



HAL
open science

Gradient based optimization of semi-numerical models with symbolic sensitivity: Application to a simple ferromagnetic MEMS switch device

Benoît Delinchant, Harijaona Lalao Rakotoarison, Vincent Ardon, Olivier
Chadebec, Orphée Cugat

► To cite this version:

Benoît Delinchant, Harijaona Lalao Rakotoarison, Vincent Ardon, Olivier Chadebec, Orphée Cugat. Gradient based optimization of semi-numerical models with symbolic sensitivity: Application to a simple ferromagnetic MEMS switch device. *International Journal of Applied Electromagnetics and Mechanics*, 2009, 30 (3-4), pp.189-200. 10.3233/JAE-2009-1021 . hal-00417616

HAL Id: hal-00417616

<https://hal.science/hal-00417616>

Submitted on 26 Nov 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Gradient based optimization of semi-numerical models

with symbolic sensitivity:

Application to a simple ferromagnetic MEMS switch device

B. Delinchant, H.L. Rakotoarison, V.Ardon, O. Chadebec, O. Cugat

G2ELab : Grenoble Electrical Engineering Lab

ENSIEG - BP 46 - 38402 Saint-Martin-d'Hères FRANCE

E-mail: Benoit.Delinchant@G2ELab.inpg.fr

Abstract: This paper deals with electromagnetic modeling dedicated to constrained optimization needs. A tool dedicated to simple shape magnetic devices is presented and applied on the field of magnetic MEMS design. It allows semi-numerical modelling and provides symbolic gradient computation. A multi-level optimization strategy is used to ensure fast and global convergence.

Keywords: Optimization, Gradient, Sensitivity, Multi-level, MEMS.

I Introduction

Designing an electromagnetic device requires optimization algorithms and adapted models. In this paper, the virtual design of a ferromagnetic MEMS switch -and especially its preliminary sizing- is addressed, using MacMMems software [1] for the modelling, and CADES framework [2] for the optimization. The modelling software can produce parameterized semi-analytical modelling of the device with symbolic sensitivity. Thanks to accuracy parameters, a multi-level strategy can be deployed in the optimization framework.

The ferromagnetic switch (also called "magnetic latch") is used as an optical switch in our case ; it is based on the rocking actuation of a ferromagnetic plate under the influence of a magnetic field variation. In our device, the static magnetic field is produced by a permanent magnet and altered by planar coils, as sketched in Fig. 1.

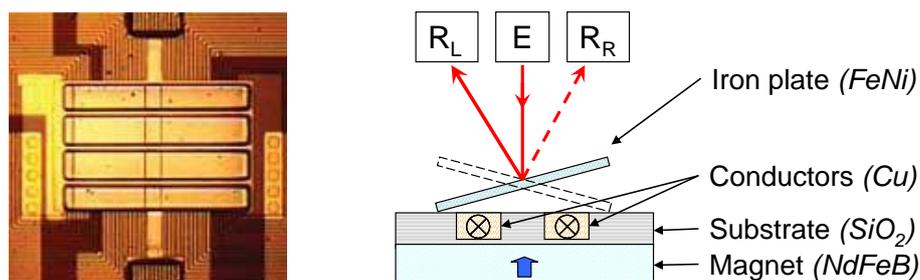


Fig. 1- Picture of a Mag-Latch switch (©MicroLab / MagFusion Inc.) and schematics.

I. A software for semi-numerical modelling with formal sensitivity

A. Semi-numerical modelling of magnetic MEMS

1. Modelling magnetic fields from permanent magnets and coils

Assumptions are constant magnetization of magnets and constant current density in conductors.

For magnets, a Coulombian equivalent charge approach is used with the following surface and volume charges :

$$\sigma_s = \vec{M} \cdot \vec{n} \quad (1)$$

$$\sigma_v = -\text{div}(\vec{M}) \quad (2)$$

The resulting magnetic excitation is the following.

$$\vec{H}(\vec{r}) = \iiint_v \frac{\sigma_v \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv + \iint_s \frac{\sigma_s \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} ds \quad (3)$$

As $\text{div}(\vec{M})=0$, volume charges are equal to zero leading to quick surface integration instead of classical volume integration.

For conductors, Biot and Savart law is used.

$$\vec{H}(\vec{r}) = \frac{1}{4\pi} \left[\iiint_v \frac{\vec{j} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv \right] \quad (4)$$

Magnetic excitations were symbolically computed, as far as possible, for some basic shapes and magnetization direction [3][4].

2. Computing forces and torques on magnet, coil and ferromagnetic bodies

For magnet, an equivalent charge method is used.

$$\vec{F} = \iint_s \sigma_s \cdot \vec{H}_{ext} ds + \iiint_v \sigma_v \cdot \vec{H}_{ext} dv \quad (5)$$

$$\vec{\Gamma} = \iint_s \vec{AP}_s \times (\sigma_s \cdot \vec{H}_{ext}) ds + \iiint_v \vec{AP}_v \times (\sigma_v \cdot \vec{H}_{ext}) dv \quad (6)$$

For conductors, Laplace law is used.

$$\vec{F} = \mu_0 \cdot \iiint_v \vec{j} \times \vec{H}_{ext} dv \quad (7)$$

$$\vec{\Gamma} = \mu_0 \cdot \iiint_v \vec{AP} \times (\vec{j} \times \vec{H}_{ext}) dv \quad (8)$$

For specifics configurations like parallelepiped magnets interaction [6], force can be fully analytical. In our

software, due to space transformation between interacting objects, and the possibility of their displacement during an optimization, force and torque computation will use numerical integration method.

For ferromagnetic material, a steady state Method of Moments (MoM) is applied. It consists in the meshed of ferromagnetic body along the X, Y and Z axes, with uniform induced magnetized elementary blocks [3]. This method doesn't require to mesh air and is particularly performing for radiating system. The issue of such an integral method is its full matrices and then its computation memory limitation.

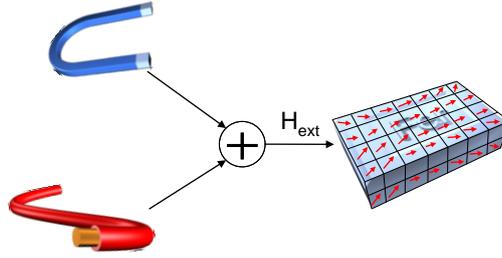


Fig. 2- Magnetization of ferromagnetic blocs with external field (eg. magnets or coils).

Figure 2 shows magnetization of each block which depends on the external field (H_{ext} produced by field sources like magnets or conductors) and the field produced by other blocks depending on their own magnetization.

$$H = H_{ext} + Q \cdot M \quad (9)$$

Blocks interaction is defined by the interaction matrix (Q in equation 9). Q is a square matrix of size $(3m) \times (3m)$ (m is the number of blocks), composed of 3×3 square matrices Q_{ab} . This matrix represented the magnetic excitation created by block (a) to the barycentre of block (b) and correspond to a well-known point matching approach.

$$Q = \begin{bmatrix} Q_{11} & \Lambda & Q_{1m} \\ M & Q_{ab} & M \\ Q_{m1} & \Lambda & Q_{mm} \end{bmatrix}, \quad Q_{ab} = \begin{bmatrix} Q_{ab\,xx} & Q_{ab\,xy} & Q_{ab\,xz} \\ Q_{ab\,yx} & Q_{ab\,yy} & Q_{ab\,yz} \\ Q_{ab\,zx} & Q_{ab\,zy} & Q_{ab\,zz} \end{bmatrix} \quad (10)$$

with $(a, b) \in [1; m]^2$

In the case of parallelepiped block [10], Q_{jk} are defined by the following equations:

$$Q_{xx} = \frac{1}{4\pi} \sum_{i,j,k=1}^2 (-1)^{i+j+k+1} \tan^{-1} \left[x_i^{-1} y_j z_k (x_i^2 + y_j^2 + z_k^2)^{-\frac{1}{2}} \right] \quad (11)$$

$$Q_{xy} = \frac{1}{4\pi} \ln \left[\prod_{i,j,k=1}^2 \left[z_k + (x_i^2 + y_j^2 + z_k^2)^{\frac{1}{2}} \right]^{-1)^{i+j+k+1}} \right] \quad (12)$$

with $x_{1,2} = x_c - x_0 \pm \frac{w_x}{2}$, $y_{1,2}$ and $z_{1,2}$ are similar.

(x_c, y_c, z_c) is barycentre of emitting block.

(w_x, w_y, w_z) is emitting block dimensions.

(x_0, y_0, z_0) is target block barycentre.

Q_{yy} , Q_{zz} are deduced from Q_{xx} and Q_{xz} , Q_{yz} are deduced from Q_{xy} .

The ferromagnetic material law is defined by $H=f(M)$ curve, parameterized by the saturation induction and initial permeability as shown in equation 13.

$$f(M) = \frac{M_{sat}}{\frac{\pi}{2} \cdot (\mu_r - 1)} \cdot \text{tg}\left(\frac{\pi}{2} \frac{M}{M_{sat}}\right) \quad (13)$$

The unknowns, ferromagnetic magnetizations, can then be found by solving the non-linear equation 14 :

$$f(\vec{M}) = \vec{H}_{ext} + \mathbf{Q} \cdot \vec{M} \quad (14)$$

To do it, a Newton-Raphson solver has been used with a relaxation method to ensure convergence.

When magnetizations are known, each block can be seen as parallelepiped magnets, to compute magnetic field or force and torque as explained in figure 3.

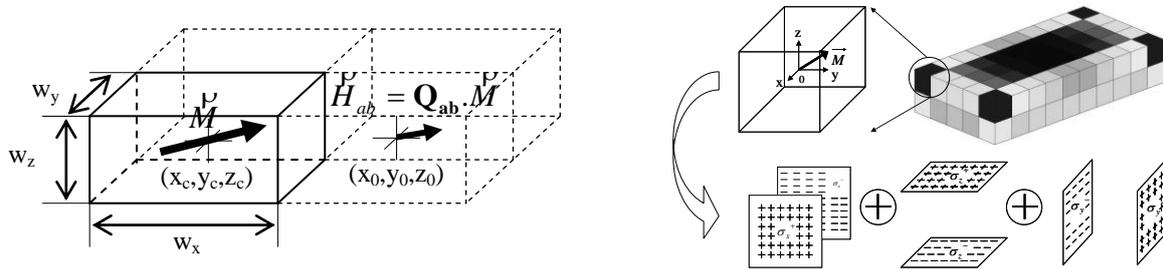


Fig. 3- Surface charge approach, used to compute force and torque with the ferromagnetic magnetizations.

B. Modelling Software

1. MacMMems : an automated semi-numeric model generator

These semi-numerical modeling techniques have been implemented in two software. The first is LOCAPI [9], a generic one based on a numeric solver where complex shapes can be introduced thanks to a meshing tool. The second, which is detailed in the following, is MacMMems [4][5] (Macro Modeller for Magnetic MEMS) dedicated to provide analytical models which aim to be derivate for optimization process. It is limited to simple shapes in order to produce symbolic derivatives.

In MacMMems, geometry and physics are described easily thanks to an intuitive GUI (Graphical User Interface). Each body owns its equations depending on the material (magnet, coil, ferromagnetic, diamagnetic) and its shape (parallelepiped block, part or full cylinder). Space transformations can be added (translations and rotations). Magnetic field, force and torques can be computed. Depending on desired computations, MacMMems

generates the global model by the assembly of equations given by bodies and space transformations. Dimensions, physics and spaces transformations can be parameterized leading to a very useful modeling.

Since a fully parameterized model, constituted of equations and call to functions, a symbolic treatment can be done to express parameter sensitivities. This task is realized in CADES Generator detailed in section C.

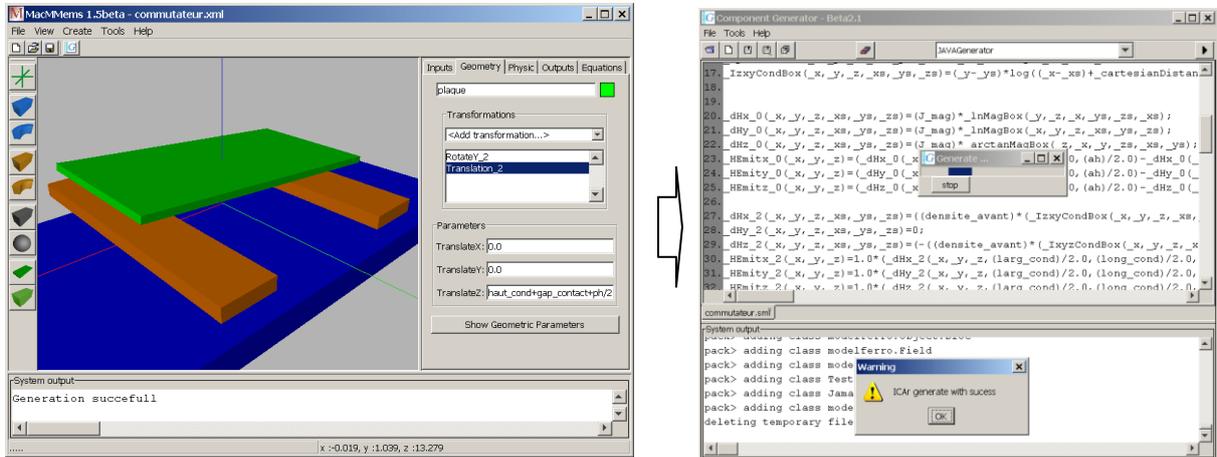


Fig. 4 - Left: MacMMems (Macro Modeller for Magnetic MEMS), Right: CADES Generator

2. Comparison with FEM Modelling

The modeling of our device with a FEM software is very uneasy because a new heavy air mesh generation is needed at each switch position. For example, two hours are needed to plot the curve on figure 5. In contrast, the semi-numerical model needs only 5 seconds for a ferromagnetic part decomposed in 6x3x1 blocks which sufficient accuracy for optimization needs.

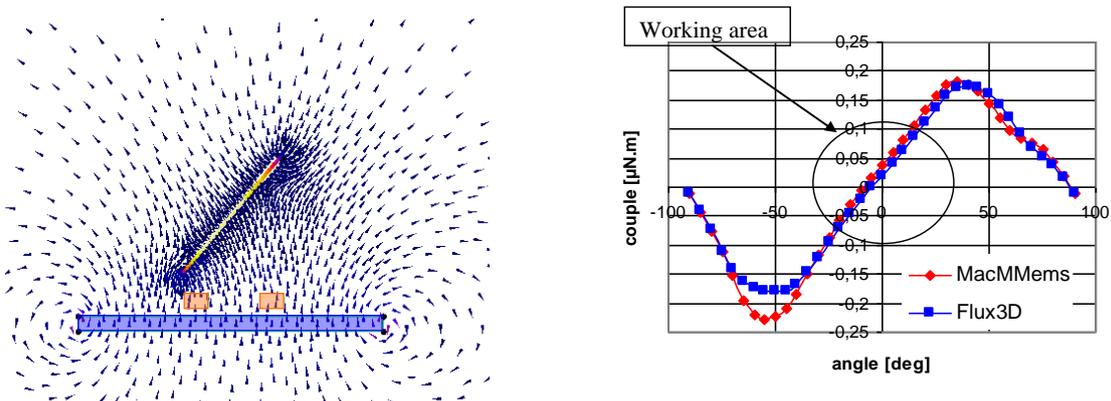


Fig. 5 - Left: 3D FEM solver (Flux3D). Right: comparison between FEM and semi-numeric solvers

C. Symbolic derivation

The following details how pure analytical model as well as mixed numeric-analytic model can be derivate in the CADES Framework [2].

1. CADES Framework with pure algebraic models

First of all, CADES framework is able to compute formal sensitivities from algebraic equations based on [7] work. To do so, model is analysed, treating equations and functions, finding parameters required for the

computation (input parameters such as geometry or physics parameters) and the resulting output parameters (such as magnetic field forces and torques). Partial derivatives for each equation or function are symbolically determined thanks to derivation rules. Model partial derivatives are then determined by the composition of equations and functions. Model's jacobian matrix is then explicitly defined by equations and functions.

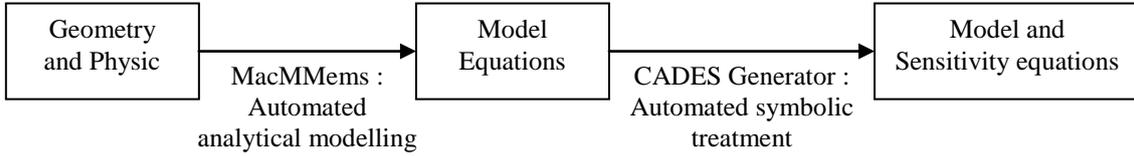


Fig. 6 - Modelling tools sequence diagram

2. CADES Framework with semi-numerical models

For numerical functions such as integrals (needed to compute torques and forces or fields produced by complex shapes) or numerical algorithm as previous ferromagnetic magnetization solver, it is easy to compute numerical sensitivities, but it is known that they brought optimization difficulties. For this reason, in CADES Framework, each numerical function is able to return partial derivatives determined symbolically as far as possible.

In the case of integration functions, partial derivatives can be fully expressed. Indeed, considering function $f(d,c,x)$ to integrate with 'x' variable, from 'a' to 'b' which can depend on geometric parameters like 'd'.

$$I(d, c) = \int_{a(d)}^{b(d)} f(d, c, x).dx \quad (15)$$

Partial derivatives expression depends on the position of the varying parameter. For instance, 'c' parameter is only on the integrand part, the expression is defined by equation 16.

$$\frac{\partial I(d, c)}{\partial c} = \int_{a(d)}^{b(d)} \frac{\partial f(d, c, x)}{\partial c}.dx \quad (16)$$

But for 'd' parameter, from which depends integration limits, the expression is defined by more complex equation 17. It can be noticed that the derivative of an integral can be computed without integral (this occurs, if the function f does not depend on parameter 'd') which is far faster than finite differences techniques.

$$\frac{\partial I(d, c)}{\partial d} = \int_{a(d)}^{b(d)} \frac{\partial f(d, c, x)}{\partial d}.dx + \frac{\partial b(d)}{\partial d} \cdot f(d, c, b(d)) - \frac{\partial a(d)}{\partial d} \cdot f(d, c, a(d)) \quad (17)$$

In the case of moment method algorithm, a symbolic derivation can be expressed for field, force or torque calculation depending on optimizable parameters (most of time geometric). This symbolic derivation is possible despite numerical algorithm, such as non linear equation solving to solve ferromagnetic magnetization

state. Indeed, torque, which can be constrained or optimized, is a function of the magnetization of the ferromagnetic material, the external field and system parameters. Its partial derivative is defined by equation 18.

$$\frac{d\Gamma(\mathbf{M}, \mathbf{H}_{ext}, p)}{dp} = \frac{\partial \Gamma}{\partial \mathbf{M}} \Big|_{H_{ext}, p} \cdot \frac{d\mathbf{M}}{dp} + \frac{\partial \Gamma}{\partial \mathbf{H}_{ext}} \Big|_{M, p} \cdot \frac{d\mathbf{H}_{ext}}{dp} + \frac{\partial \Gamma}{\partial p} \Big|_{M, H_{ext}} \quad (18)$$

$d\mathbf{M}/dp$ is the harder term to compute since it is obtained by numerical algorithm such as Newton-Raphson (see equation 14). But implicit theorem demonstrates that $d\mathbf{M}/dp$ can be symbolically expressed.

$$\frac{d\mathbf{H}_{ext}}{dp} + \frac{d\mathbf{Q}}{dp} \cdot \mathbf{M} + \mathbf{Q} \cdot \frac{d\mathbf{M}}{dp} = \frac{\partial f(\mathbf{M})}{\partial \mathbf{M}} \Big|_p \cdot \frac{d\mathbf{M}}{dp} + \frac{\partial f(\mathbf{M})}{\partial p} \Big|_M \quad (19)$$

$d\mathbf{M}/dp$ can then be found by solving the following linear system.

$$\left[\mathbf{Q} - \frac{\partial f(\mathbf{M})}{\partial \mathbf{M}} \Big|_p \right] \cdot \frac{d\mathbf{M}}{dp} = \frac{\partial f(\mathbf{M})}{\partial p} \Big|_M - \frac{d\mathbf{H}_{ext}}{dp} - \frac{d\mathbf{Q}}{dp} \cdot \mathbf{M} \quad (20)$$

Due to computation improvement requirements, the method used to compute $d\Gamma/dp$ is more complex [6]. $d\mathbf{M}/dp$ is not computed since it is not really necessary and it requires solving a linear system for each parameter 'p'. Instead of $d\mathbf{M}/dp$, an adjoint state vector, as described in [14], is computed once by solving a linear system, and $d\Gamma/dp$ is computed quickly for each parameter 'p'.

II. Optimization

A. Design specifications

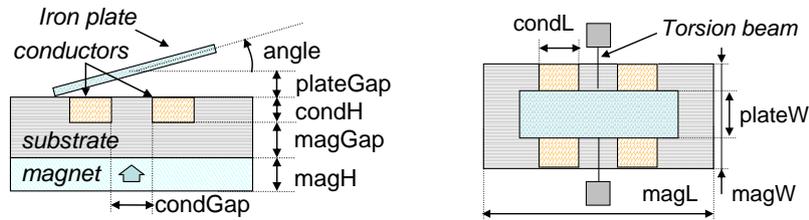


Fig. 7 – geometrical design parameters

Constant parameters are:

- magnet magnetization : 1 T,
- ferromagnetic plate dimensions : 1000x500x25 μm^3
- ferromagnetic saturation induction : 1T
- ferromagnetic initial permeability : 10000 T/(A.m)
- commutation angle : 5°
- resistive beam torque 0.227 $\mu\text{N.m}$

Table 1: Design specifications

Objective	Copper losses	To minimize
Constraints	Holding torque Actuation torque	< -0.1 $\mu\text{N.m}$ > 0.025 $\mu\text{N.m}$
Optimizabile parameters	Magnet high Magnet width Magnet length Magnet gap	[50 ;100] μm [1000 ;2000] μm [1000 ;2000] μm [1 ;100] μm

	Conductor high	[10 ;100] μm
	Conductor length	[100 ;500] μm
	Conductor gap	[100 ;1000] μm
	Conductor 1 current	[1 ;10] A
	Conductor 2 current	[1 ;10] A

B. Multi-level optimization

To improve speed convergence and accuracy, optimization strategies often use several modelling level. A two level strategy consists on using a coarse model for its calculation speed, and a fine model for its calculation accuracy. Mixing both, in an optimization strategy leads to faster results.

Our modeling method allows fine and coarse model thanks to meshing size of ferromagnetic part, and adaptive numerical integration accuracy parameter. A minimal accuracy of the coarse model has been validated in figure 5. Its calculation speed is 20 times faster than fin model calculation speed.

The coarse model can be used to find a first solution which can be followed by the fine model optimization in order to adjust solution regarding accuracy approximation of the coarse model.



Fig. 8 – discretizing for coarse model is $6 \times 3 \times 1 = 18$ blocks, for fine model $12 \times 6 \times 2 = 144$ blocks

Thanks to the formal derivation of system modeling, gradient based optimization can be done on both coarse and fine model. Unfortunately, gradient based optimization algorithms have local convergence regarding parameters initialization. It means that a better solution can be found with other initial values.

C. Global optimization strategies

1. Evolutionary Strategy optimization

Meta-heuristic algorithms, like genetic ones, try to find the global optimum but they lack of accuracy and need lot of model computations. To show this, an optimization has been done with an Evolutionary Strategy (ES) algorithm [13] (15 parents, 100 children, 75 generations leading to 7516 iterations during 47 minutes). The optimization converges to a quite good solution meeting constraints requirements, but the simulation of this solution with the fine model gave unfeasible constraints.

Table 2: Evolutionary Strategy (ES) optimization

	ES on coarse	Computation of fine
Copper losses (mW)	18,1	18,1
Holding torque ($\mu\text{N.m}$)	0,10015	0,093
Actuation torque ($\mu\text{N.m}$)	0,0254	0,0249
Iterations number	7516	1
Duration (sec)	2820	15

In our study, this solution (Fig. 9) is good enough, but this methodology is not safe regarding constraints.

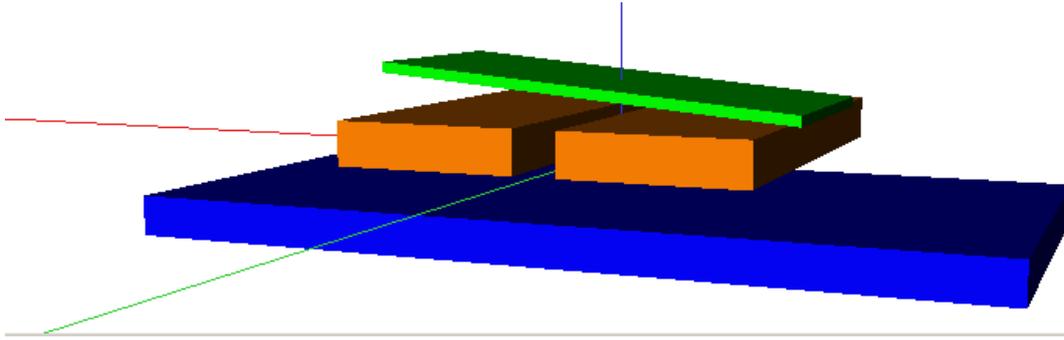


Fig. 9 – Evolutionary Strategy solution geometry

2. Multi-start gradient-based optimization

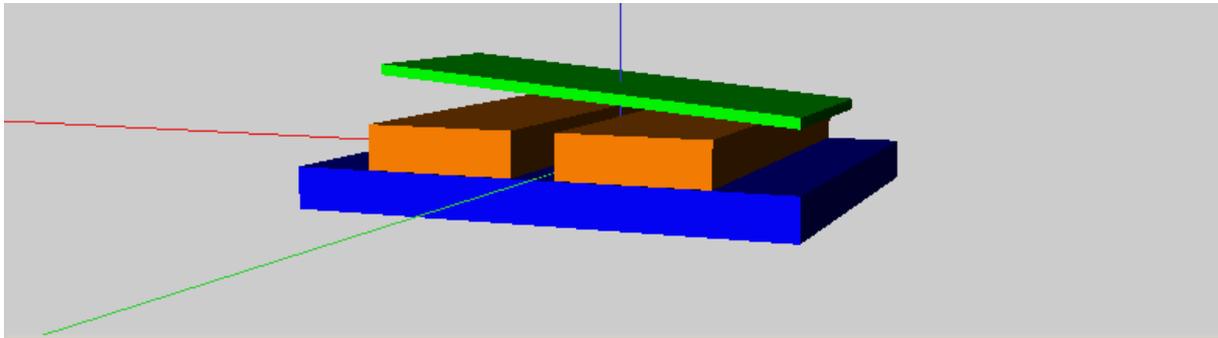


Fig. 10 – gradient based solution geometry

Figure 10 shows another solution (better) founded by another methodology. The previous optimization was done without gradients since any local information is needed for such an algorithm. The availability of gradients in our modeling allows good accuracy with gradient based algorithm such as SQP (Sequential Quadratic Programming) but its drawback is the initial configuration given to the algorithm. Considering a high speed convergence, several tries can be done with randomized initial configurations. 6 SQP were done independently and a good solution was found in few times with a so simple strategy. A good solution was founded in less than 8 minutes (6 times better). Theses results highlight potentiality of gradient based strategies in the scope of global algorithm.

Table 4: Multi-start SQP optimization strategy

	Best of 6 SQP on coarse	SQP on fine
Copper losses (mW)	13,1	14,7
Holding torque ($\mu\text{N.m}$)	0,1	0,1
Actuation torque ($\mu\text{N.m}$)	0,025	0,025
Iterations number	89	4
Duration (sec)	235	227

In the following figure, copper losses, holding torque and actuation torque are plotted during optimization iterations. 6 Solutions are marked with a circle. Each satisfies constraints but one has a better minimization than other at the iteration 47.

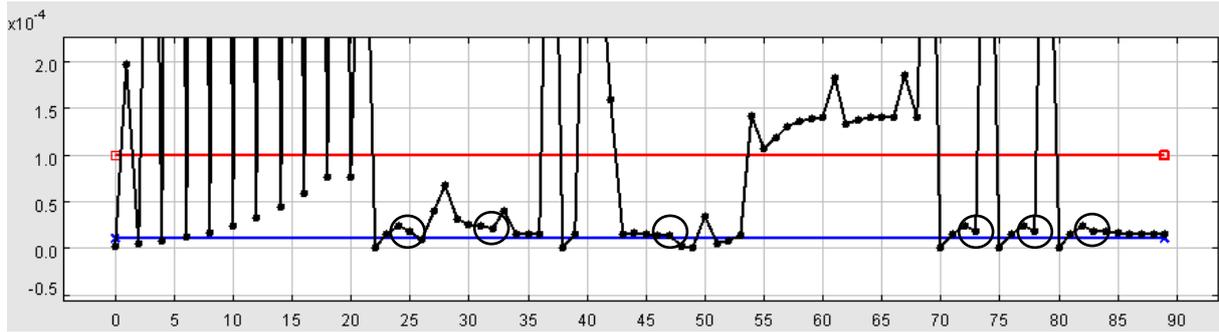


Fig. 12 – Multi-start strategy, 6 minimizations of Copper Losses (best found at iteration 47)

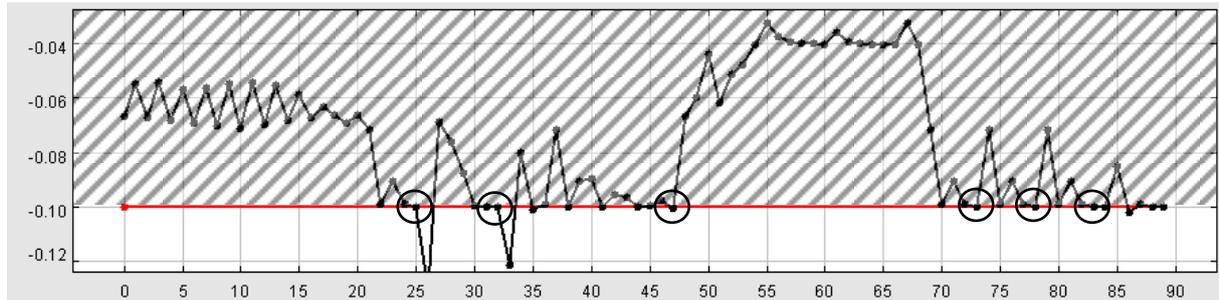


Fig. 13 – Multi-start strategy, Constraint validation (Holding torque < -0.1 $\mu\text{N.m}$)

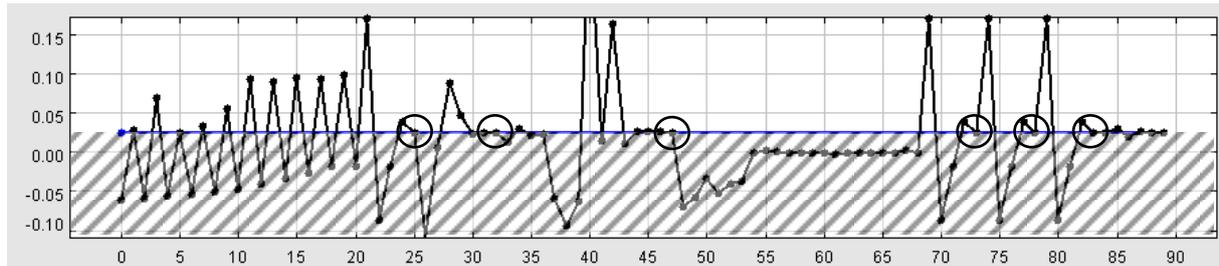


Fig. 13 – Multi-start strategy, Constraint validation (Actuation torque > 0.025 $\mu\text{N.m}$)

3. Mixed Genetic/SQP strategy

Another solution regarding SQP initialization issue is a genetic/gradient mixed strategy 0. Instead of random starting configurations, a genetic algorithm solution can be given to SQP in order to improve it. For the genetic optimization, the generation number has been reduced as well as constraints satisfaction since precision will be reached with gradients (SQP algorithm).

Table 3: Mixed Genetic-SQP optimization strategy

	Genetic on coarse	SQP on coarse	SQP on fine
Copper losses (mW)	14.1	13.1	14,7
Holding torque ($\mu\text{N.m}$)	0,099	0.1	0,1
Actuation torque ($\mu\text{N.m}$)	0,0253	0,025	0,025
Iterations number	150	8	4
Duration (sec)	400	42	227

In this last strategy, ES part remains the most time consuming (80%). Indeed, SQP on coarse model converge to a good solution very quickly as it can be seen in the following figures. But mixing both global and local

approaches allows finding a global accurate solution.

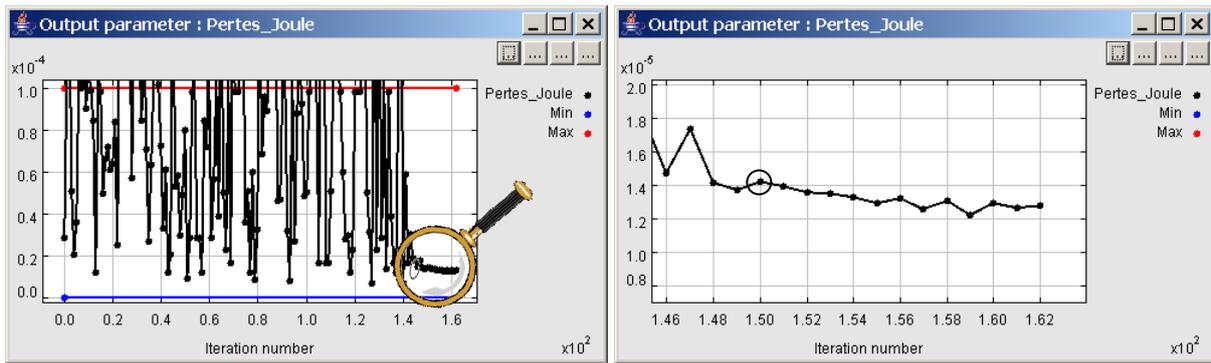


Fig. 12 – Mixed Genetic-SQP strategy- Minimizations of Copper Losses

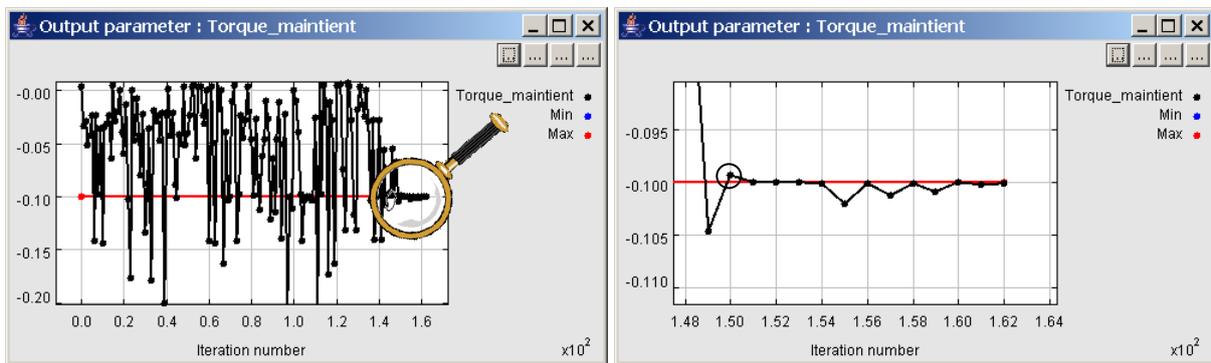


Fig. 13 – Mixed Genetic-SQP strategy- Constraint validation (Holding torque < -0.1 $\mu\text{N.m}$)

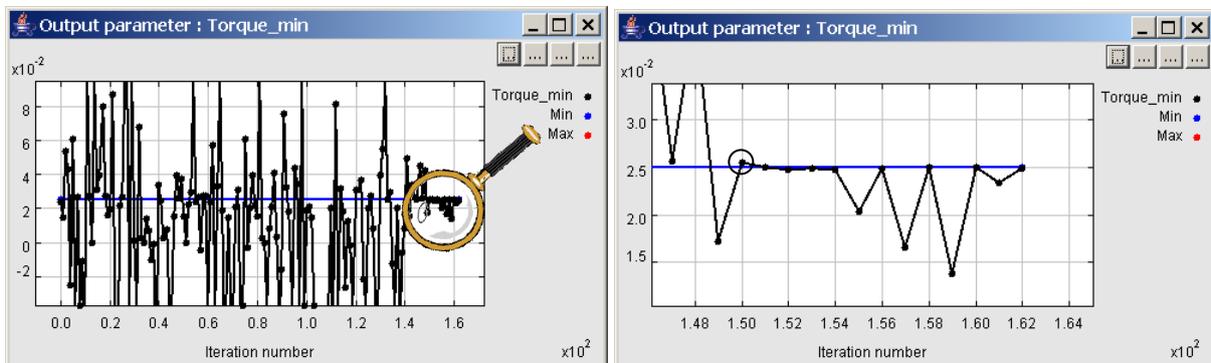


Fig. 13 – Mixed Genetic-SQP strategy- Constraint validation (Actuation torque > 0.025 $\mu\text{N.m}$)

III. Conclusion

In this paper, the use of semi-numerical modeling with symbolic sensitivity has demonstrated its usefulness in the area of gradient based optimization to design electromagnetic devices based on simple shapes like MEMS. It has been shown how electromagnetic interactions can be semi-analytically modeled. It has been also shown, from a semi-analytical (or semi-numerical) model, symbolic partial derivatives can be expressed depending on the analytical part and the numerical one. This knowledge has been capitalized in software to automatically produce modeling and sensitivities in few minutes with a simple device description.

Such a methodology, detailed on a model using integrals and method of moment solver, can be extended to

much more numerical solver. Symbolic sensitivities calculation is a hard work to do but it leads to optimization improvements regarding direct optimizations like genetic algorithms. It can be interesting to compare our methodology with other derivation techniques like automatic differentiation [15].

REFERENCES

- [1] H. L. Rakotoarison, B. Delinchant, O. Cugat "Methodology and tool for generating semi-analytical models used to pre-design electromagnetic MEMS", CEFC 2006, Miami, FL USA - April 30th - May 3rd 2006
- [2] B. Delinchant, D. Duret, L. Estrabaut, L. Gerbaud, H. Nguyen Huu, B. Du Peloux, H.L. Rakotoarison, F. Verdiere, F. Wurtz, "An optimizer using the software component paradigm for the optimization of engineering systems", COMPEL Vol 26, Issue: 2, 2007
- [3] H. L. Rakotoarison, J. P. Yonnet, B. Delinchant "Using Coulombian approach for Modeling Scalar Potential and Magnetic Field of a Permanent Magnet with Radial Polarization", IEEE Transactions on Magnetics., Vol. 43, Issue 4, april 2007, pp 1261-1264, ISSN: 0018-9464
- [4] -H.L. Rakotoarison, B. Delinchant, O. Cugat "Efficient and accurate modelling of Mag-MEMS for the optimization of system design", OIPE 2006 –September 13th – 15th 2006, Sorrento (Italy)
- [5] -H. L. Rakotoarison, B. Delinchant, O. Cugat "Methodology and tool for generating semi-analytical models used to pre-design electromagnetic MEMS", CEFC 2006, Miami, FL USA - April 30th - May 3rd 2006.
- [6] G. Akoun, J. P. Yonnet, "3D analytical calculation of the forces exerted between two cuboidal magnets", IEEE Transactions on Magnetics Vol. Mag-20 No. 5, pp. 1962-1964, September 1984.
- [7] Atienza, E.; Perrault, M.; Wurtz, F.; Mazauric, V.; Bigeon, J.; "A Methodology for the Sizing and the Optimization of an Electromagnetic Release", IEEE Transactions on Magnetics, Volume 36, Issue 4, July 2000 Page(s):1659 - 1663
- [8] H. L. Rakotoarison, V. Ardon, O. Chadebec, B. Delinchant, S. Guerin, J. L. Coulomb "Formal Sensitivity Computation of Magnetic Moment Method", to be published in IEEE Transaction on Magnetics, April 2008
- [9] O. Chadebec, J. L. Coulomb and F. Janet, "A Review of Magnetostatic Moment Method", IEEE Transactions on Magnetics, Vol: 42, No: 4, April 2006, pp.515-520.
- [10] Euleaume P., Chubar O., "Computing 3D magnetic fields from insertion devices", IEEE, 12 mai 1997, ESRF, Grenoble
- [11] B. Delinchant, F. Wurtz, D. Magot, L. Gerbaud "A component-based framework for the composition of simulation software modeling electrical systems", Journal of Simulation, Society for Modeling and Simulation International, Special Issue: Component-Based Modeling and Simulation. Jul 2004; vol. 80: pp 347 - 356.
- [12] H.H. Nguyen, F. Wurtz, N. Retiere "Composition of Optimization Algorithms by using Software Components for Optimization of Electromagnetic Devices", COMPUMAG'07, June 24-28, 2007, Aachen, Germany
- [13] T. Bäck, «Evolutionary Algorithms in Theory and Practice», Oxford University Press, New York, 1996
- [14] S. Gitosusastro, J.L Coulomb and J. C. Sabonnadiere , "Performance Derivative Calculations And Optimization Process", IEEE Transactions on Magnetics, Vol: 25, No: 4, July 1989, pp.2834 -2839.
- [15] P. Enciu, L. Gerbaud, F. Wurtz, "Practical use of automatic differentiation for optimization of electromagnetic devices", OIPE 2008, 14 - 17 September 2008, Ilmenau, Germany