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Fuzzy histogram for internal and external fuzzy directional relations

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Abstract— *Spatial relations have key point importance in image analysis and computer vision. Numerous technics have been developed to study these relations especially directional relations. Modern digital computers give rise to quantitative methods and among them fuzzy methods have core importance due to handling imprecise knowledge information and vagueness. In most fuzzy methods external directional relations are considered which are useful for small scale space image analysis but in large scale space image analysis internal cardinal directions are also important. In this paper a new function is introduced which can be equally used for both internal and external fuzzy directional relations. Trigonometric function is used to determined the directions. This approach has core importance in medical images. A practical example in medical images is discussed.*

Keywords: Spatial Relations, Direction Relations,fuzzy internal cardinal directions, Scale Space, Medical Images.

1. Introduction

One of the fundamental tenet of modern sciences is that the phenomenon can not be claimed to be well understand until it can be characterized in quantitative manners. Given our veneration for what is precise and quantitative. Advent of digital computers has resultant in a rapid expansion in the use of quantitative methods through most of human knowledge. Un precise knowledge information and vagueness gives rise to fuzzy methods. In the last decade of 19th century much attention has been paid to spatial relations between objects and directional relations has a key point importance among them. These relations are used in computer vision and image analysis [5], [1], content based image retrieval (CBIR) [7], similarity based object retrieval [14], identify forms, manage data bases, support spatial data in artificial intelligence (AI), cognitive science, perceptual psychology, geography particularly geo-information system [2], [3], [4], comparing objects scene and model and indexation space are major applications of space relations. Satellite and robotics [12], developing linguistics expressions and medical imaging applications [11].
Number of approaches has been developed to study the directional relations qualitatively and quantitatively. All of quantative methods are sensitive to distances between objects which is proportional to distribution range of angles between objects. Some methods provide only distribution rang of directions among the objects such as *statistical method* [6]. This method supports topological relation $disjoint$,$meet$ and metric relations $Near$, Far and $Faroff$. Another class of methods which capture shape information along with the distribution range of directions like *angle histogram* [10], *force histogram* [8]. Both *angle histogram* and *angle distance histogram* supports $meet$ relation along with $disjoint$ and metric relations $Near$, Far and $Far\ off$. Force histograms strictly support $disjoint$ relation and other methods support overlapping objects like *R-histogram* [14] and R^*- *histogram* [13]. These are quantitative methods but not fuzzy moreover labeling process is to complex. Main objective of this paper is to introduce a method which can compensate the overlapped objects like the $R-histogram$. It is a quantitative representation of the relative objects position. All of the above cited methods are used in small scale space but in large scale space internal object position is also important. It is needed to develop a single method which can be used in small and large scale spaces unlike separate qualitative method $internal\ 5-intersection$, $internal\ 9-intersection$ and $internal\ 13-intersection$ models introduced by Y. Liu et al. [15]. For external directions separate methods are used. Another important fact that $meet$ is a crisp point relation and only one overlapping pixel can change the entire phenomenon and stop the method of force histograms from working. Paper is structured as follows, in section 2 proposed function and calculation of histogram is described, in section 3 concerns about experiments. Section 4 concludes the work.

2. Oriented lines, Used Function and calculation of histogram

This section is concerned with object decomposition, use of function and treatment of different parts of object and finely calculation of histograms.

2.1 Oriented lines, segments and longitudinal sections

Let A and B are two objects. $(v, \theta) \in R$, where v is any real number and $\theta \in [0, 360]$ or $\theta \in [-\pi, \pi]$. $\Delta_\theta(v)$

is an oriented line at orientation angle θ as explained in figure 1(a). $A \cap \Delta_\theta(v)$ is the intersection of object A and oriented line $\Delta_\theta(v)$. It is denoted by $A_\theta(v)$, called segment of object A and its length is x. Similarly for object B where $B \cap \Delta_\theta(v) = B_\theta(v)$ is segment and z is its length. y is the difference between the maximum value of $B \cap \Delta_\theta(v)$ and minimum of $A \cap \Delta_\theta(v)$(for details see [9]). In case of polygonal object approximations, triplet (x, y, z) can be calculated from intersecting points of line and object boundary, only oriented lines which passes through vertexes of polygon are taken.

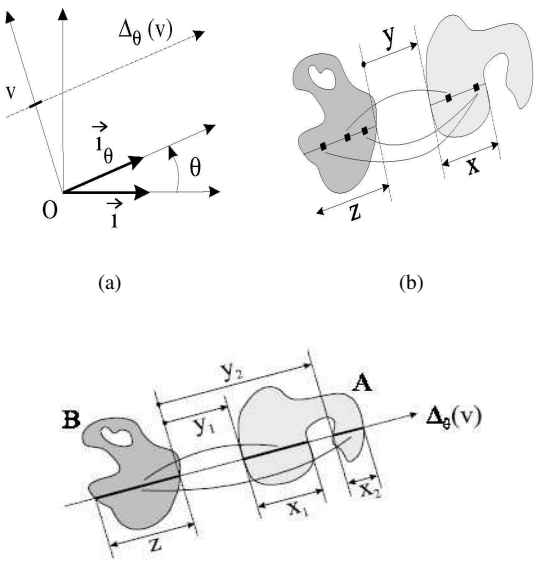


Fig. 1: (a)Oriented line,(b) Treatment of segment,(c) Treatment of longitudinal section.(Matsakis [9])

2.2 Use of function

To define the object position a function is defined. $f : R_+ \times R \times R_+ \longrightarrow [0, 1]$ mathematically

$$f(x, y, z) = \begin{cases} 0 & \text{if } y \leq \text{ - z/2} \\ \frac{2y+z}{z} & \text{if } y \in (\text{ - z/2 } \ 0\ ) \\ 1 & \text{if} y \geq 0 \end{cases}$$

where $(x, y, z) \in R_+ \times R \times R_+$. This function can easily be modeled as fuzzy membership function.

2.3 Treatment of points

If both the objects A, and B are point objects or oriented line $\Delta_\theta(v)$ passes through the vertex of polygons. Length of both segments will be zero. i.e $x = 0$,$z = 0$. If both the points are same then $y = 0$ in this case $f(x, y, z) = 0$ and if both points are not same then either A is before B or after. In first case $y < 0$ and $f(x, y, z) = 0$ and in second case $y > 0$ and $f(x, y, z) = 1$. Third possible case if $x = 0$ and $z \neq 0$ then value of$f(x, y, z)$ depends on y. If $y \in (-\frac{z}{2}, 0)$ then $f(x, y, z) \in (0, 1)$ and for $y > 0$ $f(x, y, z) = 1$ otherwise $f(x, y, z) = 0$

2.4 Treatment of segment

It is considered that the length of segments x and z are two points on real line and they form the support of a fuzzy membership function, y is taken as its evaluation point. Calculation of x,y and z are explained in above cited figure 1(b). Then given function can be modeled as fuzzy membership functions, for this purpose trapezoidal membership function can be used.

$$f(I, J) = \mu_{(-z/2,0,\infty,\infty)}(y)$$

2.5 Treatment of longitudinal sections

In decomposition process of an object into segments, there can be multiple segments depending on object shape complexity which is called longitudinal section. Each segment of a longitudinal section is at a certain distance and these distances might effect end results. Different fuzzy $T - norms$,$T - conorms$ are used fuzzy integration of available information. Here for cause of simplicity Standard $t - conorms$ is used. For standard $t - conorm$ only one value contributes.

1) **Fuzzy OR:**
   $\mu_{(OR)}(u) = max(\mu_{(A)}(u), \mu_{(B)}(u))$
2) **Fuzzy AND:**
   $\mu_{(AND)}(u) = min(\mu_{(A)}(u), \mu_{(B)}(u))$
3) **Fuzzy Algebraic sum:**
   $\mu_{(SUM)}(u) = 1 - \Pi_{i=1}^{2}(\mu_{(i)}(u))$
4) **Fuzzy Algebraic Product:**
   $\mu_{(PROD)}(u) = \Pi_{i=1}^{2}(\mu_{(i)}(u))$
5) **Fuzzy γ Operator:**
   $\mu_{(\gamma)}(u) = [(\mu_{(SUM)}(u)]^\gamma * [(\mu_{(PROD)}(u)]^{1-\gamma}$ where $\gamma \epsilon [0, 1]$

When fuzzy operator OR (respectively AND) is used, only one fuzzy value contributes for the resultant value which is $maximum$ (respectively $minimum$). For other operators both values contribute. In this case each relation has a fuzzy grade, objective is to accumulate the best available information. In case of longitudinal section, there exist number of segments and each segment has a fuzzy relation with segment of other object. Now suppose longitudinal section of object A has two segments such that $x = x_1 + x_2$ where x_1 is the length of first segment and x_2 is the length of second segment and x is length of longitudinal section. Let $\mu_1(y_1)$ defines the value of fuzzy directional relations with the first segment and $\mu_2(y_2)$ represents value of fuzzy directional relation with the second segment where y_1 and

y_2 are the distances between object A and two segments of B.
$\mu(y) = \mu_1(y_1) \odot \mu_1(y_1)$
where $\odot$ is a fuzzy operator. Existence of longitudinal section is explained in above cited figure 3(f).

2.6 Histogram calculations

The above cited function is used to detect the object. If the argument object is within the object and in the lateral half of segment then it will assign a value in $(0, 1)$. If objects are disjoint then it will assign a value 1 and if argument object is in the first half of an object then it will assign the value zero. Let A and B be two objects, and (v, θ) are two reals (I, J) are the one of argued and reference object segments. $f(I, J)$ represents the argument weight that A is in θ direction of B. Histogram will be sum of all the arguments and represented as:

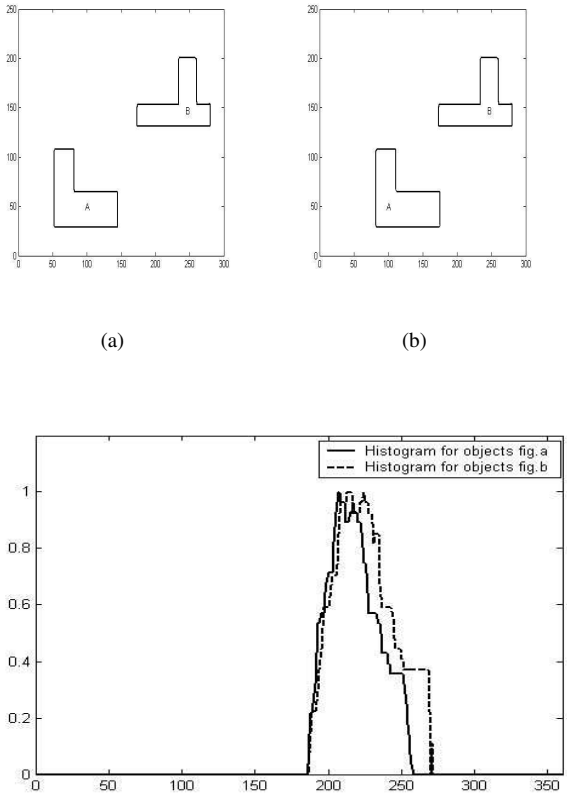
$$\int_{-\infty}^{+\infty} (F(\theta, A_\theta(v), B_\theta(v))dv = \sum_{j=1}^{n} f(I_j, j_j) \qquad (1)$$

where n is total number of segments and histogram is represented by
$\mathbb{F}^{AB}(\theta) = \int_{-\infty}^{+\infty} (F(\theta, A_\theta(v), B_\theta(v))dv,$
for normalization divide by the max value of $\mathbb{F}$.

- Position invariance: It is invariant and in general $\mathbb{F}_\theta(A, B) = \mathbb{F}_\theta(B, A) + \pi$
- Shape dependance: Histogram does not depend on object shape different objects may have same histogram.
- Distance dependance: Histogram is also sensitive to distance between objects

3. Experiments and interpretation

For experiment purpose, first of all its sensitivity to shape and distance is verified for disjoint objects. In this example position of object A is changed and proposed histogram is compared. Second set of examples is concerned with comparing different fuzzy methods for directional relations when objects are disjoint but when object A inside the object B, no fuzzy method exist, we show that *proposed histogram* $\mathbb{F}$ can be used for internal fuzzy directional relations.
In this figure objects are represented at different positions and fig2(c) shows that histogram is sensitive to shape and distance. fig 2(a) represents the objects as in source [6], fig. 2(b) represents objects when object A is translated by $(30, 0)$ and fig 2(c) represents their histograms where black solid line represents histogram between objects given in fig2(a) and dotted line represents histogram between objects given in 2(b).



(a) (b)

(c)

Fig. 2: (a)(b) objects A,B at different positions(d) Corresponding histograms

In the next example a real medical imaging problem of detection a kidney stone is considered. As this problem is related to find internal or external object position. For experiment purpose, object A is considered at different positions outside the object B. In real only situation in figure 3(b) is possible when the stone is in Ureter. In this example different methods are compared. Where F_0, F_2 represents force histograms F_0 is a histogram of constant forces and F_2 is histogram of gravitational forces. $\mathbb{F}$ is proposed histogram and $\mathcal{A}$ is angle histogram. A trigonometric function is used for evaluating directions, for example relation $Right - of$ is modeled as

$$f_{rightof} = \begin{cases} cos^2(\theta) & \text{if } \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ 0 & \text{otherwise} \end{cases}$$

Results are compared in table 1.

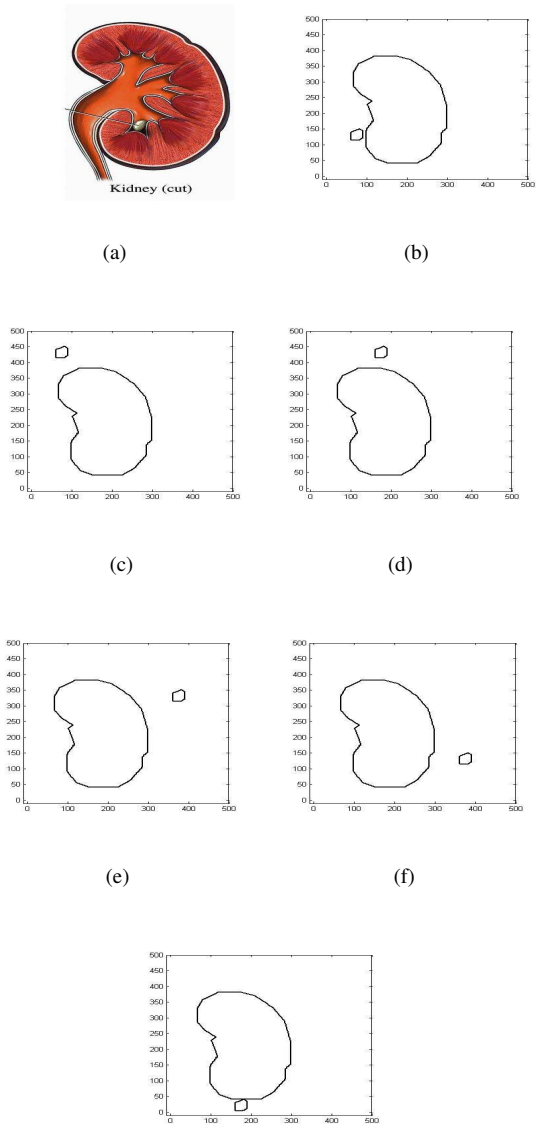


Fig. 3: (a) kidney image with a stone inside(source:www.sodahead.com). pairs of objects with object A at different positions

When the argument object A is at interior of reference object B, histogram of forces method does not work due to its defination.i.e.

$$\Phi(y) = \begin{cases} 0 & \text{if } y \leq 0 \\ \frac{1}{y^r} & \text{otherwise} \end{cases}$$

because for this case, every pair (v, θ), y is always negative. Resulting that force histogram have zero value and this restricts the application of force histogram for objects inside each other i.e. for topological relation $overlap$ and $contain$ or $contained_by$. Following different cases are considered when object A is inside object B. Results are compared in table 2.

Table 1: Comparison table when objects are disjoint

Methods comparison when objects are disjoint					
Fig	$Method$	$"above"$	$"right"$	$"below"$	$"left"$
3(b)	F_0	.05	0	.18	.77
	F_2	.1	0	.17	.73
	$\mathcal{A}$	.11	0	.44	.45
	$\mathbb{F}$	.16	0	.33	.53
3(c)	F_0	.49	0	0	.51
	F_2	.8	0	0	.2
	$\mathcal{A}$	.72	0	0	.28
	$\mathbb{F}$	.7	0	0	.3
3(d)	F_0	.64	.29	0	.07
	F_2	.84	.12	0	.04
	$\mathcal{A}$	.8	.11	0	.09
	$\mathbb{F}$	.77	.15	0	.08
3(e)	F_0	.2	.8	0	0
	F_2	.28	.72	0	0
	$\mathcal{A}$	.35	.64	.01	0
	$\mathbb{F}$	.3	.7	0	0
3(f)	F_0	.03	.87	.1	0
	F_2	.02	.86	.12	0
	$\mathcal{A}$	.04	.7	.27	0
	$\mathbb{F}$	.03	.78	.19	0
3(g)	F_0	0	.14	.47	.39
	F_2	0	.18	.58	.24
	$\mathcal{A}$	0	.11	.72	.17
	$\mathbb{F}$	0	.18	.61	.21

Table 2: Calculation of directional relations when object A is inside B by the proposed fuzzy histogram

Dir. relations object A is inside B				
$Object$	$above$	$right$	$below$	$left$
$4(a)$	.13	.15	.45	.28
$4(b)$	.1	.3	.48	.12
$4(c)$	.18	.14	.2	.48
$4(d)$	.3	.48	.15	.07
$4(e)$	.5	.3	.07	.13

Another important thing is that directional relations are concerned with the number of visual points. In quantitative methods final decision depends upon number of visible points. When objects are disjoint, range of directional distribution are limited but in case argument object is inside the reference object then every point of reference object is visible from some points of argument object. This phenomenon will change the process of reasoning. And object is visible in all directions. All the directions are calculated and for conclusion we take only two maximum directions. As in case of figure 4(a), Stone lies $below_left$ side of the kidney and in case of figure 4(b) stone lies at $below_right$ side of the kidney. Results are verified in table 2.

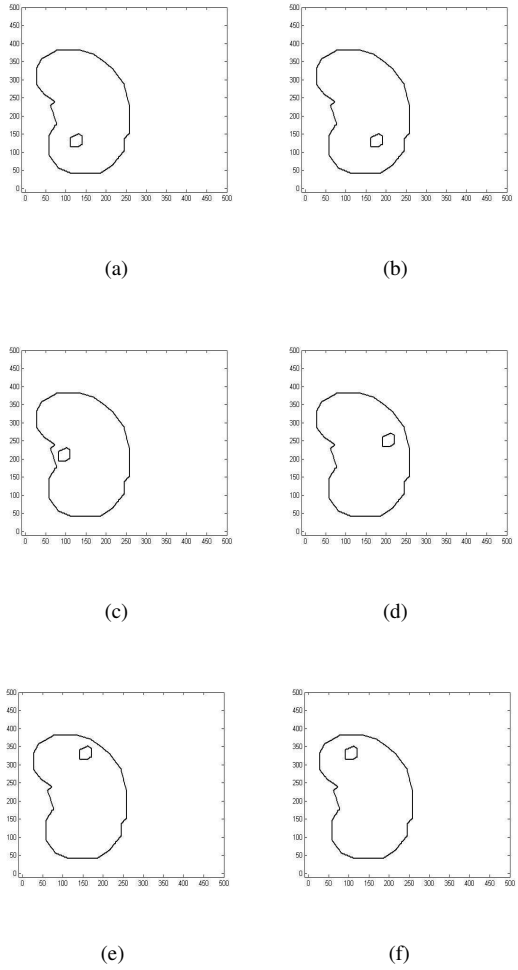


Fig. 4: Pairs of objects with object A at different positions inside the object B

4. Conclusion

Fuzzy directional relations have key point importance in image analysis. The most developed fuzzy approaches deals with $disjoint$ and $meet$ topological relations. Only $R-histogram$ deals with *overlapping* topological relation between object which is not fuzzy. We have developed a function. This function is sensitive to internal cardinal directions and equally sensitive to external cardinal directions. $Proposed\ histogram$ can be successfully used to detect object position within the object or outside the object. This approach is simple and easy to implement. When the objects are disjoint this approach of object detection is similar to the F_0 (constant histogram forces introduced by Matsakis [8]. This method can in the same manner be employed on both large and small scale image analysis.

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