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Finite Element Magnetic Models via a Coupling of Subproblems of Lower Dimensions

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Abstract— Model refinements of magnetic circuits are performed via a subdomain finite element method based on a perturbation technique. A complete problem is split into subproblems, some of lower dimensions, to allow a progression from 1-D to 3-D models. Its solution is then expressed as the sum of the subproblem solutions supported by different meshes. The procedure simplifies both meshing and solving processes, and quantifies the gain given by each model refinement on both local fields and global quantities.

I. INTRODUCTION

The perturbation of finite element (FE) solutions provides clear advantages in repetitive analyses and helps improving the solution accuracy [1]-[4]. It allows to benefit from previous computations instead of starting a new complete FE solution for any variation of geometrical or physical data. It also allows different problem-adapted meshes and computational efficiency due to the reduced size of each subproblem.

A subproblem FE method is herein developed for coupling solutions of various dimensions, starting from simplified models, based on ideal flux tubes defining 1-D models, that evolve towards 2-D and 3-D accurate models. It is an extension of the method proposed in [2]-[4], applied to refinements up to 3-D models. From the so calculated field corrections, the associate corrections of global quantities inherent to magnetic models, i.e. fluxes, magnetomotive forces (MMFs), currents and voltages, are also evaluated. The developments are performed for the magnetic vector potential FE magnetostatic and magnetodynamic formulations, paying special attention to the proper discretization of the constraints involved in each subproblem. The method is illustrated and validated on test problems.

II. COUPLING OF MAGNETIC MODELS OF VARIOUS DIMENSIONS

A. Series of coupled subproblems

Instead of solving a complete problem, generally with a 3-D model, it is proposed to split it into a sequence of subproblems, some of lower dimensions, i.e. 1-D and 2-D models. Its solution is then to be expressed as the sum of the subproblem solutions.

Each subproblem is defined in its own domain, generally distinct from the complete one. At the discrete level, this aims to decrease the problem complexity and allow distinct meshes with suitable refinements. Each subproblem approximates at best its contribution to the complete solution. The domains of

the subproblems can overlap [2], [3] or not [1], [4]. Herein, non-overlapping subdomains are considered. They are separated by interfaces $\Gamma_{f,p}$, through which a sequence of boundary conditions (BCs) or interface conditions (ICs) is to be defined.

B. Canonical magnetostatic or magnetodynamic problems

Each subproblem p is defined in a domain Ω_p , with boundary $\partial\Omega_p = \Gamma_p = \Gamma_{h,p} \cup \Gamma_{b,p}$. It is governed by magnetostatic or magnetodynamic equations with volume and surface sources or constraints. Classical volume sources fix remnant inductions in magnetic materials and current densities in stranded inductors. Similar volume sources can also express changes of permeability and conductivity from one problem to another [3]. Also, the usually homogeneous surface sources, i.e. BCs or ICs on the traces of the magnetic field \mathbf{h}_p and flux density \mathbf{b}_p , respectively $\mathbf{n} \times \mathbf{h}_p|_{\Gamma_{f,p}}$ and of $\mathbf{n} \cdot \mathbf{b}_p|_{\Gamma_{f,p}}$, with \mathbf{n} the unit exterior normal, can be extended to non-zero constraints calculated from previous problems. ICs have the general forms

$$[\mathbf{n} \times \mathbf{h}_p]_{\gamma_p} = \mathbf{j}_{f,p}, \quad [\mathbf{n} \cdot \mathbf{b}_p]_{\gamma_p} = \mathbf{b}_{f,p}, \quad (1a-b)$$

where the notation $[\cdot]_{\gamma} = \cdot|_{\gamma^+} - \cdot|_{\gamma^-}$ expresses the discontinuity of a quantity through an interface γ (with sides γ^+ and γ^-) in Ω_p . The associated surface fields $\mathbf{j}_{f,p}$ and $\mathbf{b}_{f,p}$ are generally zero, defining classical ICs for the physical fields, i.e. the continuities of the tangential component of \mathbf{h}_p and of the normal component of \mathbf{b}_p . If nonzero, they define possible surface sources that account for particular phenomena occurring in the idealized thin region between γ^+ and γ^- .

C. Sources at subproblem interfaces

Portions of a 3-D structure satisfying a translational or rotational symmetry can be first studied via 2-D models. This consists in neglecting some end effects, zeroing either $\mathbf{n} \times \mathbf{h}_p|_{\Gamma_{f,p}}$ or $\mathbf{n} \cdot \mathbf{b}_p|_{\Gamma_{f,p}}$. Besides, if the field is chosen to be zero out of Ω_p , a discontinuity of one of its traces is then voluntarily defined through $\Gamma_{f,p}$.

With such assumptions, two subproblems 1 and 2 with adjacent non-overlapping subdomains Ω_1 and Ω_2 share a common interface $\Gamma_{f,1} = \Gamma_{f,2}$ through which a field discontinuity occurs. A third subproblem, 3-D, serves then to correct the field distribution in a certain neighborhood Ω_3 on both sides of the interface, then denoted $\Gamma_{f,3}$. This is done via ICs

$$[\mathbf{n} \times \mathbf{h}_3]_{\Gamma_{f,3}} = \mathbf{j}_{f,3}, \quad [\mathbf{n} \cdot \mathbf{b}_3]_{\Gamma_{f,3}} = \mathbf{b}_{f,3}. \quad (2a-b)$$

with the surface sources

$$\mathbf{j}_{f,3} = -(\mathbf{n} \times \mathbf{h}_1|_{\Gamma_{f,1}} - \mathbf{n} \times \mathbf{h}_2|_{\Gamma_{f,2}}), \quad \mathbf{b}_{f,3} = -(\mathbf{n} \cdot \mathbf{b}_1|_{\Gamma_{f,1}} - \mathbf{n} \cdot \mathbf{b}_2|_{\Gamma_{f,2}}). \quad (3a-b)$$

These sources compensate the traces 1 and 2 to recover the continuity of the total solution. Note that $\Gamma_{f,1}$, $\Gamma_{f,2}$ and $\Gamma_{f,3}$ are similar and only differ at the discrete level due to their different supporting meshes.

The ICs (2a) and (2b) in a magnetic vector potential (\mathbf{a}) FE formulation are considered via natural and essential constraints respectively [3]. The essential constraint strongly fixes the discontinuity of the trace of \mathbf{a} through $\Gamma_{f,3}$ (continuity if $\mathbf{b}_{f,3}=0$), whereas the natural constraint weakly acts via a surface integral term in the FE formulation. This surface term, with (2a) and (3a), involves the traces of previous solutions, each one being actually involved in similar surface terms in the associated previous FE formulations, thus linked with their other volume integrals. At the discrete level, these surface integrals must be substituted with those volume integrals, limited to one single layer of FEs touching the interface [1]-[4]. Because each solution is calculated in a different mesh, mesh-to-mesh projections of solutions are necessary. They can be profitably limited to the single layers of FEs. This procedure is of key importance for ensuring consistency between all the formulations and their coupling. It will be detailed in the extended paper and it will be shown to allow the accurate calculation of the global quantities (flux, MMF, current, voltage) at each step of the series, in particular the correction due to the end effects.

III. APPLICATION EXAMPLES

As a primary illustration, two flux tubes are first separately considered before being connected in series (Fig. 1). The solutions in each separate tube are simply calculated via 1-D models. When the tubes are connected, their junction surface acts as an interface $\Gamma_{f,3}$, with continuity of the normal magnetic flux density ($[\mathbf{n} \cdot \mathbf{b}_3]_{\Gamma_{f,3}} = 0$) and discontinuity of the tangential magnetic field ($[\mathbf{n} \times \mathbf{h}_3]_{\Gamma_{f,3}} \neq 0$). This gives the requested source for a 2-D model, calculating the field correction limited to a certain neighborhood Ω_3 on both sides of the interface, with a locally refined mesh.

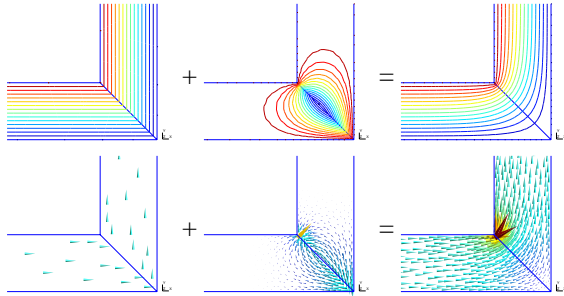


Fig. 1. Field lines (top) and magnetic flux density (bottom) of the initial problem with two ideal flux tubes in series (\mathbf{b}_1 and \mathbf{b}_2 in both tubes left), its local correction at the junction (\mathbf{b}_3 , middle) and the complete solution (\mathbf{b} , right).

A stranded inductor is then studied via the coupling of a 2-D plane model for its portion with a translational symmetry, a 2-D axisymmetrical model for its end winding and a 3-D model for the 3-D correction on both sides of the interface separating the portions (Figs. 2 and 3). Because the correction is local to the interface, the associated 3-D mesh only needs to be refined in its vicinity.

Various results and discussions will be given in the extended paper, in particular regarding the correction of both local and global quantities, the way to consider additional regions (e.g. the magnetic or conducting plate below the inductor in Fig. 2; based on [1] and [3] for magnetostatic and magnetodynamic models), the way the fields decrease at infinity

with the different models and the adaptation of the domain of each subproblem with its effect on the convergence of the complete solution. Parameterized analysis modifying some subproblems (e.g. end windings) while keeping the others constant will be shown to benefit from the developed method.

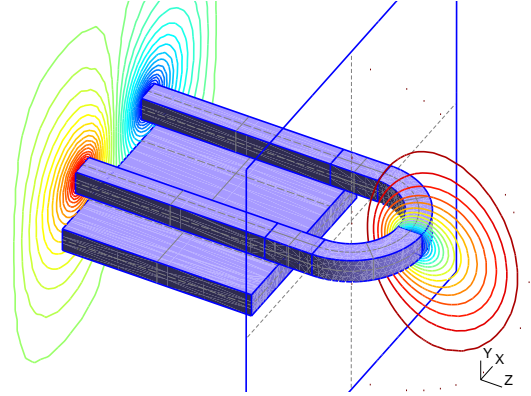


Fig. 2. Field lines generated by a stranded inductor (half geometry): solution of a 2-D plane model in the XY plane ($z=0$) (\mathbf{b}_1 , portion on the left) and of a 2-D axisymmetrical model in the YZ plane ($x=0$) (\mathbf{b}_2 , portion on the right); the interface between the two portions is shown.

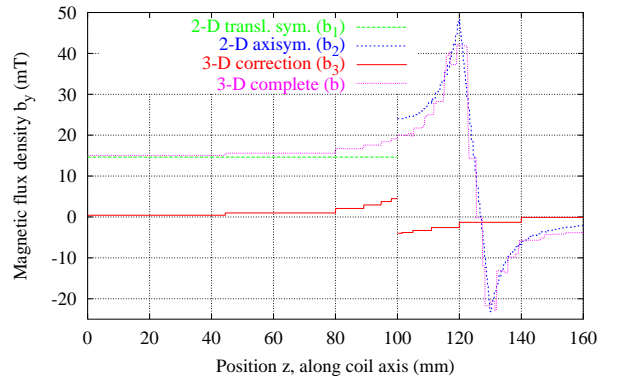


Fig. 3. Magnetic flux density along the coil axis in the 3-D system: \mathbf{b}_1 for the 2-D plane model (implicitly extended as a constant up to $z=100$ mm), \mathbf{b}_2 for the 2-D axisymmetrical model, \mathbf{b}_3 for the 3-D correction and \mathbf{b} for the complete 3-D model. The solution $\mathbf{b}_1+\mathbf{b}_2+\mathbf{b}_3$ is generally obtained with a higher accuracy than \mathbf{b} for a lower computational cost thanks to the coupling of meshes, some of lower dimensions.

IV. CONCLUSIONS

The developed subdomain FE method allows to split magnetic models into subproblems of lower complexity with regard to meshing operations and computational aspects. A natural progression from simple to more elaborate models, from 1-D to 3-D geometries, is thus possible, while quantifying the gain given by each model refinement on both local and global quantities.

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