

## Conception and Comprehension of the Contour Beam Antenna in a Satellite System by Multi-objective Genetic Algorithm with Sensitivity Analysis

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### 1. Abstract

To obtain the maximum performance of any device the designer needs to have a large experience in the entire universe where it is inserted. Consequently, more important than to find an adequate optimized solution is to comprehend the whole problem, giving to the designer opportunity to make choices and also to learn about the problem. A complete optimization procedure with sensitivity analysis is shown in this paper, which the chief aim is to increase the comprehension of complex engineering conception problems. The multi-objective genetic algorithm and two different approaches of sensitivity studies are shown running together. The contour-beam-shaped reflector antenna for specific illumination in a satellite system is solved as application problem.

**2. Keywords: Multi-objective Optimization, Sensitivity Analysis, Contour Beam Antenna, Satellite System.**

### 3. Introduction

Contour beam antennas are used on geostationary satellite to provide an efficient coverage of the service area. In many cases, their design has also to satisfy copolar and crosspolar isolation between different areas, gain slope at edges of the coverage area, and multiple frequency bands. As satellite expected life service increases, it creates a demand for reconfigurable contour beam antennas to cope with the changes in the service area. It defines a scenario where the antenna design has to simultaneously satisfy multiple objectives.

Among the possible configurations used for onboard antennas, shaped reflector antennas fed by a single feed have been considered. The reflector shaping techniques employ an optimization method to adjust the reflector surface in order to satisfy the desired specifications. The usual optimization methods are deterministic [1,2], leading to an optimum solution at the end of iteration and leaving no freedom for the designer to accommodate further adjustments.

Multi-objective Genetic Algorithm (MGA) is a stochastic-based method that yields a set of efficient solutions (Pareto-front), that simultaneously satisfies several goals [3,4]. The analysis of the Pareto-front behavior permits to understand the tradeoff between the different objectives. Compared with the deterministic optimization methods, which lead to unique solution, MGA offers the possibility to the designer to make the final choice among the set of solutions by considering additional constraints not included in the initial steps.

In this work, we apply the MGA in the synthesis of a shaped reflector antenna that is designed to provide Brazilian territory illumination and simultaneously minimize the energy level on the adjacent area. Beyond the search for the coverage specifications, the sensitivity analysis is included. This analysis consists in studying the real world inevitable deviations from project specifications, like constructing inaccuracies or different operational circumstances. We work here with two different sensitivity approaches. The first one consists in a classical approach for optimization parameters [5], but using only and directly the data from the stochastic MGA process, to study variations in the construction devices step mainly. In the second approach, the sensitivity study concerns the model specifications [6]. It means that a complementary analysis is applied to fixed parameters, which are not used as optimization parameters. For instance, this approach could to verify the effects of satellite position changes on the geostationary orbit.

The results obtained via the multi-objective optimization including sensitivity analysis help to improve the comprehension of the problem.

### 4. Multi-objective Genetic Algorithm

The Genetic Algorithm (GA) is a stochastic procedure based on the concepts of natural selection and genetics. There are many papers showing the effectiveness of the GA to solve engineering optimization problems. In most real-world problems, several goals must be satisfied simultaneously in order to obtain an optimal solution. As these objectives are usually conflicting, no single solution may exist that is best regarding all considered criteria. Multi-objective optimization (also called multicriteria, multiperformance or vector optimization) seeks to optimize the components of a vector-valued cost function. Unlike single objective optimization, the solution to this problem is not a single point, but a family of efficient points. Each point on this surface is optimal in the sense that no improvement can be achieved in a cost vector component that does not lead to degradation in at least one of the remaining components. Each element in the efficient set constitutes a non-dominated (non-inferior or non-superior) solution to the multi-objective problem. The main action of the multi-objective optimization is to determine the efficient front. With this set of solutions, it is possible to understand the dependence between each objective, which allows making efficient choices for the final solution decision. Fundamentally, in a multi-objective optimization procedure, given a set of  $n$  objective functions ( $f_i$ ) to be maximized ( $>$ ) or minimized ( $<$ ), one should find the set  $P$  of solutions defined as:

$$P = \{ \text{it does not have to exist } x \in S \mid \begin{array}{l} 1) \text{ exist } i \text{ such that } f_i(x) > f_i(x^*) \text{ and } \\ 2) \forall i \neq j \ f_j(x) \geq f_j(x^*) \end{array} \} , \quad (1)$$

where  $S$  is the feasible search space. The set  $P$  contains the *efficient solutions* ( $x^*$ ) of the problem and it is called the *Pareto-optimal set* [3,4]. With this done, we can choose one solution according to some preference (decision step).

The multi-objective genetic algorithm (MGA) used in this work is based on three current populations. Figure 1 shows the procedure. Basically, the algorithm starts with a set of solutions randomly created. These solutions are evaluated and the Pareto-optimal condition (1) is tested, giving two groups of solutions: one formed by efficient solutions, called *non-dominated* population (*NDOM*); and another by non-efficient solutions, called *dominated* population (*DOM*). An index (*IDOM*) indicating how many times each solution is dominated by others is created. This is important to execute the selection process only with solutions around the Pareto set, which makes the convergence faster. After the Pareto's check, it is time to apply the Clearing technique, whose purpose is to obtain a sparse and well-established Pareto front. If similarities among individuals are detected (in parameters or/and objectives spaces [7]), one or some of them are punished. The penalty consists in moving the penalized individual to *DOM* (by changing *IDOM* from 0 to 1).

To better control the whole evolutionary process we work with a fixed number of individuals (*nbind*) in the genetic operators actions (crossover and mutation). We call this fixed minimal size group "real population" (*REAL*), which is recreated at each generation. *REAL* is composed by all solutions of *NDOM* (after Clearing); plus  $1/4 * nbind$  solutions of *DOM* to maintain a certain diversity (chosen among those having the minor *IDOM* indexes). If the number of *REAL*'s individuals is still lesser than *nbind*, we complete with individuals of *DOM* (chosen another ones having the minor indexes once again). The opposite will be handled in the selection process. The number of selected individuals picked up from *REAL* is always *nbind*. The selection is done by pulling together a Deterministic Sampling and Tournament methods. The parents are in part selected by Deterministic Sampling (based on the population fitness average, which gives emphasis to the Pareto center area) and the remaining by Tournament (based on each objective individually *n objectives* times, which gives emphasis to the Pareto edges areas). This approach makes easier the attainment of a well-established Pareto set. After selection we apply the crossover and mutation operators for real coding schemes [8]. Generation after generation, these operators create new individuals (children) from existing ones (parents), driving to a good "search space" investigation. Attention, children do not automatically respect all the constraints (fixed maximal dimensions, for example). In this case, classical techniques are applied: for instance, values above a given limit are reduced using a simple "adjustment by saturation" [4].

The new individuals (children) are evaluated and directly inserted in *REAL*, whose *all* solutions are submitted to Pareto-optimal condition (1). This results in a modified *NDOM*, whose size fluctuates (increases and decreases) continually, while the *DOM* can only grow. Similar to a mono-objective *GA*, new good solutions can appear at any step in the MGA methodology. However, these new individuals can be lost or destroyed during the evolutionary process. The concept of Global Elitism [8] guarantees the permanence of efficient solutions. In the multi-objective algorithm proposed here, the global elitism is implicitly incorporated in the use of all *NDOM* to compound *REAL*. Finally, the evolutionary process is restarted with the new *DOM* and *NDOM*. To reduce the computational cost which is necessary to evaluate *IDOM*, the size of *DOM* population is controlled (to keep this maximum size some dominated solutions – with a bad *IDOM* index – are moved to an outside database (*DOMold*)). The end of the evolutionary process can be done by some criterion (e.g., maximum number of generations).

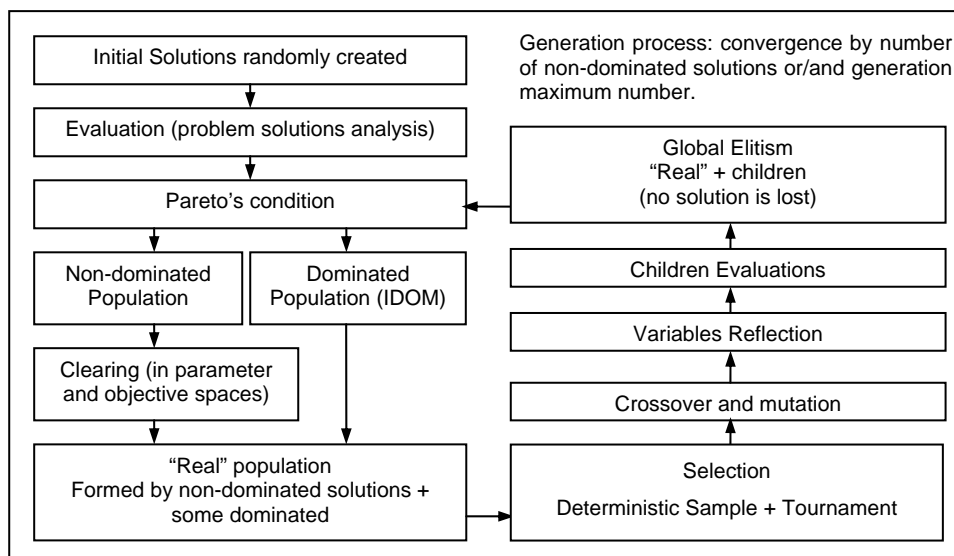


Figure 1. Multi-objective Genetic Algorithm procedure.

## 5. Sensitivity Analysis

In engineering design, the aim of the sensitivity analysis is to verify how stable a device performance is, since deviations from project specifications are inevitable in real world (imprecision of the construction, change of operational conditions due to deformation forces, ...). Two sorts of studies are done here. The first one is the sensitivity analysis in the parameters space and the second one is the sensitivity analysis to the model (predetermined parameters not present in the optimization process).

### 5.1 Sensitivity Analysis in the Parameters Space

Among several possible sensitivity estimators, the Lipschitz constant has been used. It is defined as the maximum variation rate in a finite domain, has been used [5]. Another method, based on containing ellipsoids, has been proposed [9] in order to take profit from the large amount of data generated in the stochastic search. In all cases, the precise evaluation of sensitivity quantities is intrinsically

very much time consuming. The procedure used here is based on a metric directly evaluated from the data provided by MGA. This methodology also permits to identify which solution parameter degrades more the performance when deviated. In order to validate the method, we apply it to an analytical function minimization and following to the antenna problem.

The main purpose of the MGA is to search for, among all the generated data, well-established samples of the Pareto-optimal front. If the optimization method convergence is not premature, the feasible space is well represented [7]. Many samples fall around Pareto's region while the MGA is searching for the solution, as shown in Figure 2a, so that it is reasonable to use this data in the sensitivity analysis.

In a first step, we search for the maximum variation on the values of each objective function related to deviations from the optimal parameter values ( $V^*$ ). Following, we select some samples of the Pareto-set are selected to perform the sensitivity analysis. Both metrics are evaluated using only the already acquired samples inside a perturbation domain around  $V^*$ .

Figure 2a shows the data generated by MGA in the simultaneous minimization of two quadratic objective functions ( $f$ ) with two design variables ( $x$ ):

$$f_1 = (x_1 - 1)^2 + x_2^2 \quad \text{and} \quad f_2 = (x_1 + 1)^2 + x_2^2. \quad (2)$$

The optimal region is a line connecting the parabolas vertices.

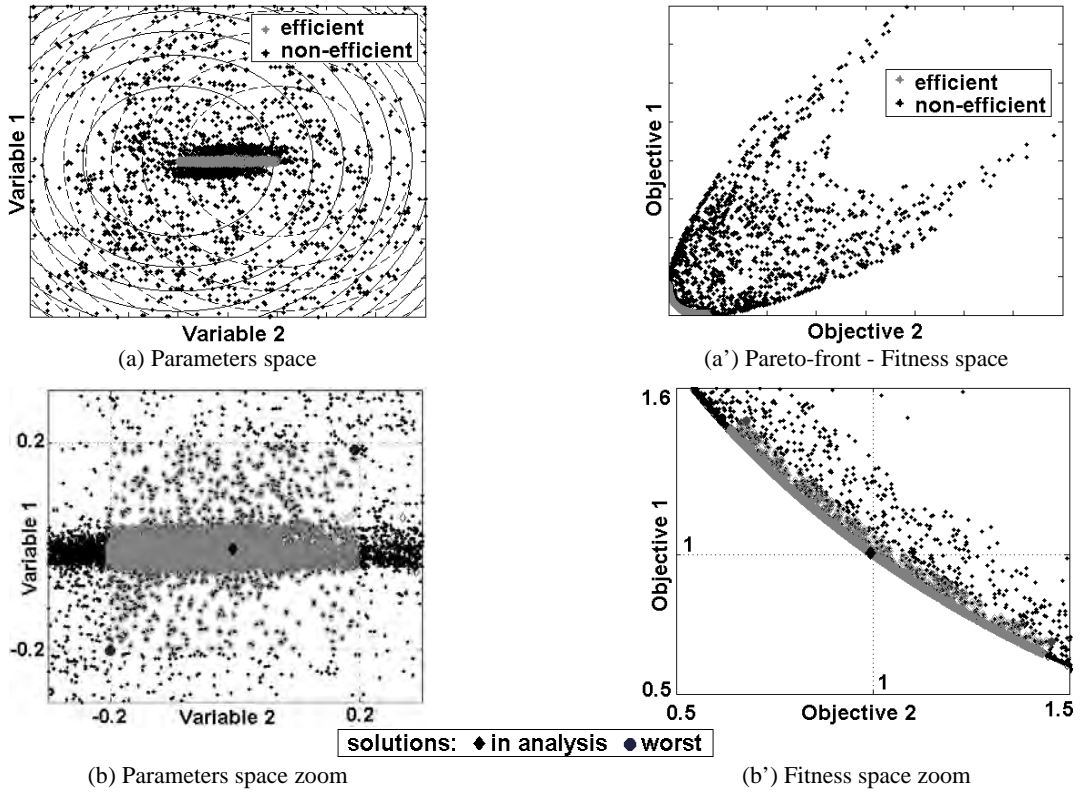


Figure 2. Two parabolas – analytical function test (2).

In this analytical example, we choose the Pareto sample  $V^* = [-0.003387, -0.004788]$ , which is the closest solution to fitness  $[1, 1]$  found by MGA. We defined the perturbation domain  $D^*$  as a deviation of  $[-0.2, 0.2]^2$  around  $V^*$ , and then took the sample with the worst fitness inside it. In this example, the worst cases for both objective functions are on the domain corners. The designer considers the solution stable if this worst fitness is acceptable.

To compare how stable is a Pareto sample in relation to others, it is essential to associate the fitness values with the distance in the parameter space. We define the behavior associated with an optimal point  $V^*$  for each objective function  $f_k(\cdot)$  inside the perturbation domain  $D^*$  as:

$$B_k(V^*) = \max_{V_i, V_j \in D^*, i \neq j} \frac{|f_k(V_i) - f_k(V_j)|}{\|V_i - V_j\|}, \quad (3)$$

where  $V_i$  and  $V_j$  are samples inside  $D^*$ . Note that  $D^*$  can be any domain containing  $V^*$ . The evaluation of the behavior is straightforward, since we use only points already sampled. Furthermore, it is a value asymptotically close to the Lipschitz constant as the number of samples tends to infinite all over  $D^*$ . That means it is an approximation to the maximum mean rate between two points inside  $D^*$ . The greater  $B_k$  is, the more unstable the function  $f_k$  at  $V^*$  will be. Since distance and objective function variation are positive values in  $B_k$  evaluation using (3), we may normalize them according to the optimal solution in analysis  $V^*$  – normalize each parameter by its respective  $V^*$  component absolute value, and function variation by  $|f_k(V^*)|$  – in order to connect directly sensitivity to significant digits.

It is enough to perform the behavior analysis to each variable separately in order to point out which solution parameter is more relevant to maintain a good device performance. In this case, to create the domain  $D^*$  we achieve a large perturbation to the parameter in analysis (like for all variables sensitivity analysis case) and small perturbations (a tiny percentage) to the others ones. The procedure shown is not exact, but it gives us a satisfactory notion of sensitivity. Moreover, no additional computation effort is necessary if the optimization stochastic method works well (allowing a good exploration of the feasible search space). The analysis method described here uses direct variables. If using indirect parameters like expansion series, the proposed analysis could not have a real meaning. In that case, it may be necessary to create a parameterization for the perturbation, which must be linked to that used in the optimization process, in order to keep on using the samples. The antenna problem solved in this paper is like that: the shaped surface is described by an expansion global series. We show that changes in results section (§7.1).

## 5.2 Sensitivity Analysis to the Model

Besides the search for the main objectives, it is also possible to include the analysis of the effects of changes in the system specifications. It is a sensitivity analysis of the model specifications [6], which means, differently from the classical parameters sensitivity study (previous section), a complementary study applied to parameters that are not used as optimization parameters. As examples, in our case, displacements of the feed and reflector positions, the effects of changes of satellite position on the geostationary orbit, etc. Hence, these are details that the designer does not control but he can estimate possible alterations.

The sensitivity analysis concerning the model can be done after the efficient solutions set is found. The procedure is: all Pareto solutions and some dominated solutions (those who are the closest to the Pareto front – IDOM index) are re-evaluated with the changes in the model, getting thus a new disposition of the front. To estimate the solution stability, we use:

$$d_1 = (x - x_1, y - y_1) , \quad d_2 = (x - x_2, y - y_2) , \quad d = |d_1| + |d_2| \quad (4)$$

where  $d$  is the deviation imposed on the solution (for instance, two perturbations:  $d_1$  and  $d_2$ ) and  $x$  and  $y$  are two objectives. Of course, it is not recommended to add  $d$  for non-correlated perturbations. In this case, the analyses must be done separately or by  $\max(|d_1|, |d_2|)$ , for example.

There is one point to pay attention: it is important to know the difference between fixed specifications and possible optimization variables or objectives. System demands must be objectives. If a fixed specification is very important to maintain the system performance, may be it is better to put this parameter as optimization variable. Model perturbations have as goal to improve the final solution quality. The sensitivity analyses can be optimization objectives, but this increases the complexity and the computational cost of the problem resolution.

In the results section (§7), both sorts of sensitivity analysis will be applied, helping in the comprehension of them.

## 6. Satellite System Arrangement

The satellite antenna configuration is a single offset reflector illuminated by a corrugated feed horn, operating at 12 GHz (Ku Band). The reflector projects a circular aperture  $A$  with diameter  $D$ , offset point  $H$  and equivalent focal distance  $F$ , like shown in Figure 3.

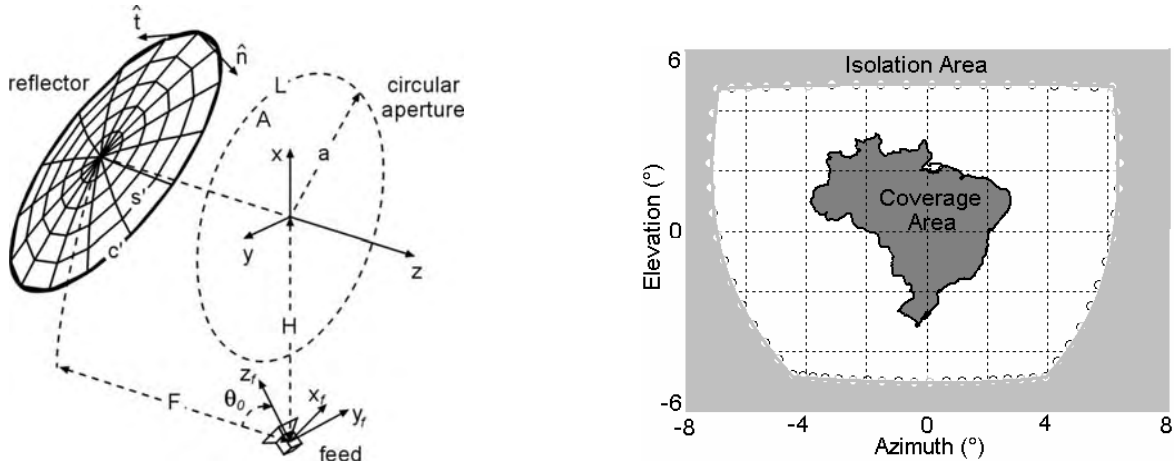


Figure 3. Geometry of the single-feed offset reflector antenna and coverage problem proposed.

The shaped reflector surface is described by an expansion of Fourier-Jacobi series [2]:

$$z'(t, \phi) = \sum_{n=0}^N \sum_{m=0}^M (C_{nm} \cos n\phi + D_{nm} \sin n\phi) F_m^n(t) . \quad (5)$$

The  $\cos^q \theta$  model describes the feed radiation pattern where  $q$  is chosen to provide a 12 dB attenuation at the reflector edge. The satellite beam points to the center of the Brazilian territory, as illustrated in Figure 3. The reflector shaping has to maximize the minimum directive gain (GT) in the coverage area and simultaneously minimize the maximum directive gain (GI) in the adjacent

area (gray), in order to avoid interference. In the synthesis, these gain specifications are attained by employing MGA to adjust the expansion coefficients of the Fourier-Jacobi series (5). In order to get information on how good is a given radiation pattern,  $N$  sampling points are placed on the whole Brazilian territory and isolation area. The distance between the sampling stations must agree with Nyquist rate and Gibbs' phenomenon [10]. The fitness function is just the search for the worst evaluation for all sampling points (to coverage area:  $\min(\text{GT})$ , to isolation area:  $\max(\text{GI})$ ). The directive gain is obtained by radiated far field computation [11]. It can be evaluated by using Physical Optics (PO):

$$\vec{J}_{PO} \approx 2\hat{n} \times \vec{H}_{inc} \quad (6)$$

where  $\vec{H}_{inc}$  is the incident magnetic field. The radiated far field of the PO can be written as

$$\vec{E}_{PO} \approx -j \frac{k\eta}{4\pi} \frac{e^{-jkr}}{r} \int_{s'} [\vec{J}_{PO} - (\vec{J}_{PO} \cdot \hat{r})\hat{r}] e^{jk\vec{r}' \cdot \hat{r}} ds' \quad (7)$$

where  $k$  and  $\eta$  are, respectively, the wave number and the intrinsic impedance of the medium,  $s'$  is the reflector surface,  $\hat{r}$  is the unit position vector of the observation point and  $\vec{r}'$  is the vector of the source point. The PO analysis is not accurate when used for predicting far-angular regions or cross-polarized fields. In order to improve the precision of (7), edge diffraction can be taken into account [12]. The total field is then given by the direct (feed field  $\vec{E}_f$ ) and indirect (PO field  $\vec{E}_{PO}$  and fringe field  $\vec{E}_{fr}$ ) components:

$$\vec{E} = \vec{E}_f + \vec{E}_{PO} + \vec{E}_{fr} \quad (8)$$

The antenna chosen and the application problem are simple to facilitate the comprehension of the optimization procedure, not being necessarily the most correct for a real communication system.

## 7. Optimization Results

Several MGA executions had been carried out to guarantee the Pareto-front found. Figure 4 displays the set of solutions where it is possible to observe that the increase in the value of GT causes the increase of GI. This is simple to explain: to augment GT, it is required to spread the energy, which results in an increase of GI. This comprehension comes from MGA – Pareto front. Understood the tradeoff between the objectives, the designer can proceed with the decision step to choose one solution according to some preference. Otherwise, beyond the attainment of the efficient solutions, we can start the sensitivity analysis complementary studies.

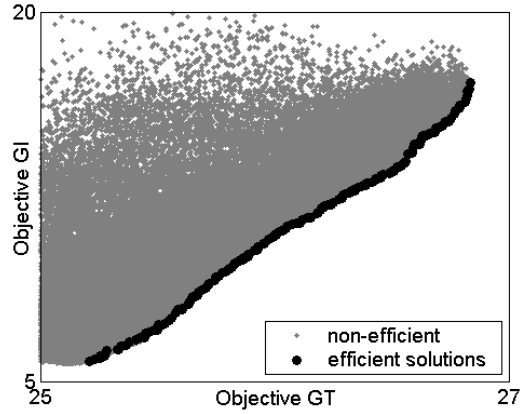


Figure 4. Pareto-front (scales in dBi).

### 7.1 Application of Sensitivity Analysis in the Parameters space

We have chosen two solutions to perform this sensitivity analysis: S1, with  $\text{GT} = 25.50\text{dBi}$  and  $\text{GI} = 7.08\text{dBi}$ ; and S2, with  $\text{GT} = 26.50\text{dBi}$  and  $\text{GI} = 13.53\text{dBi}$  (see Figure 5). It is required to change the proposed sensitivity procedure to do this study here. In this case, the parameters adjusted are not direct variables but rather a global expansion series. As a result, little changes in only one parameter could not have real meaning, for example. To control real-world perturbations on the reflector shape, like thermal deformations or mechanical efforts, we can work in a reverse approach (without using complex thermal models) like that: accepting a variation of  $0.25\text{dBi}$  in GT objective and  $0.5\text{dBi}$  in GI objective as tolerable, a finite domain of solutions is defined around the selected S1 and S2. We search into this domain the solution that has the biggest disparity of its surface in comparison with the selected solution shape. That solution having the biggest difference, with the same domain size, is considered the more stable solution. Of course it is necessary to certify if the perturbation occurred is real and possible to happen.

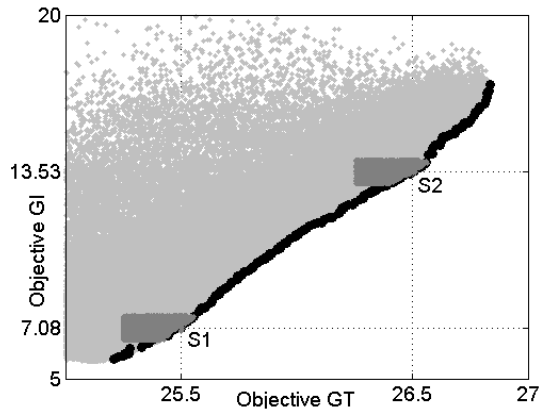


Figure 5. Selected solutions and respectively finite domains (in dBi).

Figure 6 shows the difference of the extremes solutions to S1 and S2 (only one extreme for each solution as illustration). In this case, considering the four extremes for each solution, we can say that S2 is more stable than S1. It means that S1 changes more your directive gains when little perturbations are done in your parameters than in S2. This can be a kind of complementary study to understand the problem behavior.

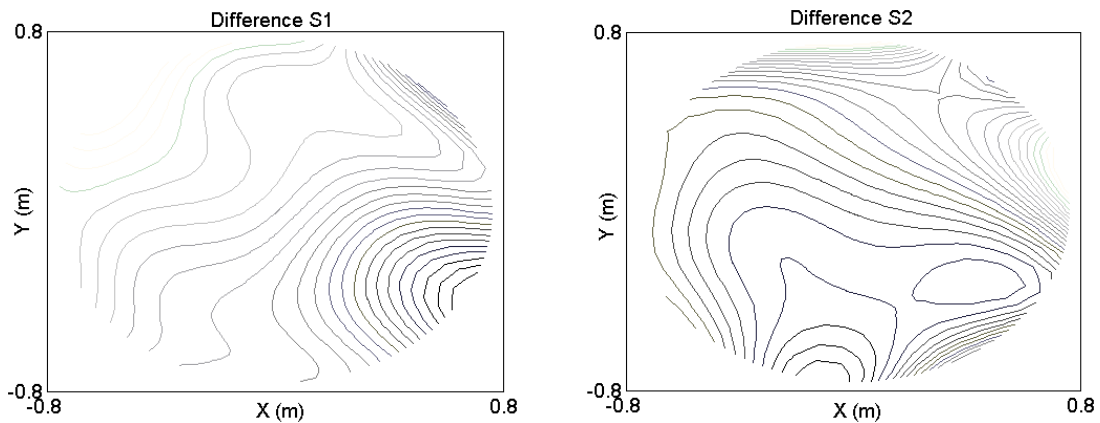


Figure 6. Differences in "z" coordinate between the selected solutions and theirs respective extremes.

## 7.2 Application of Sensitivity Analysis to the Model

For testing the procedure, we chose two perturbations:

### Satellite displacement

Due to losses, satellites position in the geo-stationary orbit shows small variations that require further correction procedure. To take into account these orbital oscillations, this sensitivity study considers  $\pm 1.0\%$  perturbation ( $\Delta r$ ) in the satellite position. Figure 7 (a) shows the solutions behavior (GI and GT are given in dBi).

### Feeder $\times$ Reflector System

Alterations due to thermal gradient, mechanical forces and other external agents can cause changes in the distances and angles of the feeder  $\times$  reflector system. In this exercise we work with the focal distance ( $F=1.524\text{m}$ ), diameter reflector ( $D=1.803\text{m}$ ), feeder displacement offset ( $H=1.278\text{m}$ ) and feed offset angle ( $\theta_0=40^\circ$ ). The perturbation was set as  $\pm 0.25\%$  in the mentioned parameters (e.g.,  $\theta_0=40^\circ \pm 0.1^\circ$ ). Figure 7 (b) shows the solutions behavior.

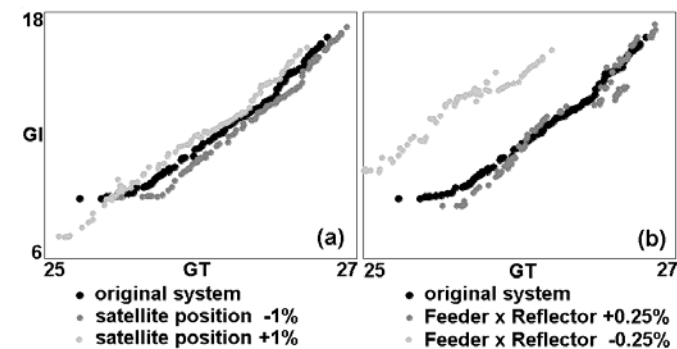


Figure 7. Pareto-fronts (in dBi) on Sensitivity Analysis to the Model.

The results displayed in Figure 7 (a) show that the  $\Delta r = -1\%$  in the satellite orbit defocuses the beam and raises the sidelobe levels, bringing, as consequence, the reduction of GT and the increase of GI. While, for the  $\Delta r = -1\%$  the opposite is verified. Figure 7 (b) shows that the GT are largely affected by lowering the initial specifications (F, D, H and  $\theta_0$ ). With the Pareto-fronts found, we can calculate the deviation (3) to find the most stable solution. Table 1 shows the minimum deviation among the solutions for each kind of perturbation and also the two alterations together (for that it is required to re-evaluate with both alterations simultaneously).

Table I. Sensitivity Analysis concerning the model.

CHANGES	MINIMAL DEVIATION	GT	GI
SATELLITE DISPLACEMENT	1.034	26.81	16.78
FEEDER $\times$ REFLECTOR	1.315	25.96	11.22
BOTH PERTURBATIONS	2.661	26.14	11.81

GT and GI in dBi, minimal deviation in uncertainty unit.

### 7.3 Final Decision Step

Based in all these analysis (Pareto-front behavior and both sensitivity studies), the designer can choose the final optimized solution or restart the optimization process with different and/or more specifications to use as advantage a better knowledge of the problem. Figure 8 shows the radiation pattern of the most stable solution found with a minimum of 26dBi direct gain in the coverage area. Figure 9 shows differences in “z” coordinate between the final solution and a perfect parabolic shape. Although the optimized reflector shows not big deviations from a parabola, the difference is more intense at the edge (0.11m maximum – approximately  $3\lambda$ ) of the reflector and it defocus the reflected rays in order to provide the adequate coverage of the desire far-field region.

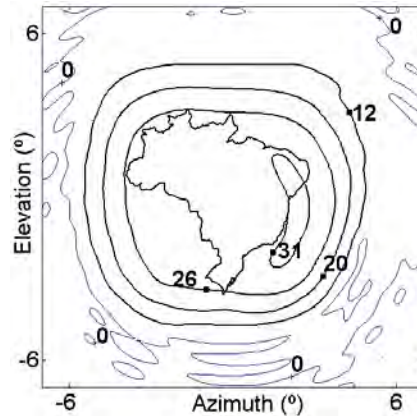


Figure 8. Footprint radiation pattern of the final solution (COPOL in dBi).

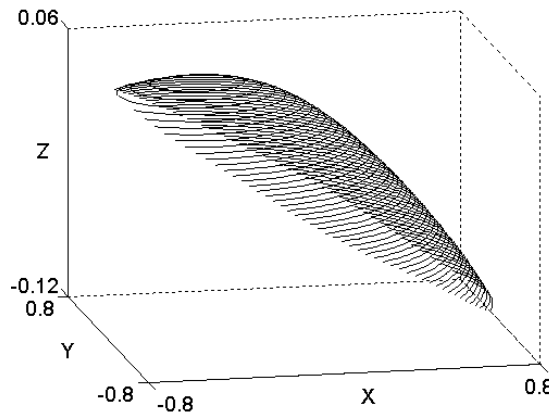


Figure 9. Differences in “z” coordinate (m) between the final solution and a perfect parabolic shape.

## 8. Conclusions

A study of multi-objective optimization with sensitivity analysis has been presented in this work. In order to explain the methodology, we considered the design of a shaped reflector antenna used in a satellite communication system. The main goal of this work is to improve the comprehension of engineering problems optimization. MGA features allow a large examination of its results and this is the biggest difference from deterministic methods. Based on all analyses and results in hand, the designer can either choose the final optimized solution in the Pareto set, or restart the optimization process considering different specifications and/or parameters. In that case, he will benefit from his better comprehension of the problem.

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