

Nonlinear Modal Analysis of Mechanical Systems with Frictionless Contact Interfaces

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Outline

Background and overview

Nonlinear modes for contact systems

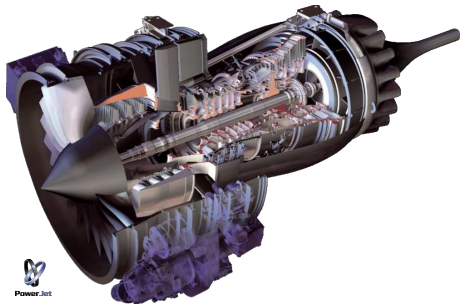
Contact analysis for compressor blades

Conclusions

Rotor-stator interaction in turbomachinery

Why does it happen ?

- ▶ optimized designs, new materials, **reduced operating tip clearances**
- ▶ performance improvements and increased vibratory stresses



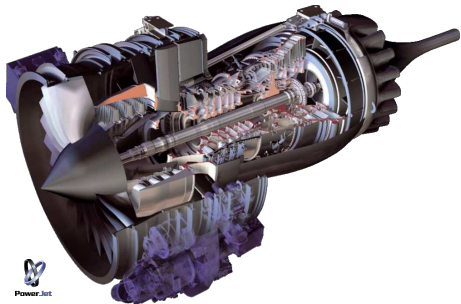
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Contact occurrences

- ▶ normal operating conditions
- ▶ not only transient but also **stationary phenomena**
- ▶ instabilities



Rotor-stator interaction in turbomachinery

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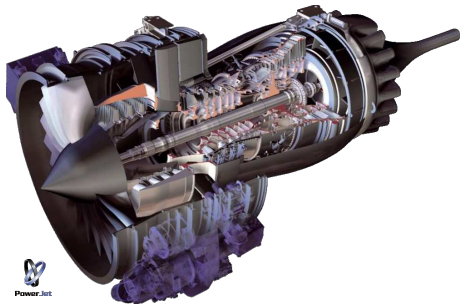
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Analysis of stationary phenomena

- ▶ time integration
- ▶ frequency-domain approaches



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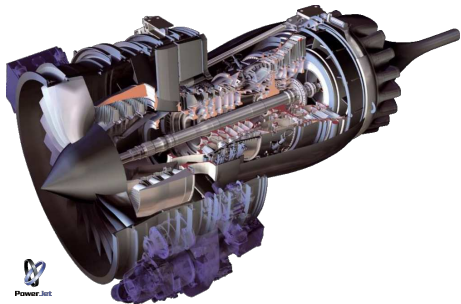
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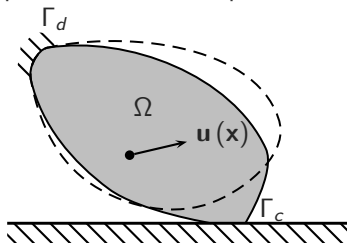
Nonlinear modes

- ▶ non-smooth nonlinearities
- ▶ large-scale systems



Contact modeling and constitutive equations

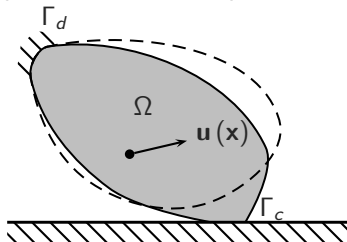
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- no external forcing
- no friction or dissipation

Contact modeling and constitutive equations

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- ▶ Gap function defined on the contact interface:

$$\mathbf{g}(\mathbf{u}) = \mathbf{u}(\mathbf{x}) \cdot \mathbf{n} - \mathbf{g}_0(\mathbf{x})$$

- \mathbf{n} , outward normal vector
- \mathbf{g}_0 , initial gap
- ▶ Contact conditions

$$\tau_N \leq 0, \quad \mathbf{g}(\mathbf{u}) \leq 0, \quad \mathbf{g}(\mathbf{u}) \cdot \tau_N = 0$$

- $\tau_N = \sigma \cdot \mathbf{n}$, contact pressure

Strong formulation of the eigenvalue problem

- ▶ Boundary value problem (Dirichlet and Signorini conditions):

$$\begin{aligned} \rho \ddot{\mathbf{u}} - \operatorname{div} \boldsymbol{\sigma}(\mathbf{u}) &= 0 & \text{on } \Omega \times \mathbb{R}_*^+ \\ \mathbf{u} &= 0 & \text{on } \Gamma_d \times \mathbb{R}_*^+ \\ \mathbf{g}(\mathbf{u}) \leq 0, \quad \tau_N \leq 0 \quad \text{et} \quad \mathbf{g}(\mathbf{u}) \cdot \tau_N &= 0 & \text{on } \Gamma_c \times \mathbb{R}_*^+ \end{aligned}$$

- ▶ Displacement field in the frequency domain:

$$\mathbf{u}(t) = \sum_{n \in \mathbb{Z}} \hat{\mathbf{u}}_n e^{jn\omega t} \quad \text{with} \quad \hat{\mathbf{u}}_n = \frac{1}{T} \int_T \mathbf{u}(t) e^{-jn\omega t} dt$$

- ▶ Eigenvalue problem:

Find $\{\omega, \hat{\mathbf{u}}\}$, with $\hat{\mathbf{u}} = \{\hat{\mathbf{u}}_n, n \in \mathbb{Z}\}$ such as:

$$\begin{aligned} -\operatorname{div} \boldsymbol{\sigma}(\hat{\mathbf{u}}_n) &= (n\omega)^2 \hat{\mathbf{u}}_n & \text{on } \Omega \times \mathbb{Z} \\ \hat{\mathbf{u}}_n &= 0 & \text{on } \Gamma_d \times \mathbb{Z} \\ \mathbf{g}(\hat{\mathbf{u}}) \leq 0, \quad \tau_N \leq 0 \quad \text{et} \quad \mathbf{g}(\hat{\mathbf{u}}) \cdot \tau_N &= 0 & \text{on } \Gamma_c \times [0, T] \end{aligned}$$

Rayleigh quotient

- ▶ Generalized Rayleigh quotient:

$$r(\hat{\mathbf{u}}) = \sum_{n \in \mathbb{Z}} \frac{k(\hat{\mathbf{u}}_n, \hat{\mathbf{u}}_n)}{m(\hat{\mathbf{u}}_n, \hat{\mathbf{u}}_n)}$$

with: $m(\mathbf{u}, \mathbf{v}) = \int_{\Omega} \rho \mathbf{u} \mathbf{v} \, d\mathbf{x}$ et $k(\mathbf{u}, \mathbf{v}) = \int_{\Omega} \boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\epsilon}(\mathbf{v}) \, d\mathbf{x}$

- ▶ Critical points $\hat{\mathbf{u}}$ of this Rayleigh quotient are eigenvectors and eigenvalues are: $\omega^2 = r(\hat{\mathbf{u}})$
- ▶ Constrained minimization (Signorini conditions):

$$\min_{\hat{\mathbf{u}}_n \in V} r(\hat{\mathbf{u}}) \quad \text{with} \quad \mathbf{g}(\hat{\mathbf{u}}) \leq 0 \quad \text{on} \quad \Gamma_c \quad \forall t \in [0, T]$$

Optimality conditions and variational formulation

- ▶ Optimality conditions: $\exists \boldsymbol{\lambda}(t) > 0$ such as:

$$\begin{aligned} \nabla_{\hat{\mathbf{u}}_n} r(\hat{\mathbf{u}}) + \int_T \langle \boldsymbol{\lambda}(t), \nabla_{\hat{\mathbf{u}}_n} \mathbf{g}(\hat{\mathbf{u}}) \rangle dt &= 0 \quad \forall n \in \mathbb{Z} \\ \boldsymbol{\lambda}(t) > 0 \text{ et } \langle \boldsymbol{\lambda}(t), \mathbf{g}(\hat{\mathbf{u}}) \rangle &= 0 \quad \forall t \in [0, T] \end{aligned}$$

- ▶ Variation formulation

Find $\{\hat{\mathbf{u}}_n, n \in \mathbb{Z}\}$ such as $\forall \mathbf{v} \in V$

$$\begin{aligned} 2 m(\hat{\mathbf{u}}_n, \hat{\mathbf{u}}_n)^{-1} (k(\hat{\mathbf{u}}_n, \mathbf{v}) - n^2 \omega^2(\hat{\mathbf{u}}) m(\hat{\mathbf{u}}_n, \mathbf{v})) \\ + \int_T \langle \boldsymbol{\lambda}, \mathbf{v} \rangle e^{in\omega t} dt &= 0 \quad \forall n \in \mathbb{Z} \\ \boldsymbol{\lambda} > 0 \text{ and } \langle \boldsymbol{\lambda}, \mathbf{g}(\hat{\mathbf{u}}) \rangle &= 0 \quad \forall t \in [0, T] \end{aligned}$$

Augmented Lagrangian

- ▶ Augmented Lagrangian:

$$L_{\kappa}(\hat{\mathbf{u}}, \boldsymbol{\theta}) = r(\hat{\mathbf{u}}) + \frac{1}{2} \int_T \|\sqrt{\kappa}(\mathbf{g}(\hat{\mathbf{u}}) + \boldsymbol{\theta})\|_+^2 dt$$

- ▶ Optimality conditions:

$$\nabla_{\hat{\mathbf{u}}_n} L_{\kappa}(\hat{\mathbf{u}}, \boldsymbol{\theta}) = 0 \quad \forall n \in \mathbb{Z}$$

- ▶ Using the variational formulation:

Find $\{\hat{\mathbf{u}}_n, n \in \mathbb{Z}\}$ *such as* $\forall \mathbf{v} \in V$

$$2m(\hat{\mathbf{u}}_n, \hat{\mathbf{u}}_n)^{-1} (k(\hat{\mathbf{u}}_n, \mathbf{v}) - n^2 \omega^2(\hat{\mathbf{u}}) m(\hat{\mathbf{u}}_n, \mathbf{v})) + \int_T \langle \kappa(\mathbf{g}(\hat{\mathbf{u}}) + \boldsymbol{\theta})_+, \mathbf{v} \rangle e^{jn\omega t} dt = 0$$

- ▶ Multiplier updates:

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \max_{\mathbf{x}, t} (\mathbf{g}(\hat{\mathbf{u}}), -\boldsymbol{\theta})$$

Numerical implementation

Discretization

- ▶ Time discretization: $t = \{t_k = kT/m, k = 1, \dots, m\}$
- ▶ Fourier series and discrete Fourier transform:

$$\hat{\mathbf{u}}_n = \frac{1}{T} \sum_{k=1}^m \bar{\mathbf{u}}_k e^{-j\frac{2\pi kn}{m}} \quad \text{and} \quad \bar{\mathbf{u}}_k = \sum_{n=-N}^N \hat{\mathbf{u}}_n e^{j\frac{2\pi kn}{m}}$$

- ▶ Gap function: $\mathbf{g}(\mathbf{U}) = \mathbf{A}\mathbf{U} - \mathbf{g}_0$
- ▶ Discretized eigenvalue problem:

$$2\mathbf{M}^{-1} (\mathbf{K} - (n\omega)^2\mathbf{M}) \hat{\mathbf{U}}_n + \mathbf{A}^T \sum_{k=1}^m \kappa \left(\mathbf{g}(\hat{\mathbf{U}}) + \boldsymbol{\theta}_k \right)_+ e^{j\frac{2\pi kn}{m}} = 0 \quad (*)$$

General algorithm

- 1 continuation with respect to the modal amplitude q
- 2 calculate $\omega^*(q)$ with $\hat{\mathbf{u}}_n(q)$ fixed
- 3 update $\hat{\mathbf{u}}_n^*(q)$ for $\omega^*(q)$

Augmented Lagrangian algorithm

Initialization

$$i = 0, i_{\max}, G^{(0)} = \infty$$

$$\boldsymbol{\theta} = \boldsymbol{\theta}^{(1)}, \kappa = \kappa^{(1)}$$

$$\alpha > 1, \beta > 1, \epsilon > 0$$

Iterations

while $G^{(i)} > \epsilon$ and $i < i_{\max}$ **do**

Find $\{\hat{\mathbf{u}}_n\}_{n=-N, \dots, N}$ solution of (*);

Evaluate $\{\mathbf{g}(\bar{\mathbf{u}}_k)\}_{k=1, \dots, m}$;

Define:

$$\mathbf{I} = \{l : |\max(\mathbf{g}(\bar{\mathbf{u}}_k), -\boldsymbol{\theta}_k)| \geq G^{(i)}/\alpha\}$$

$$\bar{G} = \max |\max(\mathbf{g}(\bar{\mathbf{u}}_k), -\boldsymbol{\theta}_k)|$$

if $\bar{G} \geq G^{(i)}$ **then** $\forall l \in \mathbf{I}, \kappa_{k,l} \leftarrow \beta \kappa_{k,l}$ and $\boldsymbol{\theta}_{k,l} \leftarrow \boldsymbol{\theta}_{k,l}/\beta$;

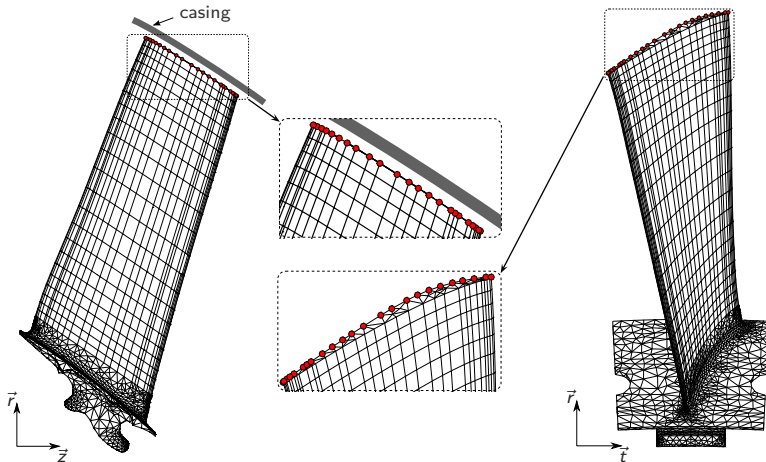
else

$$i \leftarrow i + 1 \quad \boldsymbol{\theta}_k^{(i)} \leftarrow \boldsymbol{\theta}_k, \kappa_k^{(i)} \leftarrow \kappa_k \text{ and } G^{(i)} \leftarrow \bar{G};$$

$$\boldsymbol{\theta}_k \leftarrow \boldsymbol{\theta}_k^{(i)} + \max(\mathbf{g}(\bar{\mathbf{u}}_k), -\boldsymbol{\theta}_k);$$

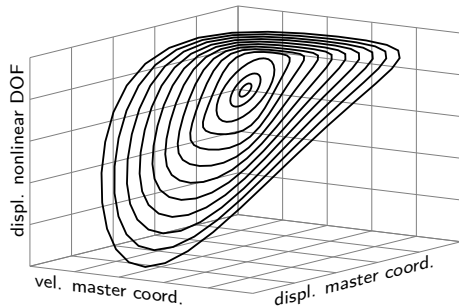
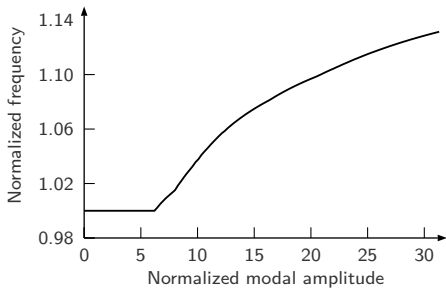
$$\mathbf{if } G^{(i)} \geq G^{(i-1)}/\alpha \text{ then } \forall l \in \mathbf{I}, \kappa_{k,l} \leftarrow \beta \kappa_{k,l};$$

Blade and contact interface modeling

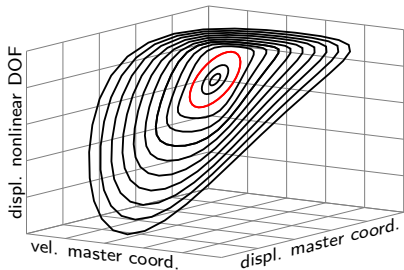
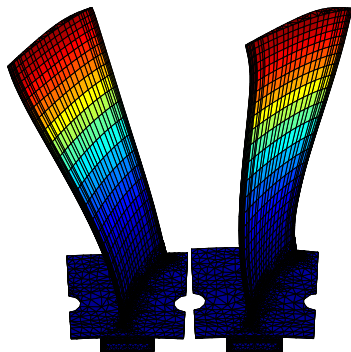
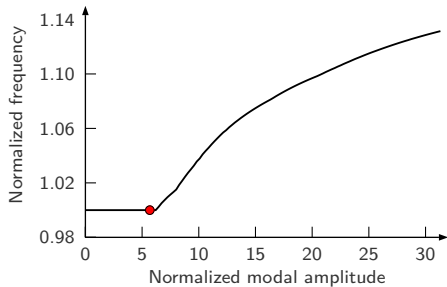


- ▶ Craig & Bampton reduced-order model
 - 24 interface nodes and 40 normal modes
- ▶ contact in radial direction \vec{r}
- ▶ uniform initial gap

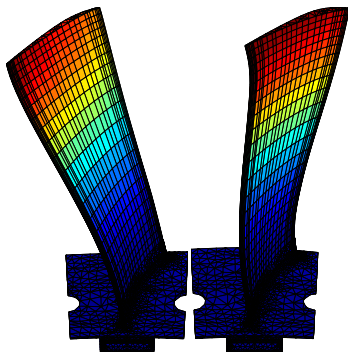
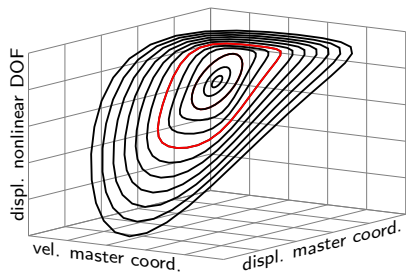
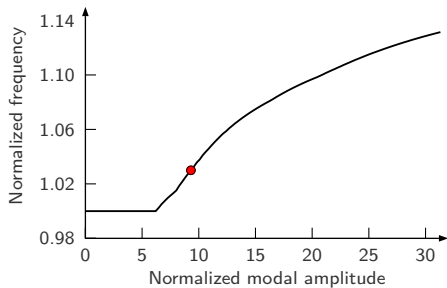
Nonlinear modes: modal parameters



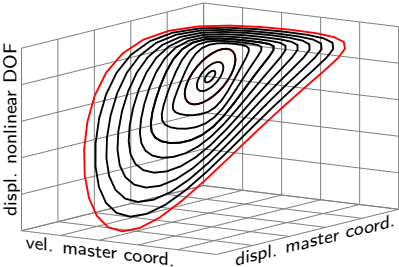
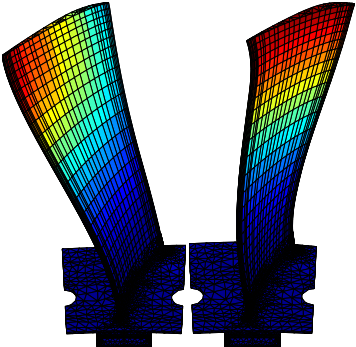
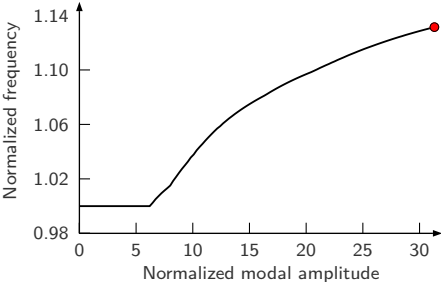
Effects of contact on mode shapes



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Concluding remarks

Summary:

- ▶ A method for nonlinear modal analysis of mechanical systems with contact interfaces has been proposed.
- ▶ Based on a mixed time-frequency formulation of the eigenvalue problem, a constrained minimization of a Rayleigh quotient is defined, which is then solved using an augmented Lagrangian approach.
- ▶ An application to a large-scale structure has been proposed. Parametric studies have shown the effects of contact on modal parameters.

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Future works:

- ▶ Introduction of friction and use of complex modes formalism
- ▶ Stability and bifurcation analysis
- ▶ Application to bladed disks with flexible casing