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► **To cite this version:**

Guy Lemarquand, Valérie Lemarquand, Slobodan Babic, Cevdet Akyel. Magnetic Field Created by Thin Wall Solenoids and Axially Magnetized Cylindrical Permanent Magnets. Progress In Electromagnetics Research Symposium 2009 Moscow, Aug 2009, Moscow, Russia. pp.614. hal-00413197

**HAL Id: hal-00413197**

**<https://hal.science/hal-00413197>**

Submitted on 3 Sep 2009

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# Magnetic Field Created by Thin Wall Solenoids and Axially Magnetized Cylindrical Permanent Magnets

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**Abstract**— This paper presents the calculation of the magnetic field created by axially magnetized cylindrical permanent magnets and thin wall solenoids in air. It emphasizes the equivalence of the source models: charged planes and current sheet. It shows that although the starting formulations, magnetic scalar potential, Coulomb's law, vector potential, Biot-Savart's law often depend in the literature on the source nature, they shouldn't. Indeed, it presents the magnetic field calculation for each point of view and explains which lead to analytical solutions. Then it presents the calculation of forces between permanent magnets and shows that it is equivalent to the calculation of the mutual inductance between two coils.

## 1. INTRODUCTION

It is well-known that coils and permanent magnets are equivalent sources of magnetic field even though, depending on the application, the latter or the former may be more interesting or advantageous. However, the physical nature of the magnetic field source influences the scientist who intends to calculate the created magnetic field. Indeed, scientists dealing with permanent magnets generally either calculate the magnetic scalar potential and derive it to obtain the magnetic field or use Coulomb's law to calculate the magnetic field directly, whereas scientists dealing with coils rather calculate the vector potential and use Biot-Savart's law.

This paper intends to emphasize the equivalence of the source models but also to highlight the fact that some formulations will lead more easily to analytical formulations than others. So, a very simple example is considered: an axially magnetized cylindrical permanent magnet in air. Indeed, it can be modelled either by a thin wall solenoid, a current sheet or two charged planes. Inversely, a thin wall solenoid could be considered, which would then be modeled by a current sheet or a cylindrical magnet axially magnetized or by two charged planes. The point is that the charge model corresponds to a coulombian approach and the coil model to an amperian approach. This paper gives the initial expressions for the magnetic field calculation for each possible point of view. Then it presents the calculation of forces between permanent magnets and thus show that it is fully equivalent to the calculation of the mutual inductance between two coils.

## 2. BASIC EQUIVALENCES

A cylindrical permanent magnet with an uniform axial polarization,  $J$ , creates the same magnetic field as the thin wall solenoid of same diameter and height as the magnet, when currents flow in it with a linear density (Fig. 1). This solenoid creates also the same field as the cylindrical current sheet,  $\vec{K}$ , which is linked to  $\vec{J}$  as follows:

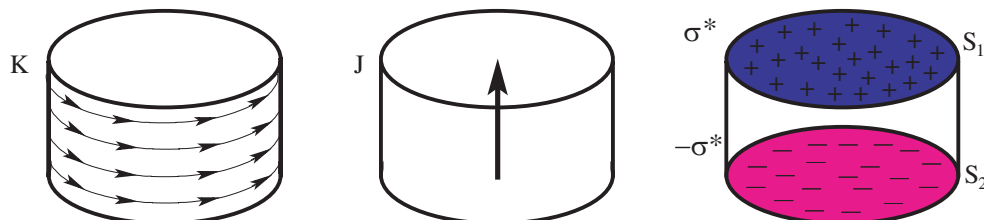


Figure 1: Equivalent source models for a cylinder of radius  $R$  and height  $h$ . Left: current sheet,  $K$ . Center: permanent magnet with axial polarization,  $J$ . Right: charged planes, charge surface density  $\sigma^*$ .

$$\vec{K} = \frac{\vec{J}}{\mu_0} \quad (1)$$

Furthermore, it also creates the same magnetic field as the two charged planes, corresponding to the top and bottom caps of the cylinder and separated by a distance equal to the cylinder height. These planes are charged with opposite charge surface densities  $+\sigma^*$  and  $-\sigma^*$  related to the polarization and defined by:

$$\sigma^* = \vec{J} \cdot \vec{n} \quad (2)$$

where  $\vec{n}$  is the vector normal to the face.

For neodymium iron boron magnets,  $J = 1.4 \text{ T}$ ,  $\sigma^* = 1.4 \text{ T}$ ,  $K = 1.11 \cdot 10^6 \text{ A/m}$ . Moreover, the magnetic sources considered in this paper are in ironless structures. This allows to obtain analytical formulations for the magnetic field.

### 3. MAGNETIC FIELD CALCULATIONS

#### 3.1. Magnetic Scalar Potential

As said in the introduction, it is striking how the method used to calculate the magnetic field depends on the source nature. When it comes to magnets, the prevalent method in literature is the calculation of the scalar potential and its subsequent derivation (gradient). Indeed, one can write the magnetic scalar potential function as

$$\varphi(\vec{r}) = \frac{J}{4\pi\mu_0} \iint_{S_1} \frac{1}{|\vec{r} - \vec{r}'|} ds_1 - \frac{J}{4\pi\mu_0} \iint_{S_2} \frac{1}{|\vec{r} - \vec{r}'|} ds_2 \quad (3)$$

where the radius  $\vec{r}$  defines the observation point  $M$  position,  $\vec{r}'$  the position of the elementary source on the charged surface,  $S_1$  the positive charged surface,  $S_2$  the negative one. The scalar potential is expressed in amperes. This calculation uses the representation of the magnetic source by charge surface densities. Of course, it supposes a linear behavior so as to calculate the total scalar potential by the application of the superposition principle.

The magnetic field is then calculated by :

$$\vec{H} = -\overrightarrow{\text{grad}}\varphi \quad (4)$$

However, the difficulty in this method lies in the fact that one has to integrate a function first and then to derive the integration result. Various methods, using Green's functions or toroidal functions have been proposed to solve this problem but few of them lead to a complete analytical formulation of the magnetic field. Indeed, a numerical calculation is often required as last step in most of them.

#### 3.2. Coulomb's Law

When the charge surface density model is chosen another way to calculate the magnetic field is to use Coulomb's law directly as in Eq. (5)

$$\vec{H}(\vec{r}) = \frac{J}{4\pi\mu_0} \iint_{S_1} \frac{1}{|\vec{r} - \vec{r}'|^2} ds_1 - \frac{J}{4\pi\mu_0} \iint_{S_2} \frac{1}{|\vec{r} - \vec{r}'|^2} ds_2 \quad (5)$$

On account of the cylindrical symmetry the magnetic field is a function on  $r$  and  $z$  only and its azimuthal component,  $H_\theta(r, z)$ , is equal to zero. Thus, the magnetic field can be expressed as follows:

$$\begin{aligned} \vec{H}(r, z) = & \frac{J}{4\pi\mu_0} \int_0^{2\pi} \int_0^R \frac{(r - r_1 \cos(\theta)) \vec{u}_r - r_1 \sin(\theta) \vec{u}_\theta + (z - z_1) \vec{u}_z}{(r^2 + r_1^2 - 2rr_1 \cos(\theta) + (z - z_1)^2)^{\frac{3}{2}}} r_1 dr_1 d\theta \\ & - \frac{J}{4\pi\mu_0} \int_0^{2\pi} \int_0^R \frac{(r - r_2 \cos(\theta)) \vec{u}_r - r_2 \sin(\theta) \vec{u}_\theta + (z - z_2) \vec{u}_z}{(r^2 + r_2^2 - 2rr_2 \cos(\theta) + (z - z_2)^2)^{\frac{3}{2}}} r_2 dr_2 d\theta \end{aligned} \quad (6)$$

where the indexes 1 (resp. 2) are related to the positive (resp. negative) charged face. Once again, the linearity is supposed, but it is consistent with the behavior of modern permanent magnets. The solution of these integrals can be fully analytically formulated with elliptic integrals of the first, second and third kind. Moreover, the three components of the magnetic field are obtained and the result is valid in all the space, inside as well as outside the magnet [1–5].

One should remark here that these methods, though mainly developed for permanent magnet sources, can also apply to coils. Indeed, the modelling of a magnetic source results from a choice and not from the nature of the source itself. Furthermore, the attention is drawn on the fact that with the charged plane model the magnetic field,  $\vec{H}$ , is calculated first. Indeed, the values obtained are valid in all the space. Then, the magnetic flux density,  $\vec{B}$ , is deduced. One shouldn't forget either that the magnetic field presents a discontinuity on the charged planes. This sometimes generates punctual numerical difficulties for the calculation of numerical values with the analytical formulations.

### 3.3. Magnetic Vector Potential

Generally, scientists dealing with coils use models naturally related to currents and calculate either the vector potential or use Biot-Savart's law. This section discusses methods related to the former [6–8]. The vector potential,  $\vec{A}$ , is defined as

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \iint_S \frac{\vec{K}}{|\vec{r} - \vec{r}'|} ds \quad (7)$$

The magnetic flux density is then deduced by derivation (curl):

$$\vec{B} = \text{rot} \vec{A} \quad (8)$$

Here again, the difficulty may lie in the fact that integrations have to be calculated to obtain formulations of the potential vector which have then to be derived. The results can be written analytically with elliptic integrals of the first, second and third kind.

### 3.4. Biot-Savart's Law

In fact, boths previous steps, expression of the vector potential and its derivation to obtain the magnetic flux density can be summarized by Biot-Savart's law. Indeed, the latter gives a way to calculate the magnetic flux density directly:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iint_S \frac{\vec{K} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} ds \quad (9)$$

At this point, some remarks have to be done. First, although these methods were traditionnaly proposed by scientists who wanted to calculate the magnetic flux density created by coils and windings in air, they present nevertheless a great interest for the calculation of the magnetic field created by permanent magnets. Indeed, as the integrals to be calculated depend on the model they don't have the same boundaries and one model may lead to an analytical solution where the other fails. This was experienced by the authors. Moreover, both methods calculate the magnetic flux density and the magnetic field, when needed, is deduced from the former. As the magnetic flux density is continuous in the space, this may be a reason why solutions are found with this modelling which the coulombian approach don't give. But it has to be noted, that the reverse situation is to be found too! The point is then, that the choice of the model is free and shouldn't depend on the source nature. The model which has to be chosen is the one that gives fully analytical formulations of the magnetic field for the geometry considered. Indeed, the final formulation is the same.

## 4. FORCE CALCULATIONS

Magnetic sources, coils or magnet, have two great kinds of applications. Of course, they are primarily used to create magnetic fields (as in MRI devices, wigglers, sensors...) but two sources are also widely used to interact and thus create forces and torques (as in electrical machines, magnetic couplings...). This section deals with the force calculation. Then, three interactions can physically exist: between two magnets, between two coils and between a coil and a magnet. But as the sources are modelled for the calculation the methods that have to be considered are rather related to the models. Therefore, Coulomb's law can be used for the charge model and Lorentz's force for the current model. Moreover, in the last case, the force calculation is also closely related to the calculation of the mutual inductance of the coils.

#### 4.1. Force and Mutual Inductance

The calculation of the magnetic attraction between two current-carrying coils is closely related to the calculation of their mutual inductance. Since their mutual energy is equal to the product of their mutual inductance and the currents in the coils, the component of the magnetic force of attraction or repulsion in any direction is equal to the product of the currents multiplied by the differential coefficient of the mutual inductance taken with the respect to that coordinate. Thus, the magnetic force may be calculated by simple differentiation in cases where a general formula for the mutual inductance is available,

$$F = I_a I_b \frac{M}{g} \quad (10)$$

where  $I_a$  and  $I_b$  are the currents in the two coils,  $M$  their mutual inductance and  $g$  the generalized coordinate [9–13].

#### 4.2. Magnetic Vector Potential and Lorentz's Force

This section considers the case when the sources are modelled by current sheets and the magnetic flux density is expressed by the vector potential. Then, the interaction between the two coils can be described by Lorentz's force. Indeed, the magnetic flux density,  $\vec{B}_a$ , created by one of the coil interacts with the current sheet of the second one,  $\vec{K}_b$ :

$$\vec{F} = \iint_S (\vec{K}_b \times \vec{B}_a) ds \quad (11)$$

For coaxial sources, only the axial force occurs because of the symmetry of the problem. As a result, the axial force exerted between two axially polarized cylinder permanent magnet which are coaxial can be expressed analytically with elliptic integral of the first, second and third kind [14, 15].

#### 4.3. Coulomb's Law

In the case of sources modelled by two charged planes, the force between the sources is described by Coulomb's law. This force is the sum of the four interactions between the four charged planes:

$$\begin{aligned} \vec{F}(\vec{r}) = & \frac{J^2}{4\pi\mu_0} \iint_{S_{1a}} \iint_{S_{1b}} \frac{ds_{1a} ds_{1b}}{|\vec{r}_{1a}' - \vec{r}_{1b}'|^2} - \frac{J^2}{4\pi\mu_0} \iint_{S_{1a}} \iint_{S_{2b}} \frac{ds_{1a} ds_{2b}}{|\vec{r}_{1a}' - \vec{r}_{2b}'|^2} \\ & - \frac{J^2}{4\pi\mu_0} \iint_{S_{2a}} \iint_{S_{1b}} \frac{ds_{2a} ds_{1b}}{|\vec{r}_{2a}' - \vec{r}_{1b}'|^2} + \frac{J^2}{4\pi\mu_0} \iint_{S_{2a}} \iint_{S_{2b}} \frac{ds_{2a} ds_{2b}}{|\vec{r}_{2a}' - \vec{r}_{2b}'|^2} \end{aligned} \quad (12)$$

where the indexes  $a$  are related to the first source, the indexes  $b$  to the second one, the radii  $\vec{r}_{1a}'$  and  $\vec{r}_{1b}'$  are the position of the elementary source on the positive charged surfaces, the radii  $\vec{r}_{2a}'$  and  $\vec{r}_{2b}'$  the position of the elementary source on the negative charged surfaces. Furthermore, the sources are supposed to be coaxial and the considered force is the axial one. In this expression, three of the four integrations in each term can be solved analytically. So, only one remains that has to be numerically calculated and the calculation is globally very fast.

### 5. STIFFNESS CALCULATIONS

Depending on the application, the knowledge of the force may not be sufficient. For example, the dimensioning of magnetic couplings require the value of the stiffness. Then, a general remark is that as long as the considered model leads to the calculation of a force, the notion of stiffness makes sense. But when the model leads to the calculation of a mutual inductance in place of the force, the notion of stiffness can't be derived. At this stage, this may be a guideline to help choosing the model.

For coaxial sources, the axial stiffness,  $K_z$  is defined by:

$$K_z = -\frac{\partial}{\partial z} F_z \quad (13)$$

where  $F_z$  is the axial force.

## 6. GENERALIZATION

This paper presents methods for calculations in a very simple case: a cylindrical source. It must be emphasized here that it thus gives the basis for more complicated geometries. Indeed, numerous geometries can be considered as the superposition of this basic geometry, but with different dimensions or orientations. For example, an axially polarized ring permanent magnet can be seen as the superposition of two cylinder permanent magnets of same heights, opposite polarizations and different radii (the radii difference giving the ring radial thickness).

## 7. CONCLUSION

In conclusion, this paper emphasizes the equivalence of the source models for the calculation of the magnetic field created by axially magnetized cylindrical permanent magnets and thin wall solenoids in air. It shows that the starting formulations should not depend on the source nature as some lead more easily to analytical solutions. Eventually, it shows that the calculation of forces between permanent magnets is equivalent to the calculation of the mutual inductance between two coils.

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