A general mixed boundary conditions definition for eddy current problems

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A GENERAL MIXED BOUNDARY CONDITIONS DEFINITION FOR EDDY CURRENT PROBLEMS

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ABSTRACT

The authors present a new method to compute the current distribution (eddy currents or conduction currents) in a solid or liquid material with boundaries made of another thin and conducting material. In this configuration numerical methods such as finite element techniques are unavailable due to the thickness of the boundary to be meshed . The authors propose to use a special finite element called "boundary element" and to introduce the characteristics of the containing medium into the boundary conditions applied to the main conducting material. The techniques for solving problems with such mixed boundary conditions are exposed and results for some conduction problems are given.

INTRODUCTION

Current distribution in conducting materials has to be defined in many problems in electrical engineering. Two kinds of applications require that distribution to be computed :

- eddy current problems : currents are generated by a magnetic flux variation. Examples can be found in NDT systems, induction heating devices, or in magnetic losses evaluation.
- conduction problems : currents are generated by a difference of electric potential. Examples can be found in fuses calculation, electrolytic systems design or in conduction ovens (glass ovens).

The governing equations of all these problems are obtained using well known theory (scalar or vector potential formulation). Numerical methods as finite element or boundary integral techniques can be used for solving such equations. However these methods are not suited to the solution of problems with very thin media.

That difficulty appears in NDT systems with thin cracks, in electrical machines analysis with small air-gap. That also appears in current problems, eddy currents in MHD systems when copper bars are used for current return, or conduction current when an electrolytic liquid is in a conducting recipient with a little thickness. For that problem we shall develop a technique to take into account these two media and especially to introduce physical characteristics (electric properties and geometric configuration) of the thin conducting medium with boundary conditions on the boundary of the other medium.

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PROBLEM DEFINITION

The general conducting materials geometry is shown on figure 1. A conducting medium with conductivity   is bounded by another thin conducting material of conductivity and thickness   .

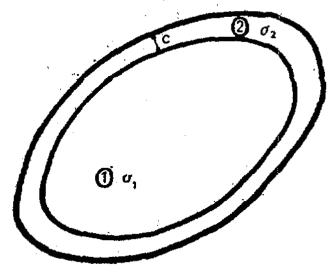

Figure 1 : Typical geometry for conducting materials association.

These two media are placed inside a non conductive environment, usually the air.

In these materials electrical currents flow. These currents are generated :

- either by electromagnetic flux variation. That is the case of MHD systems or electromagnetic pumps. Material 1 is in example made of molten metal and material 2 is made of copper.
- or by setting inside material 1 some electrodes (anode and cathode) and by imposing conduction current to flow from one to another. Material 1 is an electrolytic liquid, or molten glass. Material 2 is the recipient containing material 1 .

EQUATIONS

System equations are Maxwell equations and we shall develop the potential formulations of these equations in the 2D particular configuration. According to the problem under consideration two formulations can be used.

Electric vector potential formulation : many authors in previous papers have shown that this formulation was the best for eddy current problems. For 2D problems the equation on electric vector potential T is :

$$\nabla^2 T - \mu\sigma \frac{\partial T}{\partial t} = \mu\sigma \frac{\partial H_0}{\partial t} \qquad (1)$$

in all conducting materials.

Scalar electric potential formulation : for conduction problems a simplified formulation can be used due to the fact that no magnetic quantity is time varying. A scalar potential can be found such as :

$$\vec{J} = \sigma \vec{E} = -\sigma \vec{\nabla} V \qquad (2)$$

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and resulting equation is :

$$\nabla^2 V = 0 \quad (3)$$

Finite element method or boundary integral equation techniques give an accurate solution provided that thickness c of the surrounding domain is not too small.

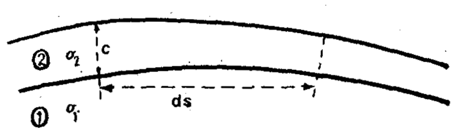
BOUNDARY CONDITIONS

Boundary and interface conditions has to be expressed on electric quantities Continuity of normal component of current density and tangential components of electric field has to be imposed. If exterior conductivity σ_2 is null boundary conditions are classical ones,

$$\frac{\partial V}{\partial n} = 0 \quad \text{on medium 1 boundary}$$

$$T = \text{constant value} \quad (4)$$

If conductivity σ_2 is not null interface conditions between 1 and 2 can be expressed as follows :
let us consider an element of that interface of length ds



by expressing current density vector

$$\vec{J}\,(J_n, J_s)$$

in local coordinates interface conditions are :

$$J_{n1} = J_{n2} \quad (5)$$

$$\frac{1}{\sigma_1} J_{s1} = \frac{1}{\sigma_2} J_{s2} \quad (6)$$

If the thickness of medium 2 is sufficiently small current density can be expressed in terms of uniform current I flowing in medium 2 . This current I will have the same direction as the middle line of the medium 2 , and by the fact parallel to the tangential component of current density in medium 1 so that :

$$J_{s2} = \frac{I}{c} \quad (7)$$

and current I variation along element ds is given by :

$$dI = J_{n2}\, ds \quad (8)$$

Introducing these relations into interface conditions gives :

$$J_{n1} = \frac{c\,\sigma_2}{\sigma_1} \frac{\partial J_{s1}}{\partial s} \quad (9)$$

that relation can be expressed in terms of electric scalar or vector potentials :

$$\frac{\partial T}{\partial n} + \frac{\sigma_1}{c\,\sigma_2} T = 0 \quad (10)$$

$$\frac{\partial^2 V}{\partial s^2} - \frac{\sigma_1}{c\,\sigma_2} \frac{\partial V}{\partial n} = 0 \quad (11)$$

that means that boundary conditions are defined on the limit of domain 1 . The problem is now well defined for a numerical method resolution :

an equation

$$\nabla^2 V = 0 \quad (12)$$

or

$$\nabla^2 T - \mu\sigma \frac{\partial T}{\partial t} = \mu\sigma \frac{\partial H_0}{\partial t} \quad (13)$$

a domain : the conducting medium

boundary conditions : mixed boundary conditions.

RESOLUTION

Solution of such a problem will be obtained in different ways if a formulation in scalar or vector potential is used.
<u>Vector potential :</u>
<u>boundary condition</u>

$$\frac{\partial T}{\partial n} + \frac{\sigma_1}{c\,\sigma_2} T = 0 \quad (14)$$

is directly introduced in the functional expression term :

$$\oint T \frac{\partial T}{\partial n} dl \quad (15)$$

of the energy functional expression of a finite element development.
That relation gives the extra equation required in a Boundary Integral Equation method . The boundary equation written for medium 1 is :

$$T(P) = \int_{\partial \Omega_i} \left[G(P,Q) \frac{\partial T(Q)}{\partial n} - T(Q) \frac{\partial G(P,Q)}{\partial n} \right] ds \qquad (16)$$

with two unknowns T and $\frac{\partial T}{\partial n}$ on each node, and the extra equation :

$$\frac{\partial T}{\partial n} + \frac{\sigma_1}{c\sigma_2} T = 0 \qquad (17)$$

Scalar potential :
in this case the boundary condition imposes an iterative process to be developed by considering this condition as a non homogeneous Neumann one :

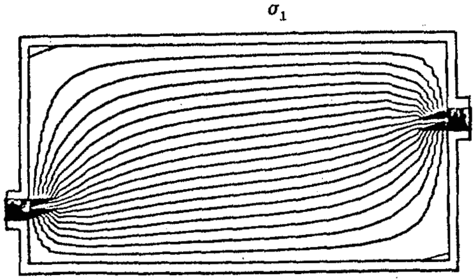
$$\frac{\partial^2 V}{\partial s^2} - \frac{\sigma_1}{c\sigma_2} \frac{\partial V}{\partial n} = 0 \qquad (18)$$

second derivative $\frac{\partial^2 V}{\partial s^2}$ being expressed with

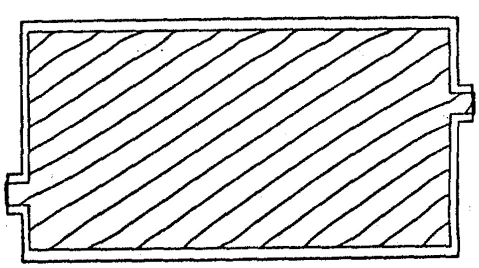
potential values V and interpolation polynomial second derivative, that imposes at least second order polynomial interpolation has to be used.
In this case the same resolution way is required either for boundary integral equation method or finite element one .

RESULTS

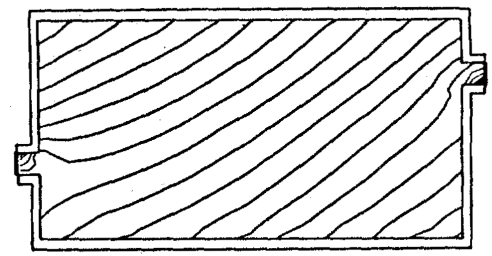
These mixed boundary conditions have been implemented in a finite element package and in a first time used for conduction problems. Some results can be found below giving current distribution in an electrolytic liquid for different conductivities of recipient. These results are characterized by the coefficient: $\alpha = \frac{c\sigma_2}{\sigma_1}$



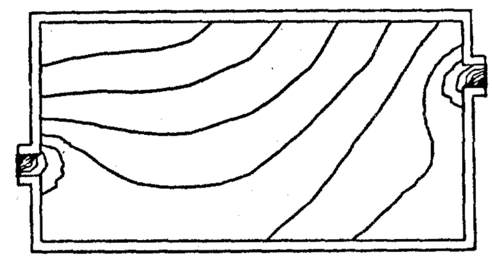
α=0 (non conductive walls)



α = 0.001



α = 0.01



α = 0.1

CONCLUSION

The method we propose in this paper allows to take into account very thin conducting media . This technique is available either using a finite element resolution method or a boundary integral equation one . The two main advantages of such a technique are first a lower computing cost (containing medium does not require to be meshed) and second an increase accuracy due to the fact that geometrically close nodes are analytically connected . For these reasons the method we propose is particularly well-suited for modelisation of conduction problems such as electrolytic problems or cathodic protection of submarine structures problems .

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