit rate maximization loading algorithm with mean BER-constraint for linear precoded OFDM

Fahad Syed Muhammad, Jean-Yves Baudais, Jean-François Hélard

Bit Rate Maximization Loading Algorithm with Mean BER-Constraint for Linear Precoded OFDM

Fahad Syed Muhammad*, Antoine Stephan, Jean-Yves Baudais, and Jean-François Hélard
Institute of Electronics and Telecommunications of Rennes (IETR), 35043 Rennes Cedex, France
Email*: fahad.syed-muhammad@insa-rennes.fr

***Abstract*—The bit loading algorithms are proposed for orthogonal frequency division multiplexing (OFDM) and linear precoded OFDM under power spectral density and mean bit error rate (BER) constraints. These algorithms maximize the bit rate of the system while respecting a mean BER. The advantages of both linear procoded OFDM and classical OFDM are analyzed under different scenarios. It is shown that the proposed allocations perform better than the allocations under peak error rate constraint, while respecting a power spectral density constraint. The performance of the proposed algorithms is also better than the previously proposed algorithm under mean BER constraint. Moreover, it is observed that the algorithms for linear precoded OFDM under mean BER constraint have significantly lower computational complexity.**

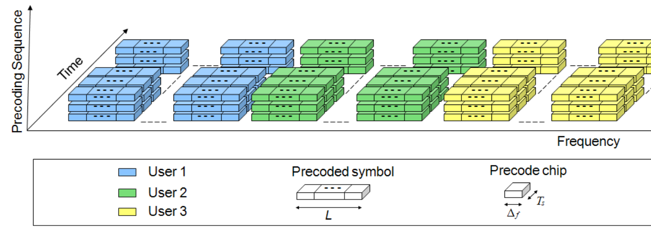

Fig. 1. LP-OFDM system description.

I. Introduction

In multicarrier modulation (MCM), with known channel state information, it is advantageous to adaptively modulate different subcarriers. This process is sometimes known as bit loading [1]. Orthogonal frequency-division multiplexing (OFDM) employs the fast Fourier transform for spectral decomposition and is the main MCM technique. Generally, each subcarrier is assigned a suitable energy, driven by the signal-to-noise ratio (SNR), and is loaded with a given modulation, such as various constellations of quadrature amplitude modulation (QAM). Linear precoded orthogonal frequency-division multiplexing (LP-OFDM) is based on classical OFDM combined with a precoding component. The idea is to group together a set of subcarriers with the help of precoding sequences. Each resulting group accumulates the energies of all of its subcarriers to achieve an equivalent SNR.

The resource allocation problem can be formulated under total power constraint [2] as well as under power spectral density (PSD) constraint [3]. In this paper, we will only discuss the formulations subject to PSD constraint, which means that the transmitted power has to obey a certain spectral mask and constant available power is assumed over all subcarriers. The bit loading is generally performed with fixed error rate on every subcarrier to attain a target error rate of the system [1]. In order to achieve a target error rate, we allow different subcarriers to carry different values of the bit error rate (BER), and we put a limit on the mean BER of an OFDM symbol, as proposed in [4] and [5]. This provides some flexibility to increase the throughput of the system under PSD constraint.

In this paper, we propose discrete rate-adaptive loading algorithms for OFDM and LP-OFDM systems under PSD and mean BER constraints. The algorithm proposed for the OFDM system is a modified version of the algorithm of Wyglinski [5]. This modified approach is further extended to the LP-OFDM system. The advantages of these systems are observed under different scenarios in a power line communication (PLC) context. The uncoded QAM is selected as the modulation scheme. This work may also be extended to the study of the coded systems and it might be a promising extension in the future.

The rest of the paper is organized as follows. In Section II, the structure of an LP-OFDM system is described. Section III gives the modified bit loading algorithm for an OFDM system. In Section IV, the bit loading algorithm for an LP-OFDM system is devised. In Section V, simulation scenarios are discussed and results are presented. Finally, Section VI concludes the paper.

II. System Description

A general LP-OFDM scheme is based on classical OFDM combined with a linear precoding component. The resulting LP-OFDM, whose precoding is applied in the frequency dimension, is known as spread-spectrum multicarrier multiple-access (SS-MC-MA) in mobile radio communications [7]. In practice, the system is modified by simply adding a precoding block in the transmission chain, thus the system complexity is not significantly increased. Furthermore, the linear precoded component can be exploited to reduce the peak-to-average power ratio (PAPR) of the OFDM system [8]. The linear precoding component improves the signal robustness against frequency selectivity and narrowband interference, since the signal bandwidth could become much larger than the coherence and interference bandwidths. It also accumulates the energies of many subcarriers by grouping them together which is useful in increasing the throughput especially under PSD constraint.

The studied LP-OFDM system is shown in Fig. 1. The entire bandwidth is divided into N parallel subcarriers which are split up into K groups S_k of L subcarriers, where k signifies the group number. The precoding function is then applied block-wise by mean of precoding sequences of length L also known as precoding factor. Note that the subcarriers in a given group are not necessarily adjacent. Each user u of the network is being assigned a group B_u of subsets S_k. We emphasize that $\forall u,\ B_u$ are mutually exclusive subsets. Consequently, multiple access between the U users is managed following a frequency division multiple access (FDMA) approach, instead of a code division multiple access (CDMA) approach. In a general approach, the generated symbol vector at the output of the OFDM modulator for an LP-OFDM system can be written as

$$s = F^H M X. \tag{1}$$

Vector s is N-dimensional, with N the number of used subcarriers. $X = [x_1, \cdots, x_K]^T$ is the output of the serial-to-parallel conversion of the K QAM modulated symbols to be transmitted. M represents the precoding matrix of size $N \times K$ applied to X, which precodes K symbols over the N subcarriers. This precoding matrix is composed of orthogonal Hadamard matrices. Finally, F^H represents the Hermitian of the unitary Fourier matrix of size $N \times N$ that realizes the multicarrier modulation. It is worthy to mention here that we are going to consider the single user case only. The number of precoding sequences used to spread information symbols on one subset S_k is denoted by $C^{(k)}$, with $0 \le C^{(k)} \le L$, since we assume orthogonal sequences. A certain amount of energy $E_c^{(k)}$ is assigned to each precoding sequence $c^{(k)}$ associated to a given modulation symbol of $b_c^{(k)}$ bits, where $1 \le i \le C^{(k)}$.

III. Proposed Allocation for OFDM

The proposed allocation maximizes the bit rate under PSD and mean BER constraint. The SNR gap, Γ, for any uncoded QAM with a target symbol error rate (SER), P^s, and for a null system margin, is given as [9]

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{P^s}{4}\right)\right]^2, \tag{2}$$

where Q^{-1} is the inverse of the well-known Q-function.

Classically, to achieve a target error rate, SER is fixed on each subcarrier which is equal to the global SER of an OFDM symbol, because all the constellation sizes of QAM have the same value of the SNR gap at constant SER, as it is clear from (2). But the value of the SNR gap varies with the constellation size at constant BER. For a fixed target BER and due to this dependence of the SNR gap on the number of bits per symbol, the target BER is rarely achieved as discussed in [5]. One solution is to fix the BER rather than SER on each subcarrier as proposed in [10], which gives the similar throughput as in [1] and [3] but does not violate the fixed target constraint. This will be known as peak BER constraint allocation in the following. In this paper, we maximize the bit rate while respecting a mean BER and PSD constraint. Thus, the allocation problem is formulated as follows

$$\max \quad \sum_{i=1}^{N} b_i\ ,$$

$$\text{subject to } E_i \le E, \quad \text{and} \quad \frac{\sum_{i=1}^{N} P_i^b \cdot b_i}{\sum_{i=1}^{N} b_i} \le P_T, \tag{3}$$

where b_i and E_i are the number of bits and energy allocated to subcarrier i, respectively and E is the given PSD limit. P_i^b is the BER of subcarrier i and P_T is the target mean BER of the system.

We consider a multicarrier system employing 6 uncoded M-QAM constellations with $M \in \{4, 8, 16, 32, 64, 128\}$. For uncoded QAM, SER is given as [11]

$$P_i^s \approx 4\ Q\left(\sqrt{\frac{3\ \gamma_i}{M_i - 1}}\right), \tag{4}$$

where P_i^s, γ_i and M_i are the SER, SNR value and the constellation assigned to subcarrier i, respectively. BER is obtained from SER using the approximation $P_i^b \approx P_i^s / b_i$ [5]. Using this approximation, the second constraint of the problem described in (2) becomes,

$$\bar{P} = \frac{\sum_{i=1}^{N} P_i^s}{\sum_{i=1}^{N} b_i}, \tag{5}$$

which shows that the mean BER depends upon the sum of the SERs of all the subcarriers of a multicarrier system and not upon the individual number of bits supported by each subcarrier. Based on this result, we modify the loading algorithm proposed in [5], where rather than searching for the worst BER to reduce the constellation size, we search for the worst SER. This approach will then further be extended to an LP-OFDM system in Section IV. The algorithm can be described as follows.

1: Initiate all the subcarriers with the highest modulation scheme (i.e. 128-QAM)
2: Calculate $\bar{P}$ from (5)
3: **while** $\bar{P}\ \ge\ P_T$ **do**
4: Search the subcarrier j with maximal SER: $j = \arg\max(P_i^s)$
5: Reduce the constellation size of the subcarrier j, null the subcarrier if it supports 2 bits
6: Calculate $\bar{P}$ from (5)
7: **end while**

In this way, we achieve the near optimal solution faster than that of [5], and the throughput is also higher, as it will be shown later. It is clear that the mean BER directly depends upon the sum of SERs (i.e. the product of BER and the number of bits per symbol) and does not directly depend upon the sum of BERs. In other words, reducing the constellation size of the subcarrier with the worst BER does not necessarily mean that it is the maximum decrease in the mean BER of the system. Therefore, in this allocation we reduce the constellation size of the subcarrier with the worst SER, which means that it is the maximum decrease in the mean BER of the system while still removing one bit.

IV. Proposed Allocations for LP-OFDM

As discussed earlier, an LP-OFDM system groups together multiple subcarriers. R_k is the number of bits supported by a group S_k of subcarriers. Here, we propose an allocation scheme with the same number of bits on all the precoding sequences of a group. In consequence, all the precoding sequences in a group have the same SER P_k^s and the sum of all the SERs of a group can be defined as group error rate, $P^k = LP_k^s$. Note that P_k is not an SER by defintion. P_k^s is the SER of all the precoding sequences of group k, and is given as

$$P_k^s = 4\; Q\left(\sqrt{\frac{3\;\gamma_k}{2^{R_k/L}-1}}\right), \qquad (6)$$

where γ_k is given as [3]

$$\gamma_k = \frac{L}{\displaystyle\sum_{i\in S_k}\frac{1}{|H_i|^2}}\cdot\frac{E}{N_0}, \qquad (7)$$

where H_i is the gain of subcarrier i. Hence using (5), the mean BER of an LP-OFDM symbol can be given as

$$\bar{P} = \frac{\displaystyle\sum_{k=1}^{K} P^k}{\displaystyle\sum_{k=1}^{K} R_k}. \qquad (8)$$

The allocation problem for an LP-OFDM system can be formulated as follows

$$\max \quad \sum_{k=1}^{K} R_k\ ,$$

$$\text{subject to } E_k \le E, \quad \text{and} \quad \frac{\displaystyle\sum_{k=1}^{K} P^k}{\displaystyle\sum_{k=1}^{K} R_k} \le P_T, \qquad (9)$$

where E_k is the energy allocated to group k. In the proposed algorithm, we search for the worst value of P_k, and reduce the constellation sizes of all the precoding sequences of the found group. It is summarized as follows.

Initiate all the precoding sequences with the highest modulation scheme (i.e. 128-QAM)
Calculate $\bar{P}$ from (8)
while $\bar{P}\ \geq\ P_T$ **do**
    Search the group of subcarriers with the worst P_k

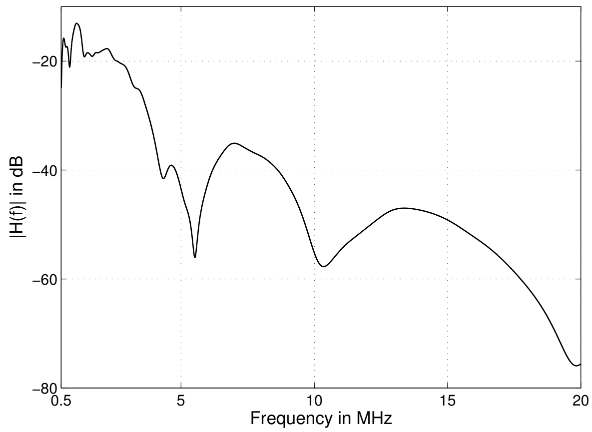

Fig. 2. 15-paths reference channel model for PLC [6].

    Reduce the constellation size of all the precoding sequences of the found group, null the precoding sequences if they support 2 bits.
    Calculate $\bar{P}$ from (8)
end while

Due to the removal of bits from all the precoding sequences of a given group, this allocation is much faster than the proposed allocation in [5] and gives higher throughput for high SNR, as it will be shown later. Constellation sizes of all the precoding sequences of a given group are reduced because the removal of bits from just one precoding sequence in a group changes negligibly the group error rate, as all the precoding sequences of a group have the same value of the transmit power. It is shown in (4) that SER depends upon the Q-function of SNR and due to the higher slope of the Q-function, SER of those precoding sequences, which do not have reduced constellation sizes, dominate in the group error rate. It is worthy to mention here that, the OFDM allocation can be obtained by taking $L = 1$ in the LP-OFDM allocation.

V. Simulations and Results

In this section, we will present simulation results for both the proposed allocation schemes. The results will be compared with the allocation schemes proposed by Wyglinski and the ones proposed under peak error rate constraint for OFDM and LP-OFDM systems. The generated signal is a baseband signal produced by dividing 20 MHz band of Zimmermann channel into 1024 subcarriers. It is assumed that the synchronization and channel estimation tasks have been successfully performed. We use the multipath model for the power line channel, proposed in [6] and presented in Fig. 2. The considered reference model is 110 m link 15-paths model whose frequency response is given by

$$H(f) = \sum_{i=1}^{N} g_i \cdot e^{-(a_0+a_1 f^k)^{d_i}} \cdot e^{-j2\pi f d_i/v_p}, \qquad (10)$$

TABLE I
PARAMETERS OF THE 15-PATH MODEL.

attenuation parameters					
$k = 1$		$a_0 = 0$		$a_1 = 2.5 \cdot 10^{-9}$	
path-parameters					
i	g_i	d_i(m)	i	g_i	d_i(m)
1	0.029	90	9	0.071	411
2	0.043	102	10	-0.035	490
3	0.103	113	11	0.065	567
4	-0.058	143	12	-0.055	740
5	-0.045	148	13	0.042	960
6	-0.040	200	14	-0.059	1130
7	0.038	260	15	0.049	1250
8	-0.038	322			

TABLE II
COMPUTATION TIMES IN MILLISECONDS, 1024 SUBCARRIERS, $P_T = 10^{-7}$ (INTEL CORE 2, 2.4-GHZ PROCESSOR).

	LP-OFDM peak BER	OFDM peak BER	Proposed LP-OFDM	Proposed OFDM	Wyglinski
Mean	5.46	21.96	0.45	41.99	43.96
Worst	30.43	34.33	1.02	176.87	194.11

where v_p depends upon the dielectric constant of the electric line. The parameters of the 15-path model are listed in Table I, and τ_i is the delay of path i. A background noise level of -110 dBm/Hz is assumed and the signal is transmitted with respect to a flat PSD of -40 dBm/Hz. Results are given for a target BER of 10^{-7}. In the literature, the mean BER approach for OFDM was used in very few papers, including [5]. Thus, in this paper, we compare our results with [5], which implicits, that the performance of our allocation is better than [2] and other algorithms working under peak SER approach.

Fig. 3 shows the achieved bit per OFDM symbol versus the average channel gain, $\text{ACG} = \frac{1}{N} \sum |h_n|^2$ which conveys the attenuation experienced by the signal through the channel. The corresponding mean SNR is then given by $\text{SNR}_{\text{dB}} = -40 + \text{ACG}_{\text{dB}} + 110$. The performance of the proposed allocation for OFDM is compared with the allocation of Wyglinski at different average channel gains, particularly for poor SNR. It is shown that the proposed scheme performs better than the allocation of Wyglinski, as the minimum bits are removed before achieving P_T. This allocation is also faster as shown in Table II.

In Fig. 4 the throughputs of the proposed allocations are compared, at higher average channel gains, with those of LP-OFDM and OFDM under peak BER constraint [3]. It is clear from the results that OFDM achieves higher throughput for poor SNR and LP-OFDM performs better at higher average channel gains. The optimal value of the precoding factor, for

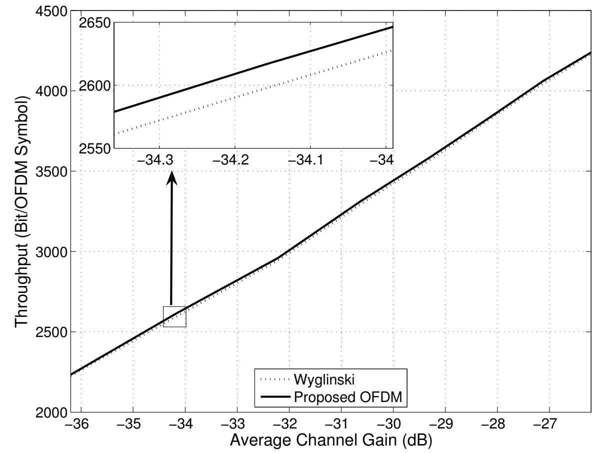

Fig. 3. Throughput comparison of the proposed allocation for OFDM and the allocation proposed by Wyglinski at various average channel gains.

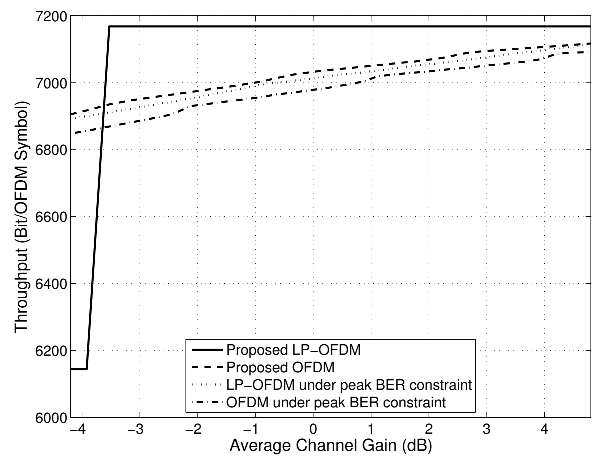

Fig. 4. Throughput comparison of the proposed allocations with the OFDM and LP-OFDM allocations under peak BER constraint at various average channel gains.

the proposed LP-OFDM allocation, is obtained by running the simulations for various possible values of L, especially for higher SNRs, which is the region of high interest. The results are shown in Fig. 5. It can be observed that in the region where LP-OFDM performs better than OFDM, the maximum spreading always obtains the highest bit rate. Therefore L was taken as N (i.e. 1024 in this case). For the peak BER constraint LP-OFDM, the optimal value of the precoding facter, $L = 32$, is used.

Under PSD constraint, the major task is to efficiently utilize the available energy, since the transmit power that is not used by a subcarrier (or a group of subcarriers) is lost and can not be used by other subcarriers (or groups of subcarriers). It can be observed in Fig. 6 that, the proposed allocations with mean BER constraint utilize more efficiently the available

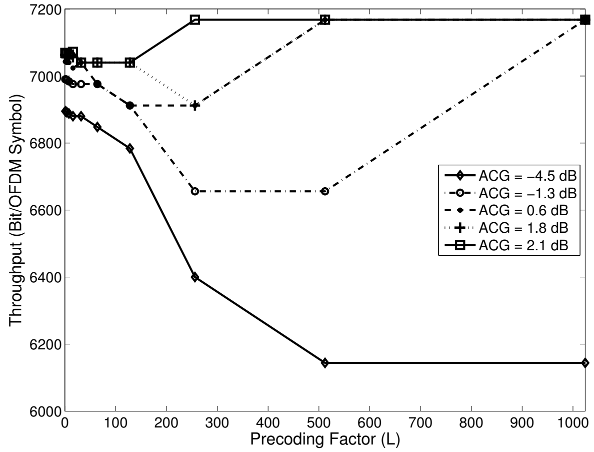

Fig. 5. Precoding factor vs bit rate at various average channel gains.

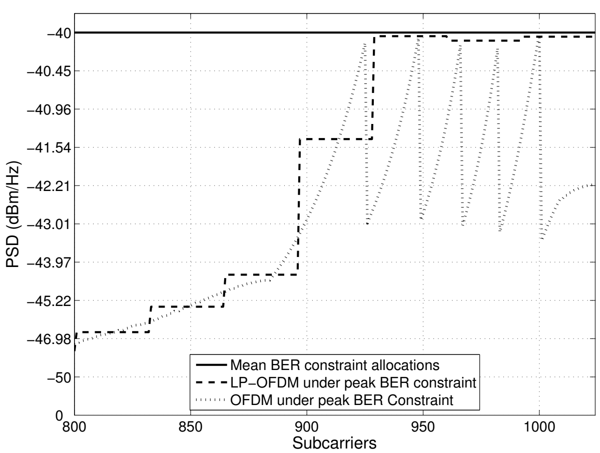

Fig. 6. Energy distribution comparison of all the four considered allocation schemes at an ACG = -3.5 dB. OFDM under peak BER constraint supports 6869 bit, LP-OFDM under peak BER constraint supports 6911 bit, proposed OFDM with mean BER constraint supports 6935 bit and proposed LP-OFDM with mean BER constraint supports 7168 bit.

energy. The spike-shaped curve of the OFDM under peak BER constraint shows the transitions of the modulation orders (i.e. decreasing the constellation sizes) when no more energy is available to sustain the target BER. LP-OFDM under peak BER constraint accumulates the energies of all the subcarriers of the group to utilize it more efficiently. It can be observed that it is using more energy than OFDM, but still not utilizing it completely. LP-OFDM with mean BER constraint is using all the available energy at higher channel gains and also accumulating the energies of different subcarriers to transmit more bits.

Consequently, it can be observed that the proposed OFDM allocation performs better at lower average channel gains while the proposed LP-OFDM allocation performs better at higher average channel gains. It has also been stated that the OFDM allocation can be obtained by taking $L = 1$ in the LP-OFDM allocation, therefore it might be an interesting solution to design an adaptive transmitter which decides between two optimal values of L (i.e. either 1 or N). Furthermore, a summary of mean and worst case computation times for a 1024-subcarrier system with a P_T of 10^{-7} is shown in Table II. These results are obtained by running the simulations for 600 different values of ACG. It can be observed that the proposed LP-OFDM allocation is the fastest one and the proposed OFDM allocation is faster than the Wyglinski's incremental allocation. Furthermore, comparing our results with [5] implicits that the performance of the proposed allocations is better than [2].

VI. Conclusion

In this paper, we propose bit allocation algorithms for OFDM and LP-OFDM systems. These algorithms provide higher throughputs and have lower computational complexity while respecting a mean BER of an OFDM symbol. The performance of these allocations is compared with that of Wyglinski and the allocations under peak BER constraint. It is shown that the proposed allocation for OFDM performs better at lower channel gains and achieves higher throughput than the algorithm of Wyglinski. This algorithm is also faster than that of Wyglinski. Furthermore, a bit loading algorithm for LP-OFDM is also proposed which performs better at higher channel gains and has a very low implementation complexity. While comparing with the allocations under peak BER constraint, it is shown that the proposed allocations have higher throughputs under PSD constraint, as they are using the available energy more efficiently.

References

[1] I. Kalet, "The Multitone Channel," *IEEE Trans. Commun.,* vol. 37, no. 2, pp. 119-124, Feb 1989.

[2] A. Leke, and J. M. Cioffi, "A maximum rate loading algorithm for discrete multitone modulation systems," in *Proc IEEE Global Telecommunications Conf.,* Phoenix, AZ, 1997, vol. 3, pp. 1514-1518.

[3] M. Crussière, J.-Y. Baudais, and J.-F. Hélard, "Adaptive spread-spectrum multicarrier multiple-access over wirelines," *IEEE Journal on Selected Areas in Communications,* vol. 24, no. 7, pp. 1377-1388, 2006.

[4] Y. George, and O. Amrani, "Bit Loading Algorithms for OFDM," in *Proc IEEE Inter. Symp. on Info. Theory.,* Chicago, USA, 2004, pp. 391.

[5] A. M. Wyglinski, F. Labeau, and P. Kabal, "Bit loading with BER-constraint for multicarrier systems," *IEEE Trans. Wireless Commun.,* vol. 4, no. 4, pp. 1383-1387, Jul. 2005.

[6] M. Zimmermann and K. Dostert, "A multipath model for the powerline channel," *IEEE Trans. Commun.,* vol. 50, no. 4, pp. 553, Apr. 2002.

[7] K. Fazel, and S. Kaiser, *Multi-Carrier and Spread Spectrum Systems,* John Wiley & Sons, 2003.

[8] S. Nobilet, J.-F. Hélard, and D. Mottier, "Spreading sequences for uplink and downlink MC-CDMA systems: PAPR and MAI minimization," *European Trans. on Telecomm.,* vol. 13, no. 5, pp. 465-474, 2002.

[9] J. Cioffi, A multicarrier primer ANSI T1E1.4/91-157, 1991, Committee contribution, Tech. Rep.

[10] F. Syed-Muhammad, J.-Y. Baudais, J.-F. Hélard, and M. Crussière, "A coded bit-loading linear precoded discrete multitone solution for power line communication," in *Proc IEEE Workshop on Signal Processing Advances in Wireless Commun.,* Recife, Brazil, 2008, pp. 555-559.

[11] J. G. Proakis, *Digital Communications,* 4th ed. McGraw-Hill, 2000.

[12] Z. Wang, and G. B. Giannakis, "Linearly precoded or coded OFDM against wireless channel fades?," in *Proc IEEE Workshop on Signal Processing Advances in Wireless Commun.,* Taoyuan, Taiwan, 2001, pp. 267-270.