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# Magnetic Pressure and Shape of Ferrofluid Seals in Cylindrical Structures

R. Ravaud, G. Lemarquand, V. Lemarquand

## Abstract

This paper presents a three-dimensional analytical model for studying the shape and the pressure of ferrofluid seals in totally ironless structures. This three-dimensional analytical approach is based on the exact calculation of the magnetic field components created by ring permanent magnets whose polarizations are either radial or axial. We assume that the ferromagnetic particles of the ferrofluid are saturated. Moreover, the permanent magnets (Neodymium Iron Boron) used in the considered applications create a magnetic field which is much higher than the magnetic field created by the magnetic particles in the ferrofluid so the latter is neglected. Nevertheless, the static behavior of the ferrofluid seal depends on both the magnetic field produced by the permanent magnets and the saturation magnetization of the ferrofluid particles. Furthermore, the accurate knowledge of the ferrofluid seal shape as well as the magnetic pressure inside the ferrofluid seal is very useful for the design of devices using both permanent magnets and ferrofluid seals. It is emphasized here that our structures are completely ironless and thus, there are no iron-base piston for these structures. Then, this paper makes a review of the main structures using ring permanent magnets and ferrofluid seals. For each ironless structure, the shape and the pressure of the ferrofluid seals are determined.

## Index Terms

ferrofluid seals, analytical approach, permanent magnets, magnetic pressure

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## I. INTRODUCTION

18  
19 The interest of using ferrofluid in the new technologies increases clearly with the understanding of  
20 its chemical and physical properties [1]-[3]. Ferrofluids are in fact a unique class of material that have  
21 been discovered in the 1960s. A ferrofluid is a stable colloidal suspension of sub-domain magnetic nano  
22 particles in a liquid carrier. These magnetic particles are coated with a stabilizing dispersing agent that  
23 prevents generally particle agglomeration. Such a property is very useful because in some applications  
24 (as the ones presented in this paper), the magnetic fields used are very intense. Besides, it is noted that  
25 we discuss the utility of using intense magnetic fields in this paper. Some industries use ferrofluids for  
26 realizing airtightness seals or in audio engineering to decrease the temperature of coils in loudspeakers.  
27 As this choice allows us to improve greatly these devices, a care study of the shape of a ferrofluid seal  
28 must be carried out because this property is directly linked to its magnetic energy and its mechanical  
29 properties. Their properties prove to be useful and efficient in various engineering areas such as damping  
30 systems [4], heat transfers, motion control systems, sensors [5][6]. They also have very promising medical  
31 applications [7] [8]. However, they are more commonly used in bearings and seals for rotating devices.  
32 The pioneering work regarding the ferrofluid lubrication was done by Tarapov [9]. Indeed, he considered  
33 a plain journal bearing lubricated by ferrofluid and submitted to a non-uniform magnetic field. Since  
34 then, numerous studies in the field of ferrofluid dynamic bearings have been carried out. Both static  
35 and dynamic characteristics of these bearings have been studied theoretically [10]-[17]. Moreover, recent  
36 trends in the ferrofluid lubrication applications are described and discussed [18]-[24]. Ferrofluids are also  
37 used in electrodynamic loudspeakers in which they fulfill several functions: they ensure the airtightness,  
38 they play a heat transfer part and they work as a radial bearing. Furthermore, a ferrofluid seal can replace  
39 the loudspeaker suspension and leads to a better linearity of the cone movement [25]-[29].

40  
41 This paper presents a three-dimensional analytical model for studying the shape and the static pressure  
42 of ferrofluid seals submitted to intense magnetic fields produced by ironless structures. We use the static  
43 magnetic pressure for studying the shape of the ferrofluid seals. Our three-dimensional analytical approach  
44 is based on the exact calculation of the magnetic field components created by ring permanent magnets  
45 whose polarizations are either radial or axial. Moreover, the approach taken is based on the coulombian

46 model of permanent magnets. Consequently, it gives a great flexibility for the review of the different  
47 structures studied in this paper.

48 In addition, it must be emphasized here that our experimental conditions in which the ferrofluid is used  
49 differ from the other experiments proposed by the authors. Indeed, our applications (ironless loudspeakers,  
50 bearings) require very high magnetic fields. Consequently, we think that the most important parameter  
51 representing the pressure in the ferrofluid seal must be determined carefully. This term is often simplified  
52 in many studies because it is difficult to determine. However, this term is not only the most important  
53 parameter that must be determined carefully but this is also probably the only term that allows us to  
54 study the ferrofluid seal shape. The gravitational energy or the surface tension energy do not influence a  
55 ferrofluid seal shape used in these conditions. This is why we think that it is more useful to obtain an exact  
56 three-dimensional analytical expression of the magnetic pressure by only considering the preponderant  
57 term in the equation of equilibrium in the ferrofluid seal rather than using several simplifying terms whose  
58 influence is questionable in our applications.

59 As the ferrofluid considered in this paper (Ferrotec APG05) has a saturation magnetization of at least  
60  $32 \text{ kA/m}$  and the magnetic field is greater than  $400 \text{ kA/m}$ , the value of this pressure is higher than  $12800$   
61  $\text{N/m}^2$ . On the other hand, the surface tension exists. But when the values of both the surface tension  
62 coefficient,  $A$ , ( $A$  equals  $0.0256 \text{ kg/s}^2$  for the used ferrofluids) and the radius of curvature are considered,  
63 the effect of the surface tension can be omitted: this latter does not deform the free boundary surface.

## 64 II. THREE-DIMENSIONAL ANALYTICAL MODEL

65 First, we present the calculation tools to study the different configurations using one or several ring  
66 permanent magnets. As the goal of this paper is to determine the shape of the ferrofluid seals for different  
67 magnet configurations, the magnetic field created by the magnets must be calculated. Then, the way of  
68 obtaining the seal shape is discussed.

69 It is emphasized here that we do not use a two-dimensional model for calculating the magnetic field  
70 created by these arc-shaped permanent magnets because such an approach is not accurate in the near-field  
71 [30]. This is the reason why we use only a three-dimensional approach.

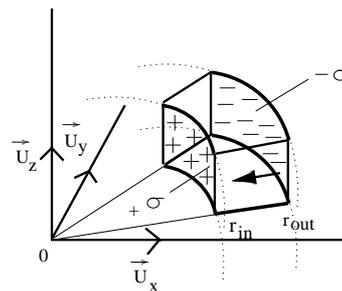


Fig. 1. Arc-shaped permanent magnet whose polarization is radial: the inner curved surface is charged with the magnetic pole surface density  $+\sigma^*$  and the outer curved surface is charged with the magnetic pole surface density  $-\sigma^*$ , the inner radius is  $r_{in}$ , the outer one is  $r_{out}$

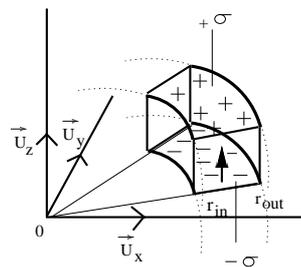


Fig. 2. Arc-shaped permanent magnet whose polarization is axial: the upper surface is charged with the magnetic pole surface density  $+\sigma^*$  and the lower surface is charged with the magnetic pole surface density  $-\sigma^*$ , the inner radius is  $r_{in}$ , the outer one is  $r_{out}$

#### 72 A. Magnetic field created by the ring magnets

73 The magnetic field created by the ring magnets is determined by using the coulombian model. The  
 74 magnets are assumed perfect (flawless) with a perfectly uniform magnetization. Consequently, each  
 75 permanent magnet is represented by two charged surfaces. In the case of a permanent magnet whose  
 76 polarization is radial, the magnetic poles are located on both curved surfaces of the ring and the magnetic  
 77 pole surface density is denoted  $\sigma^*$  (Fig. 1). The rings are assumed radially thin enough to neglect the  
 78 magnetic pole volume density related to the magnetization divergence. In the case of a permanent magnet  
 79 whose polarization is axial, the magnetic pole surface density  $\sigma^*$  is located on the upper and lower faces  
 80 of the ring (Fig. 2). The magnetic field components have been entirely determined in previous papers  
 81 [30][31]. Since the azimuthal component equals zero because of the cylindrical symmetry [32], only two  
 82 components must be calculated. Therefore, the magnetic field depends on both radius  $r$  and altitude  $z$ .

83 *B. Potential energy and magnetic pressure of the ferrofluid seal*

84 The potential energy of the ferrofluid is very important parameter for the study of ferrofluid seals.  
 85 Its calculation requires several assumptions, generally admitted by the authors. First, the ferromagnetic  
 86 particles of the ferrofluid are assumed to be small saturated spheres which can be freely oriented in all  
 87 the directions of space. Thus, all the particles of the saturated ferrofluid are aligned with the permanent  
 88 magnet orienting field. In addition, when submitted to these intense magnetic fields, the ferrofluid does  
 89 not modify the field created by the permanent magnets and the field created by the ferrofluid itself is  
 90 omitted. Furthermore, the aggregation in chains of the ferrofluid particles is omitted [33]. It is noted that,  
 91 when the device is at laboratory temperature and at rest, the aggregation phenomenon is observed. In  
 92 short, the magnetic pressure is expressed as follows:

$$p_m(r, z) = \mu_0 \mathbf{M}_s \cdot \mathbf{H}(r, z) = \mu_0 M_s \sqrt{H_r(r, z)^2 + H_z(r, z)^2} \quad (1)$$

93 where both magnetic field components  $H_r(r, z)$  and  $H_z(r, z)$  are analytically calculated [30]. Thus, the  
 94 potential energy of the ferrofluid seal,  $E_m$ , can be deducted :

$$E_m = - \int \int \int_{(\Omega)} \mathbf{H}(r, z) \cdot (\mu_0 \mathbf{M}) dV = - \int \int \int_{(\Omega)} p_m(r, z) dV \quad (2)$$

95 where  $(\Omega)$  is the ferrofluid seal volume. As a remark, the magnetic pressure is given in  $N/m^2$  and the  
 96 potential energy in  $J$ . The analytical expression of the magnetic pressure depends on the number of ring  
 97 permanent magnets used in the ironless structure. For example, if we consider a structure made of two  
 98 ring permanent magnets radially magnetized, with opposite magnetizations, the magnetic pressure  $p_m(r, z)$   
 99 inside the ferrofluid seal is given by (3).

$$p_m(r, z) = \mu_0 M_s \sqrt{H_{r,2}(r, z)^2 + H_{z,2}(r, z)^2} \quad (3)$$

100 where  $M_s$  is the intensity of magnetization of a magnetic particle. The radial component  $H_{r,2}(r, z)$  of  
 101 the magnetic field created by the two permanent magnets is defined by (4).

$$H_{r,2}(r, z) = \frac{\sigma^*}{2\pi\mu_0} (\varsigma(u_1) - \varsigma(u_2)) \quad (4)$$

102 with

$$\begin{aligned}
\zeta(u) = & \left( \frac{2i(1+u)\sqrt{\frac{d(-1+u)}{c+e_1+du}}(-(a_1d+b_1(c+e_1)))F^*\left[i\sinh^{-1}\left[\frac{\sqrt{-c+d-e_1}}{\sqrt{c+e_1+du}}\right], \frac{c+d+e_1}{c-d+e_1}\right]}{d\sqrt{-c+d-e_1}e_1\sqrt{\frac{d(1+u)}{c+e_1+du}}\sqrt{1-u^2}} \right) \\
& + \left( \frac{2i(1+u)\sqrt{\frac{d(-1+u)}{c+e_1+du}}(b_1c-a_1d)\Pi^*\left[\frac{e_1}{c-d+e_1}, i\sinh^{-1}\left[\frac{\sqrt{-c+d+e_1}}{\sqrt{c+e_1+du}}\right], \frac{c+d+e_1}{c-d+e_1}\right]}{d\sqrt{-c+d-e_1}e_1\sqrt{\frac{d(1+u)}{c+e_1+du}}\sqrt{1-u^2}} \right) \\
& + \left( \frac{2i(1+u)\sqrt{\frac{d(-1+u)}{c+e_2+du}}(-(a_2d+b_2(c+e_2)))F^*\left[i\sinh^{-1}\left[\frac{\sqrt{-c+d-e_2}}{\sqrt{c+e_2+du}}\right], \frac{c+d+e_2}{c-d+e_2}\right]}{d\sqrt{-c+d-e_2}e_2\sqrt{\frac{d(1+u)}{c+e_2+du}}\sqrt{1-u^2}} \right) \\
& + \left( \frac{2i(1+u)\sqrt{\frac{d(-1+u)}{c+e_2+du}}(b_2c-a_2d)\Pi^*\left[\frac{e_2}{c-d+e_2}, i\sinh^{-1}\left[\frac{\sqrt{-c+d+e_2}}{\sqrt{c+e_2+du}}\right], \frac{c+d+e_2}{c-d+e_2}\right]}{d\sqrt{-c+d-e_2}e_2\sqrt{\frac{d(1+u)}{c+e_2+du}}\sqrt{1-u^2}} \right) \\
& - \left( \frac{2i(1+u)\sqrt{\frac{d(-1+u)}{e_3+du}}((a_3d-b_3e_3))F^*\left[i\sinh^{-1}\left[\frac{\sqrt{-d-e_3}}{\sqrt{e_3+du}}\right], \frac{-d-e_3}{d+e_3}\right]}{d\sqrt{-d-e_3}(-c+e_3)\sqrt{\frac{d(1+u)}{e_3+du}}\sqrt{1-u^2}} \right) \\
& - \left( \frac{2i(1+u)\sqrt{\frac{d(-1+u)}{e_1+du}}(b_3c-a_3d)\Pi^*\left[\frac{-c+e_3}{d+e_3}, i\sinh^{-1}\left[\frac{\sqrt{-d+e_3}}{\sqrt{e_3+du}}\right], \frac{-d+e_3}{d+e_3}\right]}{d\sqrt{-d-e_3}(-c+e_3)\sqrt{\frac{d(1+u)}{e_3+du}}\sqrt{1-u^2}} \right) \\
& - \left( \frac{2i(1+u)\sqrt{\frac{d(-1+u)}{e_4+du}}(a_4d-b_4e_4)F^*\left[i\sinh^{-1}\left[\frac{\sqrt{-d-e_4}}{\sqrt{e_4+du}}\right], \frac{-d+e_4}{d+e_4}\right]}{d\sqrt{-d-e_4}(c+e_4)\sqrt{\frac{d(1+u)}{e_4+du}}\sqrt{1-u^2}} \right) \\
& - \left( \frac{2i(1+u)\sqrt{\frac{d(-1+u)}{e_4+du}}(b_4c-a_4d)\Pi^*\left[\frac{-c+e_4}{d+e_4}, i\sinh^{-1}\left[\frac{\sqrt{-d-e_4}}{\sqrt{e_4+du}}\right], \frac{-d+e_4}{d+e_4}\right]}{d\sqrt{-d-e_4}(-c+e_4)\sqrt{\frac{d(1+u)}{e_4+du}}\sqrt{1-u^2}} \right)
\end{aligned} \tag{5}$$

103 The axial component of the magnetic field created by the two ring permanent magnets is given by (6).

$$\begin{aligned}
H_{z,2}(r, z) = & \frac{\sigma^*}{\pi\mu_0} \left( -r_{in} \frac{K^*\left[-\frac{4rr_{in}}{(r-r_{in})^2+z^2}\right]}{\sqrt{(r-r_{in})^2+z^2}} + r_{in} \frac{K^*\left[-\frac{4rr_{in}}{(r-r_{in})^2+(z-h)^2}\right]}{\sqrt{(r-r_{in})^2+(z-h)^2}} \right) \\
& - \frac{\sigma^*}{\pi\mu_0} \left( r_{in} \frac{K^*\left[\frac{4rr_{in}}{(r-r_{in})^2+z^2}\right]}{\sqrt{(r-r_{in})^2+z^2}} - r_{in} \frac{K^*\left[\frac{4rr_{in}}{(r-r_{in})^2+(z+h)^2}\right]}{\sqrt{(r-r_{in})^2+(z+h)^2}} \right)
\end{aligned} \tag{6}$$

Parameters	
$a_1$	$r_{in}rz$
$b_1$	$-r_{in}^2z$
$c$	$r^2 + r_{in}^2$
$d$	$-2rr_{in}$
$e_1$	$z^2$
$a_2$	$-r_{in}r(z-h)$
$b_2$	$r_{in}^2(z-h)$
$e_2$	$(z-h)^2$
$a_3$	$r_{in}rz$
$b_3$	$-r_{in}^2z$
$e_3$	$r^2 + r_{in}^2 + z^2$
$a_4$	$r_{in}r(-z-h)$
$b_4$	$-r_{in}^2(-z-h)$
$e_4$	$r^2 + r_{in}^2 + (z+h)^2$

TABLE I  
DEFINITION OF THE PARAMETERS USED IN (5)

104 where  $K^*[m]$  is given in terms of the incomplete elliptic integral of the first kind by (7)

$$K^*[m] = F^*\left[\frac{\pi}{2}, m\right] \quad (7)$$

105  $F^*[\phi, m]$  is given in terms of the elliptic integral of the first kind by (8):

$$F^*[\phi, m] = \int_{\theta=0}^{\theta=\phi} \frac{1}{\sqrt{1-m\sin(\theta)^2}} d\theta \quad (8)$$

106  $\Pi^*[n, \phi, m]$  is given in terms of the incomplete elliptic integral of the third kind by (9)

$$\Pi^*[n, \phi, m] = \int_0^\phi \frac{1}{(1-n\sin(\theta)^2)\sqrt{1-m\sin(\theta)^2}} d\theta \quad (9)$$

107 The parameters used in (5) are defined in Table II-B. Moreover, when we input (4) in Mathematica, we  
 108 have to take the real part of  $H_r(r, z)$  because of the noise calculus. What's more, the parameter  $i$  used  
 109 in (5) is the imaginary number ( $i^2 = -1$ ).

### 110 C. Parameters used for describing the ironless structures

111 The further calculations are presented for magnets with 1T remanent magnetization in order to normalize  
 112 the results. In fact, the magnets used in the prototypes are Neodymium Iron Boron ones for which the  
 113 remanent magnetization can reach 1.5T. Furthermore, the interesting regions of space are the ones where  
 114 the ferrofluid goes. With these 1T normalized magnets and for the proposed configurations, the magnetic  
 115 field intensity there is greater than 400 kA/m. Of course, all the field values are proportional to remanent  
 116 magnetization value. Then, we use commercial ferrofluids, either from the company Ferrotec or Ferrolabs  
 117 . Such ferrofluids have a saturation magnetization,  $M_s$ , smaller than 32 kA/m and a particle concentration  
 118 below 5,5 %. It is to be noted that for bearing or loudspeaker applications, a great bearing effect is sought  
 119 which requires high saturation magnetizations. Therefore, the magnetic field,  $H$ , created by the permanent  
 120 magnets is far higher than the ferrofluid critical field [34]. So, the ferrofluid is totally saturated and its  
 121 magnetization is denoted  $M_s$ .

122 Color plots of the magnetic pressure will be done in the further sections in order to visualize the  
 123 characteristics of the studied configurations. The warmest color (red) is used for the regions of space  
 124 where the magnetic pressure is the greatest and the coldest color (blue) for the ones where the magnetic  
 125 pressure is the smallest. Moreover, the same color corresponds to the same value throughout this paper:  
 126 the corresponding scale is given in Table II in which the magnetic pressure and the related magnetic field  
 127 values are given.

## 128 III. SHAPE OF THE FERROFLUID SEAL IN VARIOUS MAGNETIC FIELDS

### 129 A. Description of the generic structure

130 Figure 3 shows the generic structure of all the devices studied. They consist of three outer stacked  
 131 rings, of an inner non-magnetic piston and of ferrofluid seals. The piston is radially centered with the  
 132 rings. The rings' inner radius,  $r_{in}$ , equals 25 mm and their outer radius,  $r_{out}$ , equals 28 mm. The rings  
 133 can be either made out of permanent magnet -as here the middle ring- or out of non-magnetic material  
 134 -like the upper and lower rings-. For each configuration, the ferrofluid seals are located in the air gap  
 135 between the piston and the rings; their number, their position as well as the magnetization direction of

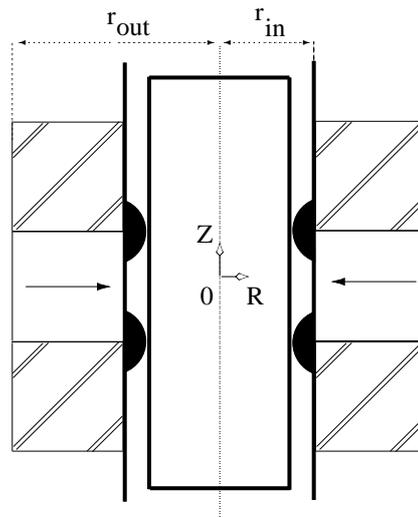


Fig. 3. Device structure: three outer rings (permanent magnet or non-magnetic) radially centered with an inner non-magnetic piston and ferrofluid seals in the air gap between the rings and the piston.  $r_{in} = 25mm$ ,  $r_{out} = 28mm$ .

136 the ring magnets will be discussed.

137

138 For each configuration, we have also represented the radial component of the magnetic field created by  
 139 the ring permanent magnets: indeed, this parameter can also be optimized by changing the ring dimensions  
 140 in electric machines.

141

142 It is assumed throughout this paper that the forming shapes of ferrofluid are considered to be proportional  
 143 to the magnetic pressure. Such an assumption is possible because the magnetic field produced by the ring  
 144 permanent magnets is very high.

145

146 The coordinate system  $(0, \vec{u}_r, \vec{u}_\theta, \vec{u}_z)$  is placed into the centre of the magnetic system in Figs 6 and  
 147 7 and is axially moved off the centre in Figs 4, 5, 8, 9, 10 and 11. In addition, the radial scale of the  
 148 permanent magnets has been divided by 2 in these figures.

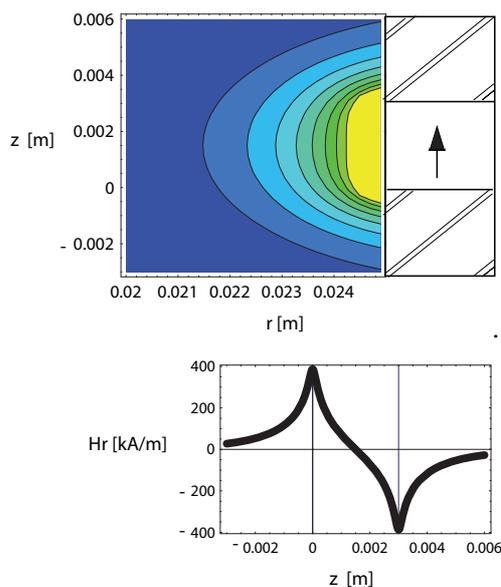


Fig. 4. Right: upper and lower non-magnetic rings, a middle ring permanent magnet axially magnetized. Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along the Z axis.

#### 149 B. Structures using one ring magnet

150 The first structure considered corresponds exactly to the configuration shown in Fig.3. All the rings  
 151 have the same square cross-section whose side equals 3 mm. The middle ring is a radially magnetized  
 152 permanent magnet and the upper and lower rings are non-magnetic. The magnetic field created by the  
 153 magnet in the air gap is calculated at a distance in length from the rings and along the Z axis which  
 154 equals 0.1 mm. Moreover, its radial component  $H_r$  shown in Fig.4 is quite uniform in front of the magnet.  
 155 Two field gradients exist in front of the edges of the magnet. We can point out that the magnetic pressure  
 156 calculated in the air gap between the rings and the piston is plotted on Fig.4 as well. The ferrofluid seeks  
 157 the regions of both intense field gradient and high magnetic energy. The color plot shows that, for seals  
 158 thicker than 0.5 mm, the seal expands along the whole magnet height. A smaller amount of ferrofluid  
 159 would give two separate seals, but they would be too thin to have interesting mechanical properties.  
 160 This result is linked to the magnet square section: were the magnet higher along Z, two separate seals  
 161 would appear. It can be noted that this structure is the simplest one which can be used in electrodynamic  
 162 loudspeakers.

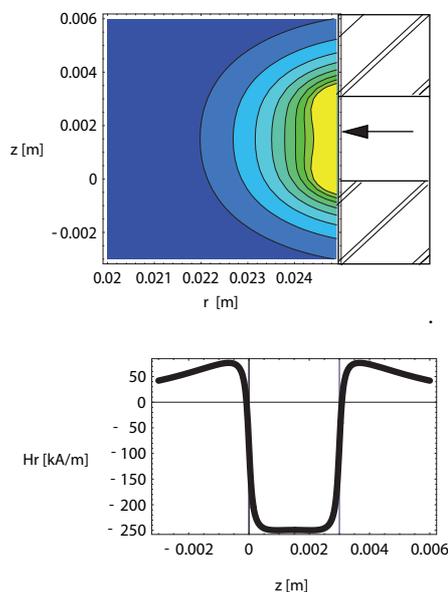


Fig. 5. Right: upper and lower non-magnetic rings, a middle ring permanent magnet radially magnetized. Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along the Z axis.

163 The second structure differs from the first one only in the magnetization direction which becomes  
 164 axial. Figure 5 shows the structure, the calculated radial field component and the corresponding magnetic  
 165 pressure. The radial component of the magnetic field is no longer uniform in front of the magnet; it shows  
 166 instead a rather large gradient all over the magnet length and the non-magnetic rings. Nevertheless, the  
 167 magnetic pressure deserves the same comments as in the first structure and is somewhat similar to the  
 168 previous one. However, it is obvious that it differs in form from one structure to the other.

### 169 C. Structures using two ring magnets

170 The structures studied here derive from the preceding ones by stacking a second permanent magnet  
 171 ring directly on the first one. Both ring magnets have the same dimensions and opposed magnetization  
 172 directions. The radial field component and the magnetic pressure are shown in Fig. 7 when the ring magnets  
 173 are radially magnetized and in Fig. 6 when the ring magnets are axially magnetized. The magnetic field

174 in both cases results from the superposition of the single magnet fields. As a consequence, each radial  
175 magnet creates a region of uniform field in front of itself and the field directions are opposite. The field  
176 intensity in each uniform region is higher than in the single magnet structure because the leakage is  
177 decreased. In effect, three field gradients exist, and the one that appears in front of the magnets' interface  
178 is twice as high as those at the edges. From the gradient point of view, Fig. 7 can be compared with  
179 Fig. 5, and the former will prove more useful because the gradient is steeper. The axial double structure  
180 creates progressive field gradients with no peculiar interest. By contrast, the radial double structure is used  
181 to design two-coil loudspeakers. The repartition of the magnetic energy density in the structure using two  
182 ring magnets does not derive from the superposition of the ones in the structures using one ring magnet.  
183 The main reason is due to the expression of the energy which depends on the square of the field. Both  
184 repartitions for radial and axial magnets seem alike at first sight, but the radial structure is in fact "more  
185 energetic" and the magnetic energy in it decreases slower at an increasing distance from the magnets.  
186 Nevertheless, the maximum energy density is in front of the magnets' interface and the ferrofluid seal  
187 will be located there. We can point out that the axial length of the seal in structures using one magnet is  
188 smaller than the one in structures using two ring magnets. Furthermore, its energy density is approximately  
189 doubled for the radial magnets.

#### 190 *D. Structures using three ring magnets*

191 All the structures considered now consist of three stacked ring permanent magnets whose magnetization  
192 directions undergo a 90 degrees rotation from one magnet to the neighboring one. Such configurations  
193 with a magnetization progressive rotation are related to Halbach cylinders [35] and to their applications  
194 in the design of electrical motors [36], sensors [37][38] or of passive magnetic bearings [39]. Contrary to  
195 electrical machines, the presented structures do not have a periodical magnet pattern.

196 Figure 8 shows the field radial component and the magnetic pressure for an assembly of a radially  
197 magnetized middle ring and two axially magnetized upper and lower rings. Moreover, the axial magnets  
198 have opposed magnetization directions. In Fig. 9, the middle ring is axially magnetized, both other rings  
199 are radially magnetized with opposed directions.

200 The energy density color plots show that two ferrofluid seals form in front of the magnets' interfaces

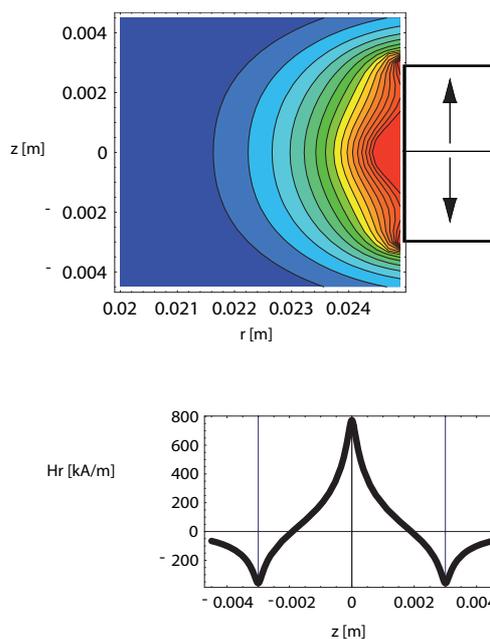


Fig. 6. Right: Two ring permanent magnets with opposed axial magnetization. Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along the Z axis.

201 and that those seals have a great magnetic energy. Consequently, they own good mechanical properties  
 202 (great radial stiffness for example). The seals are rather similar in both structures. However, the magnetic  
 203 field radial component shown in Fig. 8 is fairly uniform in the area in front of the middle magnet while it  
 204 varies with no particularly interesting properties in Fig. 9. Therefore, the structure with the radial middle  
 205 magnet is the useful one, especially for the loudspeaker design, and the field uniformity zone may be  
 206 optimized.

207 The axial height of the middle magnet can be varied regarding the axial height of the upper and lower  
 208 magnets. For example, the middle magnet is twice as high as each other magnet in Fig. 10 and half as  
 209 small in Fig. 11. As a result, the length of the field uniformity area follows closely the middle magnet  
 210 height. If the latter increases, the former increases too. We can say that the ratio middle of the magnet  
 211 height over the upper and lower magnet height is important as well. Indeed, if it increases, the field  
 212 uniformity region grows but the field intensity decreases, and inversely. It also influences the repartition

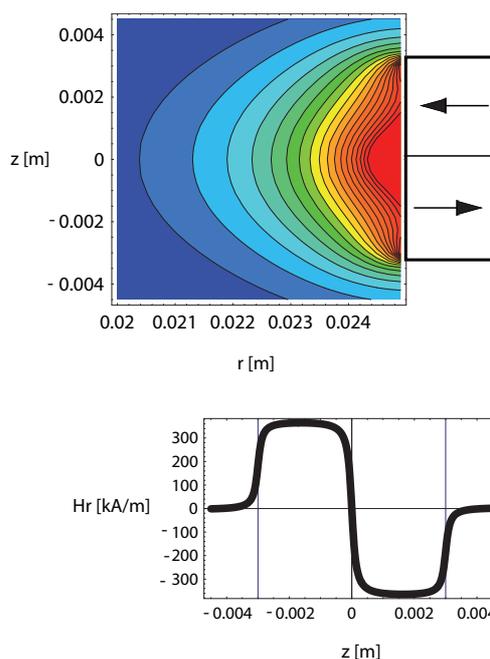


Fig. 7. Right: Two ring permanent magnets with opposed radial magnetization. Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along the Z axis.

213 of the magnetic pressure. Indeed, when the upper and lower magnets become too small, the ferrofluid  
 214 can expand over their whole axial length and the seals can become fairly voluminous, depending on the  
 215 ferrofluid available quantity. The field gradients are steep and the seal magnetic energy is quite high.  
 216 Consequently, the seals are well-fixed to the structure and have high mechanical performances. Inversely,  
 217 when the middle magnet becomes too small, both seals gather to form a single one which expands over  
 218 the whole height of the middle magnet. The energy in the seal is still high for radial thickness smaller  
 219 than the ones presented in the previous case.

#### 220 *E. Discussion*

221 So far, different magnet assemblies have been presented. Each assembly creates magnetic fields and  
 222 magnetic field gradients so as to trap and fix ferrofluid to make seals. Their characteristics will be discussed  
 223 and compared now.

224 The structures considered are either very simple or more complex. The complexity can be characterized

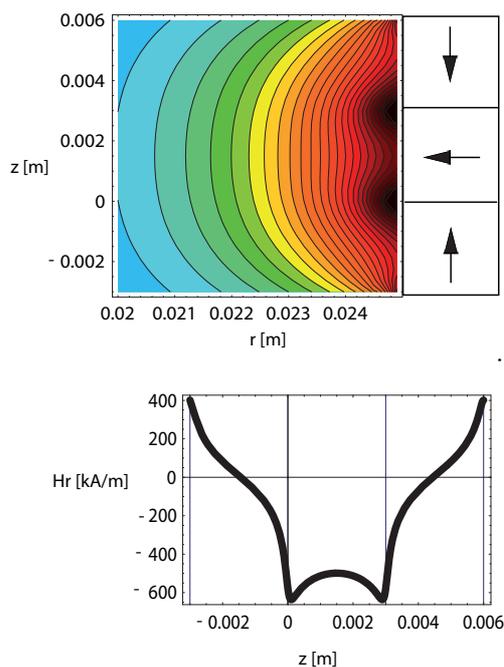


Fig. 8. Right: axially magnetized upper and lower rings, radially magnetized middle ring. Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along the Z axis.

225 by the number of separate parts necessary to build devices. We can say that from this point of view, the  
 226 single magnet structures are the simplest ones. The complexity also lies in the magnets' magnetization  
 227 direction. Indeed, an axial magnetization is easy to achieve for ring magnets whereas a radial one is  
 228 technically more difficult. Thereby, ring magnets whose polarization is radial are more expensive than  
 229 the ones whose polarization is axial. Nevertheless, some magnet manufacturers begin to deal with radial  
 230 magnetization and to sell such products.

231 The utility of a structure depends on the intended applications. For example, a device can be designed  
 232 solely to make ferrofluid seals with good mechanical properties. It can also be designed to create a radial  
 233 magnetic field or to enable a ferrofluid seal to form as it is required in loudspeakers.

234 A high magnetic field gradient and a high magnetic pressure are required to achieve robust seals. From  
 235 the preceding sections, we deduce that multi-magnet structures are more efficient than single magnet

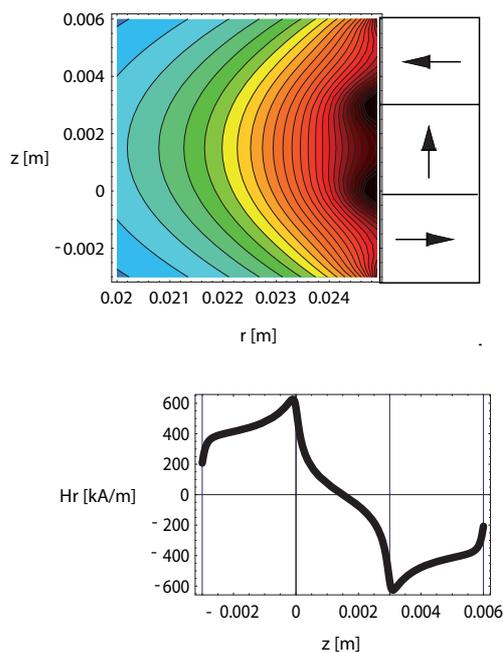


Fig. 9. Right: radially magnetized upper and lower rings, axially magnetized middle ring. Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along the Z axis.

236 structures in creating high field gradients. Furthermore, the magnetic pressure is higher in the air gap near  
 237 the magnets.

238 It can be emphasized here that the shape of the ferrofluid seal in the device depends on both the  
 239 magnetic pressure and the ferrofluid volume. It can be noted that this latter one is related to the air  
 240 gap size. The air gap exists because the inner part has to move, in rotation or in translation, inside the  
 241 outer part without rubbing against it. The air gap size depends on mechanical considerations, such as the  
 242 practical machining and the related possible tolerances. The color plots show clearly that the theoretical  
 243 size of the air gap which allows an efficient ferrofluid seal depends on the magnetic structure dimensions.

244 For example, the structures shown in Fig. 11 or in Fig. 8 lead to seals thinner than the ones shown in  
 245 Fig. 7 because the region of high magnetic pressure is radially thinner.

246 Ferrofluid seals are robust when their capacity is high; this means that they can resist to quite high  
 247 axial pressures because they exert themselves an opposing force. We can say that this force is related to

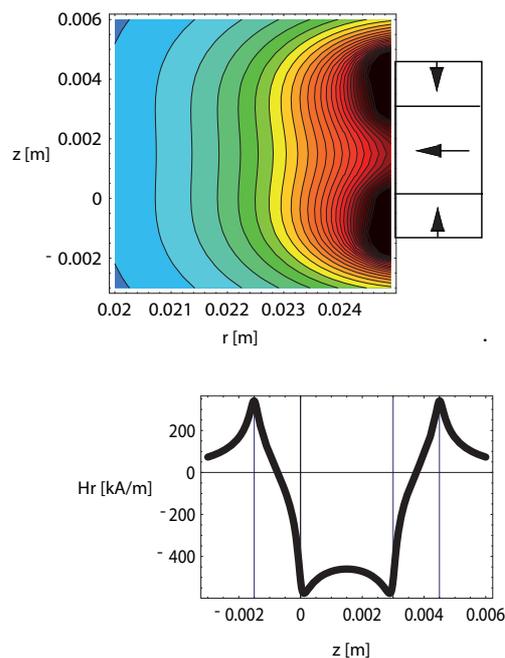


Fig. 10. Right: axially magnetized upper and lower rings, radially magnetized middle ring, each axial magnet is half as small as the radial one. Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along the Z axis.

248 the energy variation of the ferrofluid seal. The higher the energy varies with the axial displacement, the  
 249 more important the force is.

250 Moreover, a great force is attained with thin seals whose energy is high and varies extremely with axial  
 251 displacements. Therefore, the smaller the air gap is, the more favorable the mechanical structure to build  
 252 ferrofluid seals is.

253 The color plots can help optimizing both the number and the shape of ferrofluid seals for given ferrofluid  
 254 volumes.

255 For example, the plots shown in Figs. 10 and 11 point out that two seals appear in these three magnet  
 256 structures for a small volume of ferrofluid. Moreover, they show that the increase in the ferrofluid volume  
 257 results in the growth of the seals which eventually join and can form a single big one.

258 The color plots can also be considered as tools to determine the adequate quantity of ferrofluid which  
 259 will confer the intended mechanical properties on the seals. Furthermore, this study shows that some

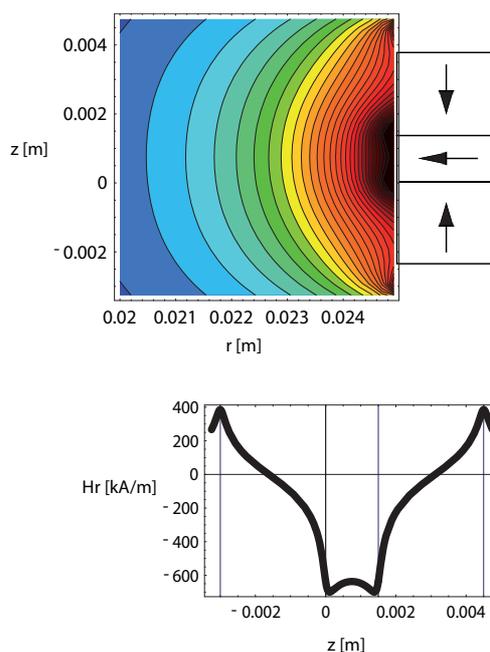


Fig. 11. Right: axially magnetized upper and lower rings, radially magnetized middle ring, each axial magnet is twice as high as the radial one. Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along the Z axis.

260 structures have greater performances than others for given magnet and ferrofluid volumes.

261 The requirements for the loudspeaker applications are different. Indeed, the magnetic structure is  
 262 expected to fulfill several purposes, namely the creation of a radial uniform magnetic field as well as  
 263 the creation of field gradients. Besides, a loudspeaker coil moves thanks to the magnetic field. The  
 264 linearity performances are related to the field uniformity and the efficiency is proportional to the field  
 265 level. Furthermore, the ferrofluid seal is used to ensure the airtightness, to transfer the heat from the  
 266 moving part to the steady one, to work as a radial bearing and to replace the loudspeaker suspension.  
 267 Thereby, it contributes to the improvement of the loudspeaker linearity. The air gaps in loudspeakers are  
 268 generally 0.1 to 0.5 mm wide: this explains why the presented magnetic field calculations have been  
 269 carried out along a path at a distance in length which equals 0.1 mm from the ring magnets. Thinner  
 270 air gaps would be more advantageous for ferrofluid seals but are not technically possible because of the  
 271 machining tolerances or the recognition of the moving part thermal dilation. Therefore, the structures

272 shown in Figs. 3, 7 and 10 are the ones that are the more useful for loudspeakers. The three magnet  
 273 structures are the more complex, but they offer the possibility of a multi-criteria optimization. We can  
 274 say that this optimization is enabled and simplified by the analytical formulations of both the magnetic  
 275 field and the magnetic energy density. Eventually, such structures show very good performances.

#### 276 IV. CONCLUSION

277 This paper has presented a three-dimensional analytical model for studying the shape of the static  
 278 pressure of ferrofluid seals subject to intense magnetic fields produced by permanent magnet ironless  
 279 structures. Our analytical approach has been applied to well-known structures composed of one, two or  
 280 three stacked ring permanent magnets whose polarization is either radial or axial. Our analytical approach  
 281 allows us to predict easily the shape and the pressure of ferrofluid seals. Then, we have discussed the  
 282 interest of using simple or complex structures with ferrofluid seals. The single magnet structures are the  
 283 simplest ones but the less efficient. The use of several magnets enhances both magnetic and mechanical  
 284 properties: when the created magnetic field is doubled, the magnetic energy is quadrupled. Furthermore,  
 285 three magnet structures enable to create one or two seals which have a high energy. The structures can  
 286 also create a high uniform magnetic field over defined regions as well as efficient ferrofluid seals: this  
 287 is useful for loudspeaker applications. The results obtained can be used for the design of cylindrical  
 288 structures using both ring permanent magnets and ferrofluid seals.

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$H$ [kA/m]	$p_m$ ( $N.m^{-2}$ )
630	23200
610	23300
590	23600
570	22800
550	22000
530	21200
510	20400
490	19600
470	18800
450	18000
430	17200
410	16400
390	15600
370	14800
350	14000
330	13200
310	12400
290	11600
270	10800
250	10000
230	9200
210	8400
190	7600
170	6800
150	6000
130	5200
110	4400
90	3600
70	2800
50	2000

TABLE II

$H$ , MODULUS OF THE MAGNETIC FIELD (kA/m);  $p_m$ , MAGNETIC PRESSURE ( $N.m^{-2}$ );  $M_s = 32kA/m$

359 Fig. 1. Arc-shaped permanent magnet whose polarization is radial: the inner curved surface is charged  
 360 with the magnetic pole surface density  $+\sigma^*$  and the outer curved surface is charged with the magnetic  
 361 pole surface density  $-\sigma^*$ , the inner radius is  $r_{in}$ , the outer one is  $r_{out}$ .

362

363 Fig. 2. Arc-shaped permanent magnet whose polarization is axial: the upper surface is charged with  
 364 the magnetic pole surface density  $+\sigma^*$  and the lower surface is charged with the magnetic pole surface  
 365 density  $-\sigma^*$ , the inner radius is  $r_{in}$ , the outer one is  $r_{out}$ .

366

367 Fig. 3. Device structure: three outer rings (permanent magnet or non-magnetic) radially centered with an  
 368 inner non-magnetic piston and ferrofluid seals in the air gap between the rings and the piston.  $r_{in} = 25mm$ ,  
 369  $r_{out} = 28mm$ .

370

371 Fig. 4. Right: upper and lower non-magnetic rings, a middle ring permanent magnet axially magnetized.  
 372 Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance  
 373 from the rings, along the Z axis.

374

375 Fig. 5. Right: upper and lower non-magnetic rings, a middle ring permanent magnet radially magnetized.  
 376 Left: magnetic pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance  
 377 from the rings, along the Z axis.

378

379 Fig. 6. Right: Two ring permanent magnets with opposed axial magnetization. Left: magnetic pressure  
 380 in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along  
 381 the Z axis.

382

383 Fig. 7. Right: Two ring permanent magnets with opposed radial magnetization. Left: magnetic pressure  
 384 in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the rings, along  
 385 the Z axis.

386

387 Fig. 8. Right: axially magnetized upper and lower rings, radially magnetized middle ring. Left: magnetic  
388 pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the  
389 rings, along the Z axis.

390

391 Fig. 9. Right: radially magnetized upper and lower rings, axially magnetized middle ring. Left: magnetic  
392 pressure in front of the rings. Bottom: magnetic field radial component at a 0.1 mm distance from the  
393 rings, along the Z axis.

394

395 Fig. 10. Right: axially magnetized upper and lower rings, radially magnetized middle ring, each axial  
396 magnet is half as small as the radial one. Left: magnetic pressure in front of the rings. Bottom: magnetic  
397 field radial component at a 0.1 mm distance from the rings, along the Z axis.

398

399 Fig. 11. Right: axially magnetized upper and lower rings, radially magnetized middle ring, each axial  
400 magnet is twice as high as the radial one. Left: magnetic pressure in front of the rings. Bottom: magnetic  
401 field radial component at a 0.1 mm distance from the rings, along the Z axis.