Analytical Expressions of the Magnetic Field Created by Tile Permanent Magnets of Various Magnetization Directions

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Abstract— This paper presents a thorough study of the magnetic field created by tile permanent magnets uniformly magnetized in air. To do so, we use the coulombian model for determining the analytical expressions of the three magnetic field components created by the tile magnets. Moreover, various magnetization directions are considered. Indeed, the direction of the magnetization can be radial, tangential or intermediate between radial and tangential. Thus, this analytical study encompasses most of the magnetization possibilities generally encountered in electrical engineering applications.

1. INTRODUCTION

The modeling of the magnetic field produced by tile permanent magnets was studied by many authors [1–3]. Several analytical methods can be used for calculating the three components of the magnetic field created by permanent magnets [4–9]. According to the coulombian model, the magnets are represented by fictitious magnetic charge densities [10, 11]. This model implies the calculation of surface and volume integrals that represent the surface charge densities and the volume charge densities. We propose in this paper to use the coulombian model for studying the magnetic field created by tile permanent magnets of various magnetization directions. For each configuration studied, all the magnetic charges are taken into account. Consequently, our analytical calculations have been performed without using any simplifying assumption. It has to be noted that such analytical calculations are possible because the tile permanent magnets considered are in air [12, 13] and the structures using these tile magnets are ironless [14].

Then, all the expressions given are expressed in a fully analytical part and a semi-analytical part. For each component, the semi-analytical part cannot be integrated analytically because all the polarizations considered are uniform. Consequently, the expressions cannot be expressed in terms of elliptic integrals of the first, second or third kind.

As a result, the given expressions allow the calculation of the three magnetic field components at any point in the space, may it be outside the magnet as well as inside it. Furthermore, the computational cost is low and so, parametric optimizations can be carried out. Indeed, tile permanent magnets can be assembled in various ways depending on the intended application. For example, radially magnetized tiles can be assembled to form a radially magnetized ring magnet and axially magnetized ring magnets can also be achieved in a similar way with axially magnetized tiles.

Finally, tile permanent magnets of different magnetizations can be associated for the design of Halbach structures and ring permanent magnets can be stacked for the design of magnetic bearings. By using such analytical expressions, the magnetic field created by such assemblies can always be determined accurately and structures using such tile permanent magnets can also be optimized with regard to criteria applying to the magnetic field values and its spatial variations.

2. NOTATION AND GEOMETRY

The geometry considered and the parameters are shown in Fig. 1. We consider one tile permanent magnet whose polarization is given by the angles θ and α. Its angular width is $\theta_2 - \theta_1$, its radial width is $r_2 - r_1$ and its height is $z_2 - z_1$. In the coulombian approach, we must determine the fictitious magnetic charges that are located on the faces of the tile permanent magnet. In our configuration, the polarization is always uniform. Consequently, there are only surface charge densities that are given by the scalar product between the polarization vector $\vec{J}$ and the four normal units. We use two coordinate systems for calculating the magnetic field produced by the tile permanent magnet. The local coordinate system is $(O', \vec{i}, \vec{j})$ and the global coordinate system is $(O, \vec{u}_x, \vec{u}_y)$. In the cartesian coordinate system $(O', \vec{i}, \vec{j})$, the vector $\vec{J}$ is expressed as follows:

$$\vec{J} = J\cos(\alpha)\vec{i} + J\sin(\alpha)\vec{j} \tag{1}$$

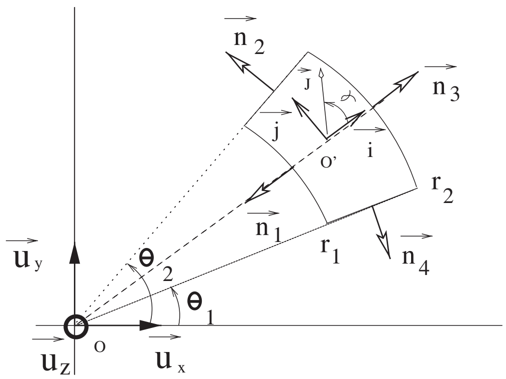

Figure 1: Tile whose polarization is directed in an arbitrary direction.

The relations between the cartesian coordinate systems $(O', \vec{i}, \vec{j})$ and $(O, \vec{u}_x, \vec{u}_y)$ are the following:

$$\begin{aligned}\vec{i} &= \cos\left(\frac{\theta_1+\theta_2}{2}\right)\vec{u}_x + \sin\left(\frac{\theta_1+\theta_2}{2}\right)\vec{u}_y \\ \vec{j} &= -\sin\left(\frac{\theta_1+\theta_2}{2}\right)\vec{u}_x + \cos\left(\frac{\theta_1+\theta_2}{2}\right)\vec{u}_y\end{aligned} \tag{2}$$

By using (1) and (2), we obtain:

$$\vec{J} = J\cos(\alpha)\left(\cos\left(\frac{\theta_1+\theta_2}{2}\right)\vec{u}_x+\sin\left(\frac{\theta_1+\theta_2}{2}\right)\vec{u}_y\right)+J\sin(\alpha)\left(-\sin\left(\frac{\theta_1+\theta_2}{2}\right)\vec{u}_x+\cos\left(\frac{\theta_1+\theta_2}{2}\right)\vec{u}_y\right) \tag{3}$$

We obtain

$$\vec{J} = J\cos\left(\alpha+\frac{\theta_1+\theta_2}{2}\right)\vec{u}_x + J\sin\left(\alpha+\frac{\theta_1+\theta_2}{2}\right)\vec{u}_y \tag{4}$$

The four normal units are defined as follows:

$$\begin{aligned}\vec{n}_1 &= -\cos(\theta)\vec{u}_x - \sin(\theta)\vec{u}_y \\ \vec{n}_2 &= -\sin(\theta_2)\vec{u}_x + \cos(\theta_2)\vec{u}_y \\ \vec{n}_3 &= +\cos(\theta)\vec{u}_x + \sin(\theta)\vec{u}_y \\ \vec{n}_4 &= +\sin(\theta_1)\vec{u}_x - \cos(\theta_1)\vec{u}_y\end{aligned} \tag{5}$$

By using the relation $\vec{J}\cdot\vec{n}_i$ for $i = 1 \ldots 4$, we obtain the four fictitous magnetic pole surface densites σ_1^*, σ_2^*, σ_3^* and σ_4^*

$$\begin{aligned}\sigma_1^* &= -J\cos\left(\theta-\alpha-\frac{\theta_1+\theta_2}{2}\right) \\ \sigma_2^* &= -J\sin\left(\frac{\theta_2-\theta_1}{2}-\alpha\right) \\ \sigma_3^* &= J\cos\left(\theta-\alpha-\frac{\theta_1+\theta_2}{2}\right) \\ \sigma_4^* &= J\sin\left(\frac{\theta_1-\theta_2}{2}-\alpha\right)\end{aligned} \tag{6}$$

Therefore, we have determined the four magnetic charge surface densities that are located on the four faces of the tile permanent magnet. The magnetic field can be determined from the scalar potential $\phi(r, \theta, z)$ produced by the four faces of the tile permanent magnet.

$$d\phi(r,\theta,z) = \sum_{z=1}^{4} G_z\left(\vec{r},\vec{r'}\right)\sigma_z^* dS_z \tag{7}$$

where $G_z(\vec{r},\vec{r'})$ is the three-dimensional Green's function applied to the four surfaces and dS_z is the elementary surface of each face of the tile permanent magnet. The associated magnetic field is determined as follows:

$$\vec{H}(r,\theta,z) = -\vec{\nabla}\phi(r,\theta,z) = H_r(r,\theta,z)\vec{u}_r + H_\theta(r,\theta,z)\vec{u}_\theta + H_z(r,\theta,z)\vec{u}_z \tag{8}$$

Thus, the three magnetic field component $H_r(r,\theta,z)$, $H_\theta(r,\theta,z)$ and $H_z(r,\theta,z)$ are calculated by projecting $-\vec{\nabla}\phi(r,\theta,z)$ on the four directions $\vec{u}_r$, $\vec{u}_\theta$ and $\vec{u}_z$.

2.1. Expression of the Radial Field Produced by a Tile Permanent Magnet of Various Magnetization Direction

The radial component $H_r(r,\theta,z)$ of the magnetic field produced by a tile permanent magnet of various magnetization direction can be expressed as follows:

$$\begin{aligned} H_r(r,\theta,z) &= \frac{J}{4\pi\mu_0}\left(\sin\left(\frac{\theta_1-\theta_2}{2}+\alpha\right)f(\theta_1)+\sin\left(\frac{\theta_1-\theta_2}{2}-\alpha\right)f(\theta_2)\right) \\ &\quad +\frac{J}{4\pi\mu_0}\sum_{i=1}^{2}\sum_{k=1}^{2}(-1)^{(i+k)}r_i(-z+z_k)\mathbf{N}[\tilde{\theta}] \end{aligned} \tag{9}$$

where

$$f(\theta_k) = \sum_{i=1}^{2}\sum_{k=1}^{2}(-1)^{(i+k)}\left(-\cos\left(\theta-\theta_j\right)\log[X_j]+\sin(\theta-\theta_j)\arctan[Y_j]\right) \tag{10}$$

$$\begin{aligned} \mathbf{N}[\tilde{\theta}] &= \int_{\theta_1}^{\theta_2}\frac{\left(r-r_i\cos\left(\theta-\tilde{\theta}\right)\right)\cos\left(\alpha+\frac{\theta_1-\theta_2}{2}-\tilde{\theta}\right)}{\left(r^2+r_i^2-2rr_i\cos\left(\theta-\tilde{\theta}\right)\right)\epsilon\left(\tilde{\theta}\right)}d\tilde{\theta} \\ & X_j = z-z_k+\epsilon(\theta_j) \\ & Y_j = \frac{(z-z_k)\left(r_i-r\cos(\theta-\theta_j)\right)}{r\sin(\theta-\theta_j)\epsilon(\theta_j)} \end{aligned} \tag{11}$$

with

$$\epsilon(\beta) = \sqrt{r^2+r_i^2+(z-z_k)^2-2rr_i\cos(\theta-\beta)} \tag{12}$$

2.2. Expression of the Azimuthal Field Produced by a Tile Permanent Magnet of Various Magnetization Direction

The azimuthal component $H_\theta(r,\theta,z)$ of the magnetic field produced by a tile permanent magnet of various magnetization direction can be expressed as follows:

$$\begin{aligned} H_\theta(r,\theta,z) &= \sin\left(\frac{\theta_1-\theta_2}{2}-\alpha\right)h(\theta_1)+\sin\left(\frac{\theta_1-\theta_2}{2}+\alpha\right)h(\theta_2) \\ &\quad +\frac{J}{4\pi\mu_0}\int_{\theta_1}^{\theta_2}\sum_{i=1}^{2}\sum_{k=1}^{2}\frac{r_i^2(-z+z_k)\cos\left(\alpha+\frac{\theta_1-\theta_2}{2}-\tilde{\theta}\right)\sin\left(\theta-\tilde{\theta}\right)}{\left(r^2+r_i^2-2rr_i\cos\left(\theta-\tilde{\theta}\right)\right)\epsilon\left(\tilde{\theta}\right)}d\tilde{\theta} \end{aligned} \tag{13}$$

$$h(\theta_j) = \frac{J}{4\pi\mu_0}\sum_{i=1}^{2}\sum_{k=1}^{2}\sin(\theta-\theta_j)\log\left[X_j\right]-\cos(\theta-\theta_j)\arctan\left[Y_j\right] \tag{14}$$

2.3. Expression of the Axial Field Produced by a Tile Permanent Magnet of Various Magnetization Direction

The axial component $H_z(r,\theta,z)$ of the magnetic field produced by a tile permanent magnet of various magnetization direction can be expressed as follows:

$$\begin{aligned} H_z(r,\theta,z) \;=\; & \frac{J}{4\pi\mu_0}\left(\sin\left(\frac{\theta_1-\theta_2}{2}-\alpha\right)g(\theta_2)+\sin\left(\frac{\theta_1-\theta_2}{2}+\alpha\right)g(\theta_1)\right) \\ & +\frac{J}{4\pi\mu_0}\int_{\theta_1}^{\theta_2}\sum_{i=1}^{2}\sum_{k=1}^{2}\frac{r_i\cos\left(\alpha+\frac{\theta_1-\theta_2}{2}-\tilde{\theta}\right)}{\epsilon(\theta_j)}d\tilde{\theta} \\ g(\theta_j) = & \sum_{i=1}^{2}\sum_{k=1}^{2}\log\left[r_i-r\cos(\theta-\theta_j)+\epsilon(\theta_j)\right] \end{aligned} \quad (15)$$

3. APPLICATION : STRUCTURE COMPOSED OF TILE PERMANENT MAGNETS WITH ROTATING POLARIZATIONS

We illustrate now the interest of using an exact analytical expression of the magnetic field produced by tile permanent magnets of various magnetization directions in the case of the structure shown in Fig. 2. The structure is composed of 16 tile permanent magnets with rotating polarizations. As each tile is uniformly magnetized, some specific effects appear on account of the curvature of the tile and the uniform polarization. In Fig. 3, the radial field is represented versus the angle θ. Consequently, the radial field produced by such a configuration is not smooth but presents some little discontinuities, as shown in Fig. 3.

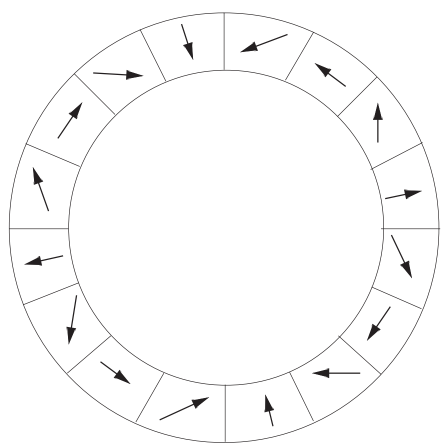

Figure 2: Structure using 16 tile permanent magnets with rotating polarizations.

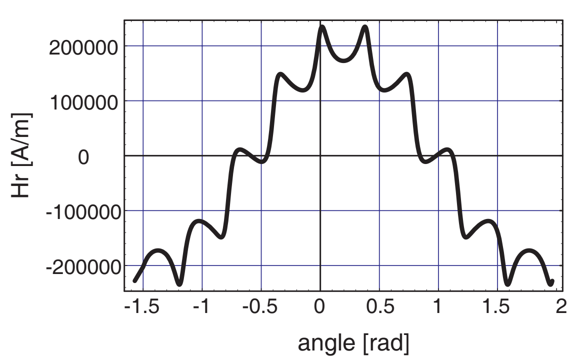

Figure 3: Exact radial field produced by a structure using tile permanent magnets with rotating polarizations, $r_2 = 0.028\,\mathrm{m}$, $r_1 = 0.025\,\mathrm{m}$, $r = 0.024\,\mathrm{m}$, $z_2 - z_1 = 0.003\,\mathrm{m}$, $z = 0.0015\,\mathrm{m}$, $J = 1\,\mathrm{T}$.

4. CONCLUSION

We have presented general analytical expressions of the magnetic field produced by tile permanent magnets of various magnetization directions. The expression are always given in an analytical part and a semi-analytical part. Such expression cannot be expressed in terms of elliptic integrals because we consider uniform polairzations, as it is generally the case in practice. Such expressions have a low computational cost and can be used for the study of Halbach structures.

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