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# Bond Graph Modeling of Current Diffusion in Magnetic Cores

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**KEYBOARD:** Bond Graphs, Variational approximation, Diffusive Representation, Current Diffusion, Maxwell's Equation.

**ABSTRACT:** The paper presents different modeling approaches that yield a same bond graph representation structure. The first one is based on a variational approximation of the Magnetic Induction Diffusion Equation and supplies various degrees of modeling starting from the simple standard electrical inductor. The second approach is based on the diffusion representation and may be simply connected to numerical simulation of the Maxwell's Equations in Frequency Domain. Finally such approaches should be applied to advanced modeling of Integrated Power Systems that operate at high switching frequency.

## 1. INTRODUCTION:

Electrical transformers and inductors are mainly based on magnetic cores. The design of such systems depend mainly on the losses they generate. These losses may be divided into hysteresis losses and current diffusion losses (induced currents), also known as Eddy's currents or Foucault's currents. Only current diffusion losses are addressed in this paper.

Current Diffusion [Pérez 1997], [Morel 2003] occurs in an electrical conductor or magnetic-conductive material. Under the assumption that the displacement currents are negligible, Maxwell's Equations combined with,

$$\vec{J} = \sigma \vec{E} \quad (1)$$

yields,

$$\Delta \vec{B} - \sigma \mu \frac{\partial \vec{B}}{\partial t} = 0 \quad (2)$$

where  $\sigma$  is the electrical conductivity,  $\vec{E}$  is the electric field,  $\vec{B}$  is the magnetic induction field,  $\vec{J}$  is the current density vector and  $\mu$  is the magnetic permeability.

We have shown [Morel 2003] that an order-2 variational approximation of equation (2) yields to a bond graph of the skin effect in a rectangular-section conductor. Equation (2) is the same, but the system is different because here, the magnetic core has not the role to conduct a current: it is only a parasitic effect.

The classical solution consists in solving the Maxwell's equations using numerical methods like finite element methods. An alternative method is the Diffusive Representation [Montseny 1998]. It is a part of the

Pseudo-differential operator theory that may be seen as an extension of transfer functions and Laplace's transform for systems of infinite order. Recent studies [Laudebat 2003] have shown the ability to apply diffusive representation in the modeling of the skin effect, or of a self-inductance having a conductive-magnetic core. In the latter cases, the transfer function is not rational, i.e. it is proportional to the square root of the frequency.

In this paper, a variational approximation of the current diffusion is constructed. Then it is compared with the diffusion representation. Both approaches yield a same bond graph representation.

## 2. MODELING CURRENT DIFFUSION IN A MAGNETIC CORE

### 2.1 PDE Modeling

The standard modeling approach is a local formulation of a Partial Differential Equation, PDE.

The Classical solution of equation (2) exists for a semi-infinite space. Practical cases correspond to parallelepiped volume as in Fig. 1. The assumption of semi-infinite space is satisfied if the geometrical domain are much larger than skin depth.

So, assuming a long and thin magnetic core, i.e.  $h, a \gg b$  (Fig. 1), the magnetic field does not depend neither on  $x$  nor on  $y$ . Therefore, the magnetic field depends only on  $z$  and on time.

Moreover, edges effects are neglected [Laudebat 2003].

The  $(oxy)$  and  $(oxz)$  planes are two planes of antisymmetry for the distribution, thus the magnetic field is in the direction of the  $x$ -axis.

Hence, Maxwell's equations in the magnetic core may be written as

$$\frac{\partial^2 B}{\partial z^2} - \sigma \mu \frac{\partial B}{\partial t} = 0 \quad (3)$$

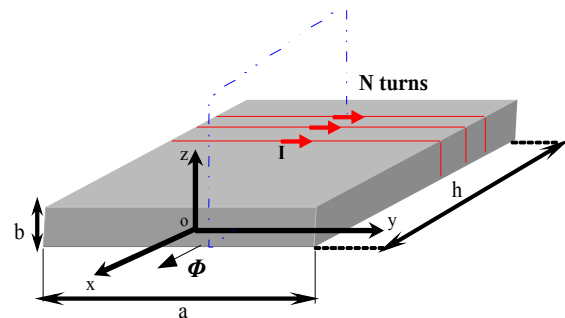


Fig. 1 The self inductance geometry

Applying the Ampere's Theorem on the loop represented in mixed line in Fig. 1 yields,

$$F = N I(t) = \int_0^h H dx = \frac{h}{\mu} B(0) \quad (4)$$

Where  $F$  is the magnetic potential difference of the magnetic core,  $N$  is the number of turns and  $I(t)$  the current flowing through the turns. Faraday's Law on electromagnetic induction enables to write the e.m.f. in one turn as

$$\lambda = \frac{v}{N} = \frac{d\phi}{dt} \quad (5)$$

Where  $\lambda$  is the magnetic flow variable,  $v$  is the voltage drop and  $\phi$  is the magnetic flow, given by

$$\phi = a \int_{-\frac{b}{2}}^{\frac{b}{2}} B(z) dz \quad (6)$$

Taking into account equation (3) and (5), the derivation with respect to time of equation (6) yields

$$\lambda = \frac{d\phi}{dt} = a \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\partial B}{\partial t} dz = \frac{a}{\sigma \mu} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\partial^2 B}{\partial z^2} dz = \frac{a}{\sigma \mu} \left[ \frac{\partial B}{\partial z} \right]_{-\frac{b}{2}}^{\frac{b}{2}} \quad (7)$$

Equations (3) and (7) define a boundary value problem. A variational approximation may be applied.

## 2.2 Variational Approximation

Our modeling approach is based on the variational approximation of "Galerkin" [Shawalter 1977].

The advantage of this numerical modelling method, known as "meshless", is to avoid the meshing of the structure what allows to decrease the simulation cost.

The idea is applied on the Magnetic Induction Diffusion Equation (2) what yields a bond graph model of Eddy's currents in the magnetic core.

Indeed, the method consists to build a finite state space model from the variational equation of relation (3). The variational equation is obtained by multiplying the two sides of equation (3) by a trial function  $s(z)$  then by integrating the result over the range  $[-b/2, b/2]$  where  $b$  is the thickness of the magnetic core. This yields the following variational equation,

$$\int_{-b/2}^{b/2} \frac{\partial^2 B}{\partial z^2} s dz - \sigma \mu \int_{-b/2}^{b/2} \frac{\partial B}{\partial t} s dz = 0 \quad (8)$$

The objective is to seek an expression of the magnetic field  $B$  using functions that verify the variational formulation (8) for any trial function  $s(z)$ . This is equivalent to verify the differential equation (3) against the magnetic field  $B$  and thus describes the physical phenomenon considered here.

Spatial distribution of the magnetic field  $B$  may be assumed to be symmetrical with respect to  $z$  because of

the symmetry of the geometry. Notice that this assumption is no longer valid if it exists several magnetic core or other electrical conductive elements in the system. Fortunately, in the later case only second order effects occur. Function  $s$  is also assumed to be symmetrical with respect to  $z$ , so (7) implies

$$\left[ \frac{\partial B}{\partial z} s(z) \right]_{-b/2}^{b/2} = s\left(\frac{b}{2}\right) \left[ \frac{\partial B}{\partial z} \right]_{-b/2}^{b/2} = s\left(\frac{b}{2}\right) \frac{\sigma \mu \lambda}{a} \quad (9)$$

so using (9), (8) may be rewritten as

$$\sigma \mu \frac{d}{dt} \int_{-b/2}^{b/2} B s dz + \int_{-b/2}^{b/2} \frac{\partial B}{\partial z} \frac{\partial s}{\partial z} dz = s\left(\frac{b}{2}\right) \frac{\sigma \mu \lambda}{a} \quad (10)$$

Which is a variational equation of the boundary value problem (3,7).

In order to resolve the problem (10), we apply a variational approximation to the magnetic field  $B$  which consists in a projection of the solution of the problem (10) on a finite-dimension space, i.e.,

$$\tilde{B}(t, z) = \sum_{j=0}^n \xi_j(t) w_j(z) \quad (11)$$

A difficult task is to define the decomposition functions,  $w_j$ . A good choice yields an accurate approximation with a small number of functions.

We have chosen eigen-functions of the steady state operator in (3) as

$$w_i(z) = \cosh\left(\frac{2iz}{b}\right) \quad 0 \leq i \leq n \quad (12)$$

Indeed, the hyperbolic cosine functions answer correctly to the constraints of the spatial distribution of the magnetic field  $B$ : they are symmetrical with respect to  $z$ . Classically in variational approximation techniques, the same base of functions  $w_i$  are chosen for the trial functions  $s(z)$ . Finally, combining equations (10-12) yields,

$$\frac{2}{b} \sum_{j=0}^n \xi_j i j \int_{-1}^1 \sinh(iu) \sinh(ju) du + \sigma \mu \frac{b}{2} \sum_{j=0}^n \xi_j \int_{-1}^1 \cosh(iu) \cosh(ju) du = \cosh(i) \frac{\sigma \mu \lambda}{a} \quad (13)$$

Moreover, equation (4) gives the output relation of the model,

$$F = N I(t) = \frac{h}{\mu} B(0) = \frac{h}{\mu} \sum_{j=0}^n \xi_j \cosh(0) = \frac{h}{\mu} \sum_{j=0}^n \xi_j \quad (14)$$

Defining,

$$F_i = \frac{h}{\mu} \xi_i \quad (15)$$

Equation (14) becomes

$$F = \sum_{j=0}^n F_j \quad (16)$$

Equation (13) may be rewritten as,

$$\frac{2}{b} \sum_{j=0}^n \xi_j M_{ij}^d + \sigma \mu \frac{b}{2} \sum_{j=0}^n \xi_j M_{ij}^s = \cosh(i) \frac{\sigma \mu \lambda}{a} \quad (17)$$

where,

$$M_{ij}^s = \int_{-1}^1 \cosh(iu) \cosh(ju) du \quad (18)$$

$$M_{ij}^d = ij \int_{-1}^1 \sinh(iu) \sinh(ju) du \quad (19)$$

defining

$$s_0 = 1, \quad s_k = \frac{\sinh(k)}{k} \quad k > 0 \quad (20)$$

the matrix elements may be written as

$$M_{ii}^s = 1 + s_{2i}, \quad M_{ij}^s = s_{i+j} + s_{i-j} \quad i \neq j \quad (21)$$

and

$$M_{ii}^d = i^2 (-1 + s_{2i}), \quad M_{ij}^d = ij (s_{i+j} - s_{i-j}) \quad i \neq j \quad (22)$$

Notice that  $M^s$  is symmetrical and

$$M_{0j}^d = 0 \quad 0 \leq j \leq n \text{ and } M_{i0}^d = 0 \quad 0 \leq i \leq n \quad (23)$$

The first values of the elements of these matrices are

$$M^s = \begin{bmatrix} 1+s_0 & 2s_1 & 2s_2 & \dots \\ 2s_1 & 1+s_2 & s_1+s_3 & \dots \\ 2s_2 & s_1+s_3 & 1+s_4 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \quad (24)$$

and

$$M^d = \begin{bmatrix} 0 & 0 & 0 & \dots \\ 0 & s_2-1 & 2(s_3-s_1) & \dots \\ 0 & 2(s_3-s_1) & 4(s_4-1) & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \quad (25)$$

Equations (14) and (17) define a state space model of the magnetic core.

Equation (17) may be rewritten as

$$2r_0 N^d F + \frac{P_0}{2} N^s \dot{F} = \lambda u_1 \quad (26)$$

Where

$$r_0 = \frac{a}{\sigma b h} \quad (27)$$

is a parameter that has the physical dimension of a resistance.

$$P_0 = \frac{\mu ab}{h}$$

is a parameter having the physical dimension of a permeance or an inductance.

$N^d$  is a matrix of coefficients:

$$N_{ij}^d = \frac{M_{ij}^d}{\cosh(i)} \quad (28)$$

$N^s$  is a matrix of coefficients:

$$N_{ij}^s = \frac{M_{ij}^s}{\cosh(i)} \quad (29)$$

$u_1$  is the vector of dimension  $n$ ,

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix} \quad (30)$$

Finally  $F$  is the state vector whose components  $F_i$  are defined in equation (16).

$$F = \begin{bmatrix} F_0 \\ F_1 \\ \dots \\ F_n \end{bmatrix} \quad (31)$$

In the case of the self inductance (Fig.1), variational approximation yields the following finite state space model:

$$\dot{F} = \lambda (P^{-1} \cdot u_1) - (P^{-1} \cdot r) \cdot F \quad (32)$$

$$I(t) = \frac{1}{N} \sum_{k=0}^n F_k \quad (33)$$

where :

$$P = \frac{P_0}{2} N^s$$

$$r = 2 * r_0 N^d$$

This state space model is analyzed in next section.



The following figures shows the results of the simulations:

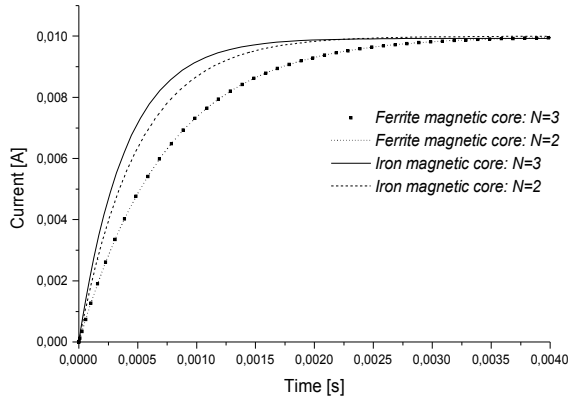


Fig. 7 Simulation of the current waveform in iron and ferrite magnetic cores for models with  $n=2$  and  $n=3$ .

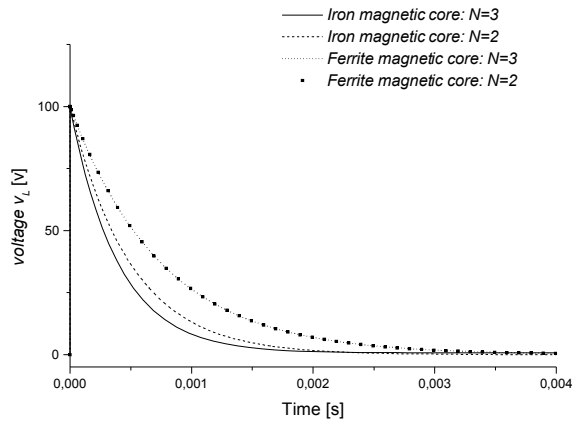


Fig. 8 Simulation of the voltage waveform across an iron and a ferrite magnetic cores for models with  $n=2$  and  $n=3$ .

As it is shown in figures 7 and 8, Foucault's currents phenomena are well described by the developed variational model.

The impact of Foucault's currents is visible by comparing the current waveform in the ferrite magnetic core model with the current waveform in the iron magnetic core model. The same thing, if we compare the voltage drop waveform across these two magnetic cores.

Indeed, the magnetic phenomenon is described by a fast dynamics of the voltage drop across the magnetic core at the beginning, due to the appearance of the Eddy currents. These currents, opposed to the magnetic flow, push the magnetic field outside the magnetic core. When the dynamic phenomena slow down. There is a progressive disappearance of the Foucault's currents that allows a progressive and slow diffusion of the magnetic induction inside the core. Thus, a phenomenon of "long memory" appears. Apparent inductance of iron is lower than in the case of ferrite where in the absence of Eddy currents,

induction is distributed uniformly through the section.

Moreover, it is clear on figures 7 and 8 that if we increase the order of the variational model, current and voltage drop waveforms across the iron magnetic core are improved.

#### Frequency Analysis:

The Fig. 9 shows the comparison of the normalized inductance solution of the variational model with  $n=2$  and the classical solution given by [Laudebat 2003]

$$L_r = \frac{L}{L_0} = \frac{\tanh(\sqrt{i\omega_r})}{\sqrt{i\omega_r}} \quad (38)$$

where "i" is the complex operator,

$$L_0 = \frac{N^2 ab \mu}{h} = N^2 P_0 \quad (39)$$

is the self-inductance of the magnetic core at low-frequency, and

$$\omega_r = \frac{\omega}{\omega_0} \quad (40)$$

is the normalized frequency with

$$\omega_0 = \frac{4}{b^2 \sigma \mu} = 4 \frac{r_0}{P_0}$$

It is clear that the self-inductance decreases when the frequency increases. This is due to the Eddy's current.

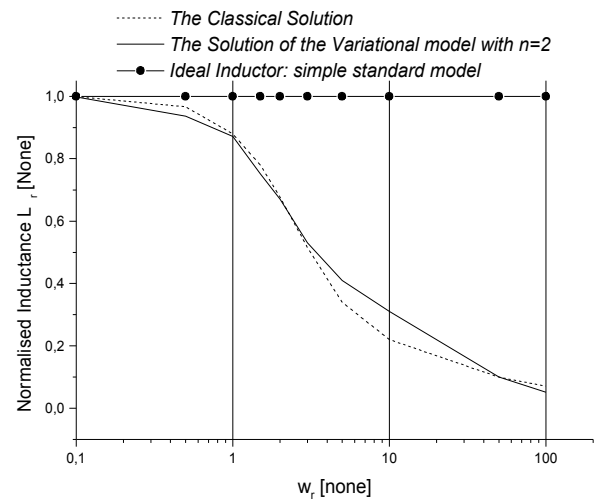


Fig. 9 Modeling the magnetic core - normalized inductance with and without current diffusion modeling.

### 2.3 Diffusive Representation Model

In the case of the self inductance (Fig. 1), diffusive representation yields the following infinite order. This is

an exact model [Laudebat 2003].

$$\frac{d\phi_k}{dt} = \frac{-4k^2\phi_k}{b^2\sigma\mu} + v \quad K = 0, 1, 2, \dots, \infty \quad (41)$$

$$I(t) = \frac{h}{N^2\mu ab} \sum_{k=0}^{\infty} \phi_k \quad (42)$$

Defining

$$F_k = \frac{h}{N\mu ab} \phi_k$$

$$N_{ij}^d = i^2 \delta_{ij} \quad (43)$$

and

$$N_{ij}^s = \delta_{ij} \quad (44)$$

Equation (41) and (42) are equivalent to (32) and (33). So, the diffusive representation yields to the same bond graph (Fig. 5). Differences exist in the matrix values. They are due to different assumptions, i. e. equations (41) and (42) are based on the choice of residue values of the diffusive representation symbol [Laudebat 2003]. The main interest of the diffusive representation, is the ability to build a finite-order approximated state space model from a frequency domain representation. The frequency domain representation may be obtained numerically, by classical FEM simulations.

### 3. CONCLUSION

The paper presents different modeling methods that yield to the same bond graph structure of the model of the current diffusion in magnetic core. The first approach

yields a first order model, the standard electrical inductor model. The second order takes into account current diffusion (induced currents). The N-order model equations have the same structure than the diffusive representation model. However diffusive representation enables an efficient coupling with powerful numerical simulators based on FEM and should yield an accurate bond graph model of the current diffusion. The approach may be extended to any phenomenon based on a diffusion equation.

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