Impact of Tetrahedral Mesh Quality for Electromagnetic and Thermal Simulations

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***Abstract*—Finite element simulation can be directly affected by mesh quality. The accuracy of finite element calculations is dependent upon of the mesh quality. Previously, we have introduced a novel approach to the construction of high-quality, isotropic tetrahedral meshes from segmented medical imaging data. This article proposes an experimental evaluation of the impact of our tetrahedral meshes on electromagnetic and thermal simulations with finite elements.**

I. Introduction

In this paper, we show the impact of the geometrical modelling on the numerical simulation of electromagnetic and thermic phenomena in the human body exposed to radiofrequency field. We present the electromagnetic [1] and thermal Finite Element Models (FEM). The time harmonic formulation is directly written in term of total electric field $\mathbf{E}$ (1). It is obtained by applying the Galerkin method to the wave equation. Coupling to a first order Engquist-Majda Absorbing Boundary Conditions (ABC) taking into account the open boundary:

$$-\int_v \nabla W \times \nabla \times \mathbf{E}.dv - \int_v W k_0{}^2 \varepsilon_t{}^* \mathbf{E}.dv + \int_v W g_{ABC}(\mathbf{E}).ds = -j\omega\mu_0 \int_\tau \mathbf{J_e}.dv \quad (1)$$

with $g_{ABC}(\mathbf{E}) = j k_0 \mathbf{E}_t$, where $\mathbf{E}_t$ is the tangential field, k_0 is the propagation constant of the electromagnetic field, W the weight function, $\varepsilon_t{}^*$ the tissue complex permittivity values and $\mathbf{J_e}$ the electric current density. Space discretization is performed using incomplete first order edge elements.

Temperature calculation is based on the solution of the instationary Pennes bio-heat equation (2) with $\mathbf{E}$ as source term.

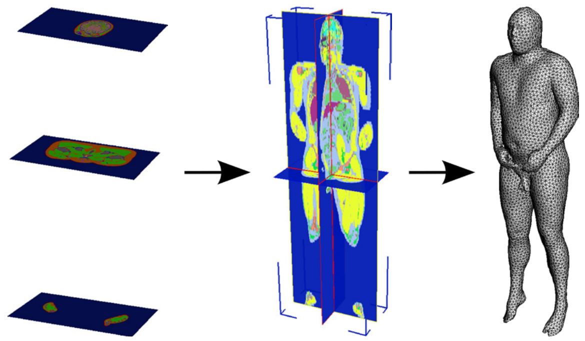
$$\rho C \frac{\delta T}{\delta t} = \nabla.(k\nabla T) - C_b\omega_b(T - T_a) + \frac{1}{2}\sigma|\mathbf{E}|^2 + Q_m \quad (2)$$

with ρ, the tissue density, C the specific heat of the tissue, k the thermal conductivity, ω_b the blood flow, C_b the specific heat of the blood, T_a the temperature of the arteries, σ the electrical conductivity and Q_m the amount of heat produced by metabolism. Thermal modelling is carried out with classic nodal FEM. We use conjugate gradient solver with various preconditioning techniques: diagonal, SSOR and Gauss.

II. Tetrahedral Meshes Generation

A. State of the Art

There are basically three approaches for tetrahedral meshing: greedy approaches, Delaunay-based methods and hierarchical decomposition approaches. The greedy approaches start from a boundary and move a front from the boundary towards the empty space within the domain. Delaunay approaches generate triangulations using Delaunay criteria. Unfortunately, in 3D and higher dimensions, the Delaunay property alone is insufficient to guarantee well-shaped elements. Finally, hierarchical decomposition approaches recursively subdivide the cube containing the geometric model until the desired resolution is reached. Elements that lie outside the meshing domain, and the elements inside the domain are split into tetrahedra. Most of the previously cited approaches share a common point : they take an input surface and enrich it with new vertices to generate the tetrahedra. This can be problematic around the objects boundaries, as the vertices of the input surface can be an important constraint for the resulting mesh, and induce tetrahedra with bad aspect ratio.



(a) (b) (c)

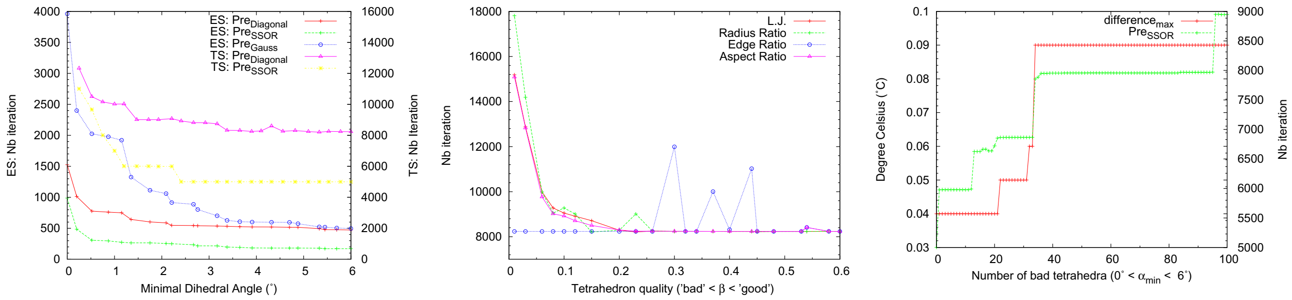
Fig. 1. Meshing process: the segmented ”*Visible Human*” Male Dataset (a), tridimensional discrete set (b), tetrahedral meshing (c) of (b) with our approach.

B. The Proposed Approach

We have proposed a novel tetrahedral mesh generation algorithm in [2]. Our algorithm directly processes voxels of segmented volumes coming from Tomographic Scanners or Magnetic Resonance Images (MRI) (Fig. 1). No polygonal input surface is needed. Our approach provides a robust mesh design tool for discrete data that can accommodate requirements on the final budget of vertices and on the mesh gradation, for arbitrary domain complexity.

C. Tetrahedra Quality Measure

In our case, the ideal isotropic tetrahedral element is the equilateral tetrahedron. An important parameter in this study



(a) (b) (c)

Fig. 2. Effect of the tetrahedral quality: (a) on the electromagnetic and thermal simulations, respectively ES and TS, for several preconditioning techniques; (b) on the thermal simulation for several quality criteria; (c) on the difference between the thermal problem simulation and the analytical solution.

is the choice of the quality criterion. Several quality measures are available in the literature. In this paper, we use five quality criteria : The first criterion is the Minimal Dihedral Angle (MDA) α_{min} of a tetrahedron, which has a maximal value of $\arccos(1/3) = 70,5°$, for a regular tetrahedron. Table I tabulates several quality measures for tetrahedral elements quality evaluation.

Criteria	MDA	L.J.	Radius ratio	Edge ratio	Aspect ratio
Formula	α_{min}	$\frac{12\sqrt[3]{9V^2}}{\sum l_{i,j}^2}$	$3\frac{r_{in}}{r_{out}}$	$\frac{l_{min}}{l_{max}}$	$\frac{12}{\sqrt{6}}\frac{3V}{l_{max}\sum A_i}$

TABLE I
QUALITY MEASUREMENT FACTORS

In this Table I, l is the length of the edges, A the areas of the faces of a tetrahedron, r_{in} the radius of its inscribed sphere, r_{out} the radius of its circumscribed sphere and V its volume. The criterion L.J. is cited in [3]. The criteria are normalized between 0 and 1 (expeted MDA), where 0 denotes a bad element and 1 an equilateral tetrahedron.

III. Effect of the Tetrahedral Quality

A. Theoretical Impact

The quality of geometric discretization is crucial for the effectiveness of these applications [4]. Coarse discretization and poor shape of the elements can introduce incorrect results and numerical errors. Degenerate elements with small volumes and small dihedral angles may lead to large local errors of the solution. Small dihedral angles can have a negative effect on the condition number of the stiffness matrix and large dihedral angles cause large interpolation errors. The worst impact results in an unsolvable system of equations.

B. Numerical Impact

The calculations are realized on a spherical geometry generated by our approach. The minimal dihedral angle is of $16.31°$. Figure 2.a shows the impact of the addition of a single bad tetrahedron on the electromagnetic (incident wave at 433 MHz) and thermal (temporal model for convection transfer heat) simulations for several preconditioning techniques. Figure 2.b presents the impact of the addition of a bad tetrahedron on the thermal simulation for several quality criteria. Figure 2.c shows the difference between the thermal problem simulation and the analytical solution [5] when several bad elements are

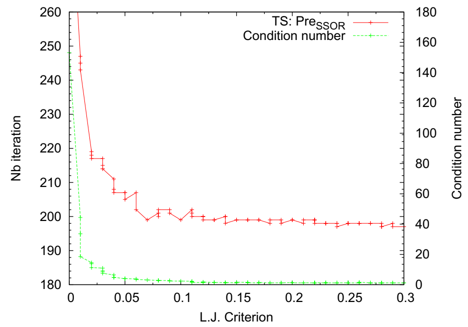


Fig. 3. Influence of the poor quality elements on the condition number for the thermal simulation.

added. The condition number for a positive definite matrix is defined by the ratio of the maximum eigenvalue to the minimum eigenvalue. Figure 3 shows the condition number and the number of iterations are highly correlated.

IV. Discussion and Conclusions

Certain geometrical quality measures are not adapted to evaluate the shape of an element. As an example, a bad-quality tetrahedron has a too great radius-edge ratio and a sliver has a good radius-edge ratio but nearly zero volume (slivers). The radius-edge ratio is not a proper measure for slivers. We have shown in a practical way the impact of the quality of tetrahedral meshes on electromagnetic and thermal finite element models. Our results clearly show that the FEM accuracy can be directly affected by mesh quality. Experimental findings suggest dramatic run time reductions using our high-quality mesher. According to this study, we chose the criteria based on the MDA and cited in [3] for the evaluation and the optimization of our mesh generation algorithm.

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