BACKWARD DOUBLY STOCHASTIC INTEGRAL EQUATIONS OF THE VOLTERRA TYPE

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Abstract

In this paper, we study backward doubly stochastic integral equations of the Volterra type (BDSIEVs in short). Under uniform Lipschitz assumptions, We establish an existence and uniqueness result.

Keywords. Volterra integrals, backward stochastic integral, backward doubly stochastic Volterra integral equations.

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1 Introduction

Backward doubly stochastic differential equations (BDSDEs for short) are equations with two different directions of stochastic integrals, i.e., the equations involve both a standard (forward) stochastic integral dW_t and a backward stochastic integral $\overleftarrow{dB_t}$:

$$Y(t) = \xi + \int_t^T f(s,Y(s),Z(s))ds + \int_t^T g(s,Y(s),Z(s))\overleftarrow{dB_s} - \int_t^T Z(s)dW_s. \tag{1.1}$$

This kind of equation was introduced by Pardoux and Peng [2] in 1994. They proved the existence and uniqueness of solutions for BDSDEs under uniform Lipschitz conditions. Many others investigations concerned BDSDEs were made with weaker conditions namely by Zhou and al. [9] in 2004 with non-Lipschitz assumptions witch in turn were weakened by Han Baoyan and al. [3] in 2005 and recently by N'zi and Owo [4] (2008). In [10] (2005), Shi and al. weaken the uniform Lipschitz assumptions to linear growth and continuous conditions by virtue of the comparison theorem that is introduced by themselves. They obtain the existence of solutions to BDSDE but without uniqueness. Pursuing their investigations on BDSDEs, N'zi and Owo [5] (2009) obtained recently an existence result with discontinuous conditions. Meanwhile, an other line of researches concerned with backward stochastic integral equations of Volterra type (BSIEVs for short) i.e., equations in form:

$$Y(t) = \xi + \int_t^T f(t,s,Y(s),Z(t,s))ds - \int_t^T Z(t,s)dW_s, \tag{1.2}$$

are lead by Lin [6] in (2002) under global Lipschitz condition on the drift witch was recently weakened by A. Aman and M. N'zi [1] in (2005) to local Lipschitz condition.
Recently, general case of BSIEVs (1.2), has been studied by J. Yong [[7],[8]] (2006).

The purpose of this paper is to generalize the theory of Volterra equations to backward doubly stochastic integral equations.
Thus, we consider the following equation:

$$Y(t)=\xi+\int_t^T f(t,s,Y(s),Z(t,s))ds+\int_t^T g(t,s,Y(s),Z(t,s))\overleftarrow{dB_s}-\int_t^T Z(t,s)dW_s, \quad (1.3)$$

that we call backward doubly stochastic Volterra integral equations (in short BDSVIEs).

Note that when f and g do not depend on t, backward doubly stochastic Volterra integral equations (BDSVIEs) coincide with backward doubly stochastic differential equations (BDSDEs) .

The present paper is organized as follows : in section 2, we deal with notations and set up our framework assumptions and give the definition of adapted solutions to BDSVIEs. The section 3 is concerned with the main result.

2 Preliminaries

2.1 Notations

The Euclidean norm of a vector $x\in\mathbb{R}^k$ will be denote by $|x|$, and for an element $z\in\mathbb{R}^{d\times k}$ considered as a $d\times k$ matrix, we define its Euclidean norm by $||y||=\sqrt{Tr(zz^*)}$ and $<z,y>=Tr(zy^*)$, where y^* is the transpose of y.
Let $(\Omega,\mathcal{F},\mathbb{P})$ be a probability space and T be a fixed final time. Throughout this paper $\{W_t;0\le t\le T\}$ and $\{B_t;0\le t\le T\}$ will denote two mutually independent standard Brownian motion processes, with values $\mathbb{R}^d$ and $\mathbb{R}^l$, respectively, defined on $(\Omega,\mathcal{F},\mathbb{P})$.
Let $\mathcal{N}$ denote the class of P-null sets of $\mathcal{F}$. For each $(t,s)\in[0,T]^2$, we define

$$\mathcal{F}_{t,s}=\mathcal{F}_t^W\vee\mathcal{F}_{s,T}^B,\quad \mathcal{F}_t=\mathcal{F}_{t,t} \text{ and } \mathbb{F}=\{\mathcal{F}_t\}_{t\ge0},$$

where for any process $\{x_t\}$; $\mathcal{F}_{u,t}^x=\sigma\{x_r-x_u;u\le r\le t\}\vee\mathcal{N}$, $\mathcal{F}_t^x=\mathcal{F}_{0,t}^x$.

Also, we set $\mathbb{F}_{\cdot s}=\{\mathcal{F}_{t,s}\}_{t\ge0}$ and $\mathbb{F}_{t\cdot}=\{\mathcal{F}_{t,s}\}_{s\ge0}$

For $S\in[0,T]$, set $\mathcal{D}_{S,T}=[S,T]^2$; $\mathcal{D}=\mathcal{D}_{0,T}$ and denote by $\mathcal{P}$ the σ-algebra of $\mathcal{F}_T$-measurable subsets of $\Omega\times\mathcal{D}$.
For any $n\in\mathbb{N}$, let $\mathcal{M}^2(S,T,\mathbb{R}^n)$ denote the set of (class of $dP\otimes dt$ a.e. equal) $n-$dimensional jointly measurable random processes $\varphi:\Omega\times[S,T]\to\mathbb{R}^n$ which satisfy:

(i) $||\varphi||^2_{\mathcal{M}^2(S,T)}=\mathbb{E}\left(\int_S^T|\varphi(t)|^2dt\right)<\infty$

(ii) $\varphi(t)$ is $\mathcal{F}_t-$measurable, for a.e. $t\in[S,T]$.

Similarly, we denote by $\mathcal{M}^2(\mathcal{D}_{S,T},\mathbb{R}^n)$ the set of (class of $dP\otimes ds\otimes dt$ a.e. equal) $n-$dimensional jointly measurable random processes $\psi:\Omega\times\mathcal{D}_{S,T}\to\mathbb{R}^n$ satisfying:

(i) $||\psi||^2_{\mathcal{M}^2(\mathcal{D}_{S,T})} = \mathbb{E}\left(\int_S^T \int_S^T |\psi(t,s)|^2 dsdt\right) < \infty$

(ii) $\psi(t,s)$ is $\mathcal{F}_s-$measurable, for a.e. $s \in [S,T]$, and any $t \in [S,T]$.

Finally, we set

$$\mathcal{H}^2(\mathcal{D}_{S,T}) = \mathcal{M}^2(S,T,\mathbb{R}^k) \times \mathcal{M}^2(\mathcal{D}_{S,T},\mathbb{R}^{k\times d}),$$

with the norm

$$||(y(.),z(.,.))||^2_{\mathcal{H}^2(\mathcal{D}_{S,T})} = \mathbb{E}\left\{\int_S^T |y(t)|^2 dt + \int_S^T \int_S^T |z(t,s)|^2 dsdt\right\} < \infty.$$

Furthermore, let $\mathrm{L}^2(\Omega,\mathcal{F}_T,\mathbb{P},\mathbb{R}^k)$ be the set of k-dimensional $\mathcal{F}_T$-measurable random vector ξ such that

$$\mathbb{E}\left(|\xi|^2\right) < \infty.$$

We will denote by $\mathcal{B}_k$ the Borel σ-algebra of $\mathbb{R}^k$.

2.2 Assumptions and definition

Let

$$f : \Omega \times \mathcal{D} \times \mathbb{R}^k \times \mathbb{R}^{k\times d} \to \mathbb{R}^k \text{ and } g : \Omega \times \mathcal{D} \times \mathbb{R}^k \times \mathbb{R}^{k\times d} \to \mathbb{R}^{k\times l}$$

be $(\mathcal{P} \otimes \mathcal{B}_k \otimes \mathcal{B}_{k\times d} \,/\, \mathcal{B}_k)$ resp. $(\mathcal{P} \otimes \mathcal{B}_k \otimes \mathcal{B}_{k\times d} \,/\, \mathcal{B}_{k\times l})$ -measurable functions such that for any $(y,z) \in \mathbb{R}^k \times \mathbb{R}^{k\times d}$,

(H_1)

$$f(.,.,y,z) \in \mathcal{M}^2(\mathcal{D},\mathbb{R}^k) \text{ and } g(.,.,y,z) \in \mathcal{M}^2(\mathcal{D},\mathbb{R}^{k\times l}).$$

We assume moreover that there exist constants $C > 0$ and $0 < \alpha < 1$ such that for any $(\omega,(t,s)) \in \Omega \times \mathcal{D}$ and any $(y_1,z_1),(y_2,z_2) \in \mathbb{R}^k \times \mathbb{R}^{k\times d}$,

(H_2)

$$| f(\omega,t,s,y_1,z_1) - f(\omega,t,s,y_2,z_2) |^2 \leq C(| y_1 - y_2 |^2 + || z_1 - z_2 ||^2)$$

$$| g(\omega,t,s,y_1,z_1) - g(\omega,t,s,y_2,z_2) |^2 \leq C | y_1 - y_2 |^2 + \alpha || z_1 - z_2 ||^2$$

Furthermore, let

(H_3)

$$\xi \in \mathrm{L}^2(\Omega,\mathcal{F}_T,\mathbb{P},\mathbb{R}^k).$$

Now, we consider the following BDSVIE: $0 \leq t \leq T$

$$Y(t) = \xi + \int_t^T f(t,s,Y(s),Z(t,s))ds + \int_t^T g(t,s,Y(s),Z(t,s))\overleftarrow{dB_s} - \int_t^T Z(t,s)dW_s. \quad (2.1)$$

Definition 2.1. A pair of processes $(Y(.),Z(.,.))$ where $Y : \Omega \times [0,T] \to \mathbb{R}^k$ and $Z : \Omega \times \mathcal{D} \to \mathbb{R}^{k\times d}$ is called adapted solution of (2.1) if $(Y(.),Z(.,.)) \in \mathcal{H}^2(\mathcal{D})$ and satisfies (2.1).

3 Existence and uniqueness of the adapted solution to BDSVIE

To reach the main result, we consider first the equation (2.1) where f and g do not depend on y and z. That is

$$Y(t) = \xi + \int_t^T f(t,s)ds + \int_t^T g(t,s)\overleftarrow{dB_s} - \int_t^T Z(t,s)dW_s,\ t \in [0,T] \tag{3.1}$$

Lemma 3.1. *Let (H_1), (H_2) and (H_3) hold. Then, BDSVIE (3.1) admits a unique adapted solution $(Y(.),Z(.,.)) \in \mathcal{H}^2(\mathcal{D})$.*

Moreover, the following estimate holds:

$$\begin{aligned} &\mathbb{E}\int_S^T |Y(t)|^2 dt + \mathbb{E}\int_S^T \int_S^T |Z(t,s)|^2 dsdt \\ \leq\ & 9(T-S)\mathbb{E}|\xi|^2 + 9\big[(T-S)\vee 1\big]\mathbb{E}\int_S^T \int_S^T \Big(|f(t,s)|^2 + |g(t,s)|^2\Big)dsdt, \end{aligned} \tag{3.2}$$

for any $S \in [0,T]$.

Proof. For any $t \in [0,T]$, consider the process $M(t,.)$ defined by: $r \in [0,T]$,

$$M(t,r) = \mathbb{E}\left(\xi + \int_0^T f(t,s)ds + \int_0^T g(t,s)\overleftarrow{dB_s} \mid \mathcal{F}_{r,0}\right).$$

The process $M(t,.)$ as defined, is $\mathbb{F}_{.0}-$square integrable martingale. Therefore, by the extension of Itô's martingale representation theorem, there exists a $\mathcal{F}_{r,0}-$progressively measurable process $Z(t,.)$ with valued in $\mathbb{R}^{k\times d}$ such that

$$\int_0^T |Z(t,s)|^2 ds < \infty$$

and

$$M(t,r) = M(t,0) + \int_0^r Z(t,s)dW_s,\ \ \forall\, r \in [0,T].$$

Hence,

$$M(t,r) = M(t,T) - \int_r^T Z(t,s)dW_s,\ \ \forall\, r \in [0,T]. \tag{3.3}$$

By definition, $M(t,T) = \xi + \int_0^T f(t,s)ds + \int_0^T g(t,s)\overleftarrow{dB_s}$ and

$$M(t,r) = N(t,r) + \int_0^r f(t,s)ds + \int_0^r g(t,s)\overleftarrow{dB_s},$$

where

$$N(t,r) = \mathbb{E}\left(\xi + \int_r^T f(t,s)ds + \int_r^T g(t,s)\overleftarrow{dB_s} \mid \mathcal{F}_{r,0}\right).$$

Therefore

$$N(t,r) = \xi + \int_r^T f(t,s)ds + \int_r^T g(t,s)\overleftarrow{dB_s} - \int_r^T Z(t,s)dW_s. \tag{3.4}$$

Note that $\mathcal{F}_{r,0} = \mathcal{F}_r^W \vee \mathcal{F}_{0,T}^B = \mathcal{F}_r \vee \mathcal{F}_r^B$. Then

$$N(t,r) = \mathbb{E}\left(\theta(\xi,t,r,T) \mid \mathcal{F}_r \vee \mathcal{F}_r^B\right),$$

where $\theta(\xi,t,r,T) = \xi + \int_r^T f(t,s)ds + \int_r^T g(t,s)\overleftarrow{dB_s}$ is $\mathcal{F}_T^W \vee \mathcal{F}_{r,T}^B-$measurable. Consequently, $\mathcal{F}_r^B$ is independent of $\mathcal{F}_r \vee \sigma(\theta(\xi,t,r,T))$ and

$$N(t,r) = \mathbb{E}\left(\theta(\xi,t,r,T) \mid \mathcal{F}_r\right).$$

Define

$$Y(t) = N(t,t) = \mathbb{E}\left(\xi + \int_t^T f(t,s)ds + \int_t^T g(t,s)\overleftarrow{dB_s} \mid \mathcal{F}_t\right). \tag{3.5}$$

Obviously, $Y(.)$ is $\mathbb{F}-$adapted and satisfies the following relation:

$$Y(t) = \xi + \int_t^T f(t,s)ds + \int_t^T g(t,s)\overleftarrow{dB_s} - \int_t^T Z(t,s)dW_s,\ t \in [0,T].$$

On the other hand, we know from above that, for any $t \in [0,T]$, $Z(t,.)$ is $\mathbb{F}_{.0}-$adapted and satisfies equation (3.4), so

$$\int_r^T Z(t,s)dW_s = \theta(\xi,t,r,T) - N(t,r),\ r \in [0,T]. \tag{3.6}$$

Since, $\theta(\xi,t,r,T)$ (resp. $N(t,r)$) is $\mathcal{F}_T^W \vee \mathcal{F}_{r,T}^B-$ (resp. $\mathcal{F}_r^W \vee \mathcal{F}_{r,T}^B-$) measurable, it follows that $\int_r^T Z(t,s)dW_s$ is $\mathcal{F}_T^W \vee \mathcal{F}_{r,T}^B-$measurable, for any $r \in [0,T]$.

Hence, from Itô's martingale representation theorem, $\{Z(t,s), r < s < T\}$ is $\mathbb{F}_{.r}-$adapted. Consequently, $Z(t,s)$ is $\mathcal{F}_s^W \vee \mathcal{F}_{r,T}^B$-measurable, for any $r < s < T$. It follows that $Z(t,s)$ is $\bigwedge_{r<s}(\mathcal{F}_s^W \vee \mathcal{F}_{r,T}^B)-$measurable.

But,

$$\bigwedge_{r<s}(\mathcal{F}_s^W \vee \mathcal{F}_{r,T}^B) = \mathcal{F}_s^W \vee (\bigwedge_{r<s}\mathcal{F}_{r,T}^B) = \mathcal{F}_s^W \vee \mathcal{F}_{s,T}^B.$$

Thus, $Z(t,s)$ is $\mathcal{F}_s^W \vee \mathcal{F}_{s,T}^B-$measurable, for any $(t,s) \in \mathcal{D}$.

Now, let us prove that $(Y(.),Z(.,.)) \in \mathcal{H}^2(\mathcal{D})$.

To this end, we use the relations (3.5) and (3.6). Then, we obtain:

$$\mathbb{E}\int_0^T |Y(t)|^2 dt \leq 3\mathbb{E}\left(T|\xi|^2 + \int_0^T\int_0^T \left(T\,|f(t,s)|^2 + |g(t,s)|^2\right) dsdt\right).$$

and

$$\mathbb{E}\int_0^T\int_0^T |Z(t,s)|^2\,dsdt \le 6\mathbb{E}\left(T|\xi|^2+\int_0^T\int_0^T\left(T|f(t,s)|^2+|g(t,s)|^2\right)dsdt\right).$$

Hence, it follows from conditions (H_1)-(H_3), that $(Y(.),Z(.,.))\in\mathcal{H}^2(\mathcal{D})$.

Therefore, $(Y(.),Z(.,.))$ is an adapted solution to (2.1).

For the uniqueness, let us suppose that $(Y'(.),Z'(.,.))\in\mathcal{H}^2(\mathcal{D})$ is an other adapted solution. Then we have, $\forall\, t\in[0,T]$

$$Y(t)-Y'(t)+\int_t^T\left[Z(t,s)-Z'(t,s)\right]dW_s=0. \tag{3.7}$$

Taking $\mathbb{E}[.|\mathcal{F}_t]$ in (3.7), we get $Y(t)-Y'(t)=0$, $\forall t\in[0,T]$.

On the other hand, still using (3.7), we deduce

$$\mathbb{E}\int_0^T\int_0^T |Z(t,s)-Z'(t,s)|^2dsdt=\mathbb{E}\int_0^T|Y(t)-Y'(t)|^2dt=0.$$

□

Our main result in this paper is the following Theorem.

Theorem 3.2. *Let (H_1), (H_2) and (H_3) hold. Then the BDSVIE (2.1) admits a unique adapted solution $(Y(.),Z(.,.))\in\mathcal{H}^2(\mathcal{D})$.*

Proof. For simplicity of notations, we write Y for $Y(.)$ and Z for $Z(.,.)$, throughout the proof.

For any $(y,z)\in\mathcal{H}^2(\mathcal{D}_{S,T})$, we consider the following BDSVIE: $t\in[S,T]$

$$Y(t)=\xi+\int_t^T f(t,s,y(s),z(t,s))ds+\int_t^T g(t,s,y(s),z(t,s))\overleftarrow{dB}_s-\int_t^T Z(t,s)dW_s. \tag{3.8}$$

Thus, by lemma 3.1, Eq. (3.8) admits a unique adapted solution $(Y,Z)\in\mathcal{H}^2(\mathcal{D}_{S,T})$ and

$$\begin{aligned}
&\mathbb{E}\int_S^T|Y(t)|^2dt+\mathbb{E}\int_S^T\int_S^T|Z(t,s)|^2dsdt\\
\le\;& 9(T-S)\mathbb{E}|\xi|^2+9\big[(T-S)\vee 1\big]\mathbb{E}\int_S^T\int_S^T\left(|f(t,s,0,0)|^2+|g(t,s,0,0)|^2\right)dsdt\\
+\;& 9C(T-S)\big[(T-S)+1\big]\mathbb{E}\int_S^T|y(t)|^2dt+9\big[(T-S)C+\alpha\big]\mathbb{E}\int_S^T\int_S^T|z(t,s)|^2dsdt\\
\le\;& 9(T-S)\mathbb{E}|\xi|^2+9\big[(T-S)\vee 1\big]\mathbb{E}\int_S^T\int_S^T\left(|f(t,s,0,0)|^2+|g(t,s,0,0)|^2\right)dsdt\\
+\;& \Gamma_{S,T}\left[\mathbb{E}\int_S^T|y(t)|^2dt+\mathbb{E}\int_S^T\int_S^T|z(t,s)|^2dsdt\right],
\end{aligned}$$

where $\Gamma_{S,T} = \max\big\{9C(T-S)\big[(T-S)+1\big]; 9\big[(T-S)C+\alpha\big]\big\}$.
Hence,

$$\begin{aligned} & ||(Y,Z)||^2_{\mathcal{H}^2(\mathcal{D}_{S,T})} \\ \leq\ & 9(T-S)\mathbb{E}|\xi|^2 + 9\big[(T-S)\vee 1\big]\mathbb{E}\int_S^T\int_S^T \Big(|f(t,s,0,0)|^2 + |g(t,s,0,0)|^2\Big)dsdt \\ +\ & \Gamma_{S,T}||(y,z)||^2_{\mathcal{H}^2(\mathcal{D}_{S,T})}. \end{aligned}$$

Let us consider the map $\Theta : \mathcal{H}^2(\mathcal{D}_{S,T}) \to \mathcal{H}^2(\mathcal{D}_{S,T})$ defined by

$$\Theta(y,z) = (Y,Z), \quad \forall\ (y,z) \in \mathcal{H}^2(\mathcal{D}_{S,T}), \tag{3.9}$$

where $(Y,Z) \in \mathcal{H}^2(\mathcal{D}_{S,T})$ is the adapted solution to BDSVIE (3.8).

The map Θ, as defined, is a contraction when $T-S>0$ is small.

Indeed, let $(y',z') \in \mathcal{H}^2(\mathcal{D}_{S,T})$ and $(Y',Z') = \Theta(y',z')$ the corresponding solution to BDSVIE (3.8) on $[S,T]$.

Let now define:

$$\bar{Y} = Y - Y',\ \ \bar{Z} = Z - Z' \text{ and } \bar{y} = y - y',\ \ \bar{z} = z - z'.$$

Then,

$$\bar{Y}_t = \int_t^T F(t,s,\bar{y}(s),\bar{z}(t,s))ds + \int_t^T G(t,s,\bar{y}(s),\bar{z}(t,s))\overleftarrow{dB}_s - \int_t^T \bar{z}(t,s)dW_s, \tag{3.10}$$

where F and G are defined by

$$F(t,s,u,v) = f(t,s,u+y'(t),v+z'(t,s)) - f(t,s,y'(t),z'(t,s))$$

$$G(t,s,u,v) = g(t,s,u+y'(t),v+z'(t,s)) - g(t,s,y'(t),z'(t,s)).$$

It is easy to check that F and G verify hypotheses (H_1) and (H_2) with

$F(t,s,0,0) = 0$ and $g(t,s,0,0) = 0$, for any $(t,s) \in \mathcal{D}$.

Therefore, by lemma 3.1, $(\bar{Y},\bar{Z}) \in \mathcal{M}^2(S,T,\mathbb{R}^k) \times \mathcal{M}^2(\mathcal{D}_{S,T},\mathbb{R}^{k\times d})$ and

$$\mathbb{E}\int_S^T |\bar{Y}(t)|^2dt + \mathbb{E}\int_S^T\int_S^T |\bar{Z}(t,s)|^2dsdt \leq \Gamma_{S,T}\Big[\mathbb{E}\int_S^T |\bar{y}(t)|^2dt + \mathbb{E}\int_S^T\int_S^T |\bar{z}(t,s)|^2dsdt\Big].$$

Hence,

$$||(\bar{Y},\bar{Z})||^2_{\mathcal{H}^2(\mathcal{D}_{S,T})} \leq \Gamma_{S,T}||(\bar{y},\bar{z})||^2_{\mathcal{H}^2(\mathcal{D}_{S,T})}.$$

Consequently,

$$||\Theta(y,z) - \Theta(y',z')||^2_{\mathcal{H}^2(\mathcal{D}_{S,T})} \leq \Gamma_{S,T}||(y,z) - (y',z')||^2_{\mathcal{H}^2(\mathcal{D}_{S,T})},$$

for any $(y,z),\ (y',z') \in \mathcal{H}^2(\mathcal{D}_{S,T})$.

Thus, the map $\Theta : \mathcal{H}^2(\mathcal{D}_{S,T}) \to \mathcal{H}^2(\mathcal{D}_{S,T})$ is a contraction when $S \in [0,T]$ is chosen such that $\Gamma_{S,T} < 1$. Hence, Θ admits a unique fixed point $(Y,Z) \in \mathcal{H}^2(\mathcal{D}_{S,T})$ which is the unique adapted solution of BDSVIE (2.1) on $[S,T]$.

To end the proof, let $S' \in [0,S]$.
From above, we have the existence of a unique adapted solution on $[S,T]$. Therefore, there exists a unique $Y(S)$.
Now, for any $t \in [S',S]$, let us consider the following equation with terminal condition $Y(S)$:

$$y(t) = Y(S) + \int_t^S f(t,s,y(s),z(t,s))ds + \int_t^S g(t,s,y(s),z(t,s))dB_s - \int_t^S z(t,s)dW_s. \quad (3.11)$$

Using the same procedure as above, we conclude that the equation (3.11) admits a unique adapted solution $(y,z) \in \mathcal{H}^2(\mathcal{D}_{S',S})$ on $[S',S]$. Therefore, we can deduce by induction, the existence and uniqueness of an adapted solution in $\mathcal{H}^2(\mathcal{D})$ to BDSVIE (2.1) on $[0,T]$. □

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