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► **To cite this version:**

Laurent Krähenbühl, Alain Nicolas. Efficient techniques for boundary integral equation methods. IEEE Transactions on Magnetics, 1983, 19 (6), pp.2667 - 2669. 10.1109/TMAG.1983.1062805 . hal-00414087

HAL Id: hal-00414087

<https://hal.science/hal-00414087v1>

Submitted on 8 Sep 2009

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EFFICIENT TECHNIQUES FOR BOUNDARY INTEGRAL EQUATION METHODS

by L. KRAHENBUHL and A. NICOLAS

Abstract : The authors expose some useful efficient techniques for Boundary Integral Equations in application package development. Problems particular to BIE techniques are solved :

- Physical singularity in geometry corners where uniqueness of normal flux density cannot be obtained. A double flux formulation gives the exact values of flux.
- The need for good accuracy on solid angle value at each point of the mesh has been proved. An original technique allows automatic computation of this value.

Finally the authors show how to use the features of the BIE method (computation of both flux density and potential) to plot at a minimum computing cost the equipotential lines in a system.

INTRODUCTION

Boundary Integral Equation methods have been theoretically developed during past years [1]. However a set of problems particular to this technique have been exposed by some authors and these must be resolved in order to develop packages for potential problems.

PHYSICAL SINGULARITIES

The BIE technique requires two unknowns on each point of the mesh : normal flux density and potential. The potential value is continuous on the boundary, flux or related quantities have not a unique value on geometry corners. Some authors [1] take a unique value associated with an average of the normal to the boundary, others take a constant value of flux on each finite element of the mesh [2], others consider two physically separate points to describe the corner [3].

By assuming two flux values on each corner point with two normal component values as shown on fig. 1, discontinuity of flux density can be taken into account.

When discretising BIE equations by the finite element technique an extra equation has to be written for each corner point. In order to avoid numerical instability this equation has to be expressed on a point different to the corner point, but sufficiently near this point (fig. 2).

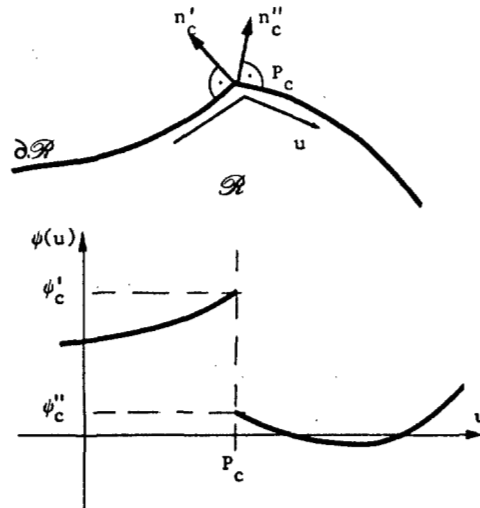


Fig. 1 : double normal and normal flux density values on a corner point.

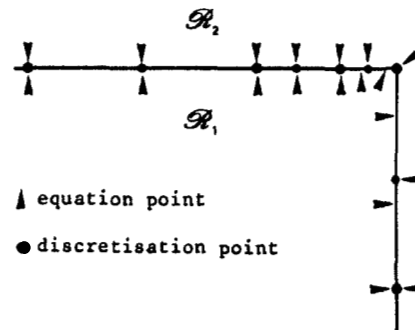


Fig. 2 : Discretisation and equations near the corner.

SOLID ANGLE COMPUTATION

In BIE methods the diagonal terms of matrix expression are formed from the value of solid angle on this point of the boundary. Some experiments have shown that the value of this angle had to be expressed with a very good accuracy : a few percent error makes the calculation divergent. This coefficient c can be automatically computed by noting :

$$c = \iint_{\partial R} \frac{\partial G}{\partial n} ds$$

where G is the Green function associated with the Laplacian operator.

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The term $\frac{\partial G}{\partial n}$ has to be computed for matrix generation, the extra computation time for integral evaluation is in fact very low.

This automatic computation appears very useful when approximating a curved boundary by parabolic segments : false angular points appear at each junction of these segments (fig. 3).

Largest error on	$c = 2\pi$ on each point	$c = -\int \frac{\partial G}{\partial n} ds$
Potential	1.6 %	0.04 %
Normal flux density	8.8 %	0.1 %

Fig. 3 : Correction of "false angular points"; magnetic sphere in a constant field discretised by 4 second order finite elements.

EQUIPOTENTIAL LINES PLOTTING

It is often useful to obtain flux line plots in order to know both the concentration of flux on different parts of the system, and the direction of flux density. This can be done by computing the values of potential on the nodes of a grid. If only one equipotential line is needed, classical method such as finite elements requires the computation of all the potential values.

BIE method gives the value of potential and flux on the boundary. These two values state the equipotential line and tangential direction of this line. An original algorithm has been developed to follow this equipotential line along the tangential direction making correction step by step, as shown on fig. 4.

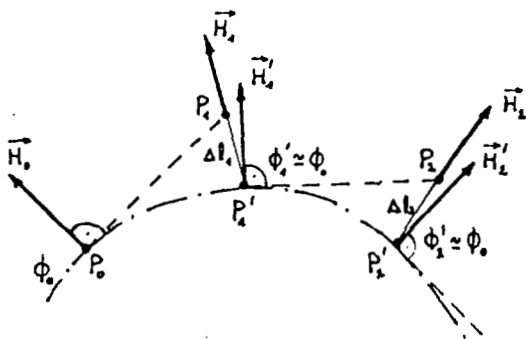


Fig. 4 : Principle of equipotential line plotting.
 -P₀ is the starting point.
 -on P₁ field and potential are computed;
 -linear correction gives point P'₁ on equipotential line;
 -if necessary a second (quadratic) correction is computed ...
 -and so on step to step.

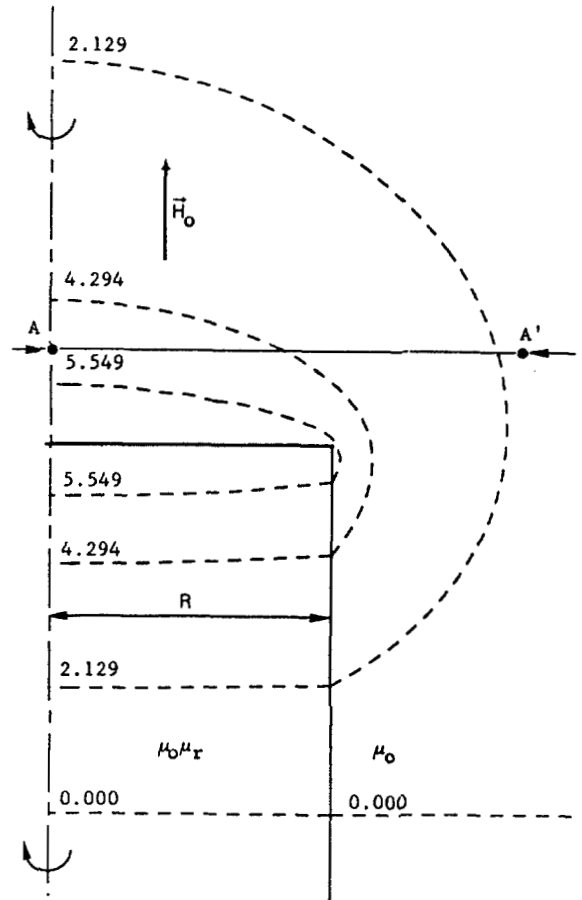


Fig. 5 : Magnetic cylinder in a constant field equipotential lines plot for $\mu_r = 20$.

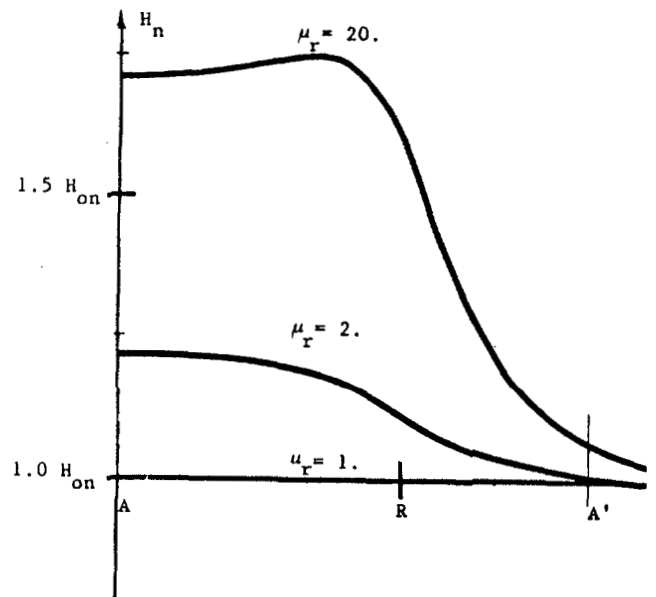


Fig. 6 : Normal field in the straight line AA' for different values of μ_r

This technique allows the plotting of an equipotential line with a good accuracy with a small number of field evaluations.

EXAMPLE

All these techniques have been implemented in the PHI2D (2D problems) and PHIAX (axisymmetric problems) [4] packages.

As an example, we present the equipotential lines plot (fig. 5) and the computation of the normal magnetic field on an external straight line (fig. 6) for a magnetic cylinder in a constant excitation field.

CONCLUSION

These techniques give excellent results with BIE methods. Association of these methods with variational formulations and finite element solutions is now possible. It would also be possible to solve non linear problems efficiently.

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