

Fault diagnosis of uncertain linear system using structural knowledge

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Abstract

This paper presents an algebraic approach of fault diagnosis in linear dynamical system. Additive actuator and sensor faults are considered. Using tools and results of algebraic identification and pseudospectra analysis, a method is proposed for the generation of a residual sensitive to faults. The main advantage of this new approach is to realize fault diagnosis only from knowledge of input and output measurements and without identifying explicitly model parameters. A numerical example is provided and discussed to illustrate the efficiency of the proposed fault diagnosis method.

Keywords: distribution theory, spectra analysis, actuator fault, sensor fault, FDI

1. INTRODUCTION

Fault diagnosis methods include some actions implemented in order to detect, isolate and identify any abnormal phenomenon on a system.

One can find in (Isermann [2006]), (Patton et al. [2000]) and the references therein the classical approaches using analytical information and which allow to do robust fault diagnosis in the presence of unknown entries and parametric uncertainties, namely parity space, observer and parameter estimation. These methods depend not only on structural knowledge of the system, but also require knowledge of system parameters that can be more or less accurate.

The fault diagnosis approach proposed in this paper, contrary to the usual approaches, does not need a process identification procedure. In particular, the only assumptions that we will make concern:

- the process structure,
- the fault structure.

It means that:

- for the first assumption, we consider only known the degree of the differential equation modelling the system although its coefficients are assumed to be unknown and constant.

- for the second assumption, we consider structured additive actuator and sensor faults (Fliess and Sira-Ramirez [2003]) which take a large variety of signals into account.

For the design of the fault diagnosis algorithm, we adopt a distributional formulation (also used in (Fliess and Sira-Ramirez [2003]) and (Belkoura and Richard [2006])). This formulation allows us to transform the diagnosis problem into spectrum analysis problem from which we can detect and isolate faults without estimating system parameters. The paper is organized as follows. Section 2 is devoted to the recall of mathematical tools which will be used along

the paper. In section 3, different assumptions are made on the system structure and the fault signal structures before defining and solving the fault diagnosis problem. Section 4 illustrates our approach with an academic example.

2. MATHEMATIC TOOLS

2.1 Distribution theory

We recall in this section some definitions and results from the distribution theory (Schwartz [1966]) and fix the notations we will use later.

Let \mathcal{K} be an open set of \mathbb{R} , the support of a function f defined on set \mathcal{K} is the closure of the set of points where the function is not zero and is written $Supp(f)$. The space of smooth functions (infinitely differentiable) with compact support in \mathcal{K} is denoted by $\mathcal{D}(\mathcal{K})$, $\mathcal{D}'(\mathcal{K})$ is the space of distributions on \mathcal{K} , i.e. the space of linear and continue functionals on $\mathcal{D}(\mathcal{K})$. Distributions extend the concept of derivative to all locally integrable functions. If a function f is locally measurable on \mathcal{K} , we define the regular distribution T_f in $\mathcal{D}'(\mathcal{K})$ for all $\phi \in \mathcal{D}(\mathcal{K})$ by $\langle T_f, \phi \rangle = \int f(s)\phi(s)ds$. In addition, if function f is continuous except at point x with a finite jump s_x , the associated distribution derivative is given by $\dot{T}_f = \dot{f} - s_x\delta_x$, where \dot{f} is the usual derivative of function f . The complement of the largest open subset of \mathcal{K} in which the distribution T vanishes¹ is called the support of T ; we denote it also by $Supp(T)$. A subspace of distributions of great interest in control theory is the subspace \mathcal{D}'_+ of distributions with left bounded support (contained in $[0, +\infty[$). Equipped with convolution and addition operations, this subspace becomes a convolution algebra with Dirac delta distribution δ as convolution identity. The Dirac distribution with delay τ is written

¹ We said that a distribution T vanishes on $\Omega \subset \mathcal{K}$ if $T(\phi) = 0$ for all $\phi \in \mathcal{D}(\Omega)$

as δ_τ . The distribution T is said to be finite order if there exists a natural number m so that, for all compact subsets $\Omega \subset \mathcal{K}$ with no empty interior

$$\exists C > 0 : \forall \phi \in \mathcal{D}(\Omega), |T(\phi)| \leq C \sup_{0 \leq i \leq m} \|\phi^{(i)}\|_\infty$$

The smallest number m satisfying the above inequality is called the order of the distribution T and is written as $Ord(T)$.

Example 2.1. $Supp(\delta_\tau) = \{\tau\}$ and $Ord(\delta_\tau) = 0$.
 $Supp(T_f) = Supp(f)$ and $Ord(T_f) = 0$.

Derivation, integration and translation can be formed from the convolution product $\dot{y} = \delta^{(1)} * y$, $\int_0^t y = H * y$, $y(t - \tau) = \delta_\tau * y$, where H is the unit step function (Heaviside distribution). The next theorem is the main result from which we will design our fault diagnosis algorithm.

Theorem 2.1. (Schwartz [1966]) If a distribution T has a compact support $Supp(T)$ and a finite order m then $\alpha T = 0$ whatever smooth function α and all its derivatives of order $\leq m$ vanish on $Supp(T)$.

According to the above theorem, it follows

$$t^k \delta^{(n)} = \begin{cases} 0 & \text{if } k > n \\ (-1)^k \frac{n!}{(n-k)!} \delta_\tau^{(n-k)} & \text{else} \end{cases} \quad (1)$$

Finally, if the convolution product of distributions S and T is well defined, we have the following equality

$$t^n (S * T) = \sum_{k=0}^n C_n^k (t^k S) * (t^{n-k} T), \quad \forall t \in \mathbb{R}, n \in \mathbb{N} \quad (2)$$

2.2 Spectral analysis

Given matrices $\tilde{A}_0, \tilde{A}_1, \dots, \tilde{A}_k \in \mathbb{R}^{m \times n}$, we assume that there exists at least one vector of reals $\tilde{\beta}$ which reduces the rank of the pencils $\tilde{A}_0 - \tilde{\beta}_1 \tilde{A}_1 - \tilde{\beta}_2 \tilde{A}_2 - \dots - \tilde{\beta}_k \tilde{A}_k$, i.e., there exists $\tilde{X} \in \mathbb{R}^n / \{0\}$ so that

$$(\tilde{A}_0 - \tilde{\beta}_1 \tilde{A}_1 - \tilde{\beta}_2 \tilde{A}_2 - \dots - \tilde{\beta}_k \tilde{A}_k) \tilde{X} = 0 \quad (3)$$

Consider now matrices A_0, A_1, \dots, A_k originated from matrices $\tilde{A}_0, \tilde{A}_1, \dots, \tilde{A}_k$ by perturbation (additive noises). The problem we want to solve in this section is the following: Given perturbed matrices A_0, A_1, \dots, A_k (without knowing matrices $\tilde{A}_0, \tilde{A}_1, \dots, \tilde{A}_k$), we search vector $\beta = (\beta_1, \dots, \beta_k)$ solution of the problem

$$\begin{cases} \min_{\|X\|=1, \lambda} \|(A_0 - \lambda_1 A_1 - \lambda_2 A_2 - \dots - \lambda_k A_k) X\|^2 \\ (\tilde{A}_0 - \lambda_1 \tilde{A}_1 - \lambda_2 \tilde{A}_2 - \dots - \lambda_k \tilde{A}_k) X = 0 \end{cases} \quad (4)$$

Note that if $\|\bullet\|$ is the 2-norm, then

$$\min_{\|X\|=1} \|(A_0 - \lambda_1 A_1 - \dots - \lambda_k A_k) X\| = \sigma_{\min}(A_0 - \lambda_1 A_1 - \dots - \lambda_k A_k)$$

and β can be characterized by

$$\beta = \underset{\lambda}{\operatorname{argmin}} \sigma_{\min}(A_0 - \lambda_1 A_1 - \dots - \lambda_k A_k)$$

where $\sigma_{\min}(\bullet)$ denotes the smallest singular value of \bullet .

Let, for each $\epsilon \geq 0$, $\Lambda_\epsilon(A_0, \dots, A_k)$ be the set

$$\{\lambda \in \mathbb{R}^k : \sigma_{\min}(A_0 - \lambda_1 A_1 - \dots - \lambda_k A_k) \leq \epsilon\}$$

Sets $\Lambda_\epsilon(A_0, \dots, A_k)$ are most likely to be empty for very small values of ϵ . As this value grows, we obtain a family of nonempty sets $\Lambda_\epsilon(A_0, \dots, A_k)$. Then, β can be estimated

as local minimum of this family of sets. The case $k = 1$ which has been the subject of a wide literature ((Boutry et al. [2005]) and (Wright and Trefethen [2002])) is called generalized eigenvalue problem. If $k > 1$, we will show that, with a suitable choice of dimensions m and n , one can always bring back to the case $k = 1$.

Definition 2.1. (cokernel)

The cokernel of matrix M , denoted by $\operatorname{coker}(M)$, is a matrix N such that matrix NM is the zero. A matrix can admit more than one matrix as cokernel.

Let $m, n \in \mathbb{N}$ with $m > n$ and A a matrix in $\mathbb{R}^{m \times n}$. There exists an orthogonal matrix $Q \in \mathbb{R}^{m \times m}$ and an upper triangular matrix $R \in \mathbb{R}^{m \times n}$ so that $A = QR$. This decomposition is called qr decomposition. It also gives a cokernel of dimension $((m-n) \times m)$ of matrix A , it's matrix N which rows are the transpose of the $m-n$ last columns of matrix Q (i.e. $N = Q(:, n+1 : m)^T$). Indeed Q being orthogonal, we have

$$\begin{aligned} NQ &= (0_{m-n, n} I_{m-n}) \text{ and} \\ NA &= NQR \\ &= [0_{m-n, n} I_{m-n}] R \\ &= 0_{m-n, n} R(1 : n, :) + I_{m-n} R(n+1 : m, :) \\ &= 0_{m-n, n} \end{aligned}$$

We give now a solution of the above problem. Let $m > (k-1)n$ and consider N_k the cokernel of matrix A_k with dimension $(m-n) \times m$ computed from qr decomposition. Then, we can write

$$\begin{aligned} \sigma_{\min}(A_0 - \beta_1 A_1 - \dots - \beta_k A_k) &= \\ \sigma_{\min}(N_k A_0 - \beta_1 N_k A_1 - \dots - \beta_{k-1} N_k A_{k-1}) & \end{aligned}$$

because σ_{\min} is invariant under orthogonal transformations.

Now, let $N_{k-1} \in \mathbb{R}^{(m-2n) \times (m-n)}$ be the cokernel of matrix $N_k A_{k-1}$. We have

$$\begin{aligned} \sigma_{\min}(N_k A_0 - \beta_1 N_k A_1 - \dots - \beta_{k-1} N_k A_{k-1}) &= \\ \sigma_{\min}(N_{k-1} N_k A_0 - \dots - \beta_{k-2} N_{k-1} N_k A_{k-2}) & \end{aligned}$$

and so on, this process gives the following equality

$$\begin{aligned} \sigma_{\min}(A_0 - \beta_1 A_1 - \dots - \beta_k A_k) &= \\ \sigma_{\min}(P_{1,k} A_0 - \beta_1 P_{1,k} A_1) & \end{aligned}$$

where $P_{1,k} = N_2 N_3 \dots N_k$ and N_j is a cokernel of dimension $(m - (k-j+1)n) \times (m - (k-j)n)$ of matrix $N_{j+1} N_{j+2} \dots N_k A_j$, $2 \leq j \leq k-1$ (that justifies the choice $m > (k-1)n$). These operations allow to determinate β_1 . The estimations of β_2, \dots, β_k can be achieved by applying the same steps.

The obvious algorithm to calculate $\underset{\lambda}{\operatorname{argmin}} \sigma_{\min}(A - \lambda B)$

where A and B are matrices of same dimension is to evaluate $\sigma_{\min}(A - \lambda_i B)$ for values of λ_i on a grid in the complex plane and then to generate a contour plotted from this data. Finally, we can take the possible minima on the intersection of the real axis and the contour. The number of possible local minima depends on the matrix rank. For example, in the above study, for each element β_i ,

the number of distinct solutions varies from 1 to $\inf(n, m - (k - 1)n)$. Additional constraints on β_i permit to search particular solution of the problem.

In the next section, we show the interest of concepts presented in this section. The design of the fault diagnosis algorithm leads to solve a problem expressed as (4), where matrices A_i are composed of process signal measurements and vector β contains the useful parameter to accomplish the fault diagnosis task.

3. FAULT DIAGNOSIS

Before detailing the algorithm stages, we make in the next subsections some assumptions about the system structure and the fault signals.

3.1 Model structure

Consider system (Σ) where control signal $u_r(t)$ and output signal $y_r(t)$ satisfy a differential equation described by

$$\begin{cases} a_n y_r^{(n)} + a_{n-1} y_r^{(n-1)} + \dots + a_0 y_r = b u_r \\ y_r^{(i)}(0), i = 0, \dots, n-1 \end{cases} \quad (5)$$

Parameter a_i , b and the initial conditions are supposed unknown and constant. In the distributional formulation (by kipping the same notation for the signals and their associated distributions), the system (5) becomes

$$a_n y_r^{(n)} + a_{n-1} y_r^{(n-1)} + \dots + a_0 y_r = b u_r + \phi_0 \quad (6)$$

where ϕ_0 is a linear combination of derivatives Dirac distribution of order less or equal to $n - 1$ and contains the contributions of the initial conditions.

When an actuator fault denoted f_a and a sensor fault denoted f_c occur, then the system is controlled by $u(t) + f_a$ and generates output $y(t) - f_c$ where $u(t)$ is the true control computed by the controller and $y(t)$ is the measured output, as illustrated in figure (1). Then, signals u and y satisfy an equation of the form:

$$a_n y^{(n)} + \dots + a_0 y = \phi_0 + b u + b f_a + a_n f_c^{(n)} + \dots + a_0 f_c \quad (7)$$

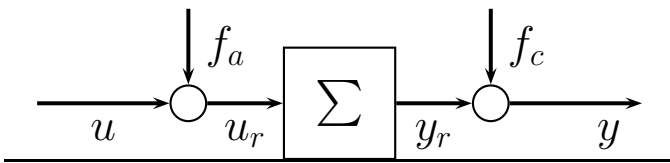


Figure 1. System with actuator and sensor faults.

3.2 Fault signal structure

We deal with fault signals modelled by structured signals (Fliess and Sira-Ramirez [2003]). A structured signal can be defined, in an informal way, as a solution of a linear differential equation. The main fault signals found in literature (abrupt, ramp, intermittent faults) can be modelled as structured signals (Fliess et al. [2010]). For this presentation, we deal with the case of abrupt faults f_a and f_c modelled as delayed step signals, i.e.

$$f_a = l_a H(t - \tau_a) \quad (8)$$

$$f_c = l_c H(t - \tau_c) \quad (9)$$

where τ_a , l_a and τ_c , l_c are respectively the occurrence time and the magnitude of faults f_a and f_c . This assumption concerning the structure of faults f_a and f_c is not a restriction since the proposed algorithm can be applied for all types of faults modelled by structured signals.

3.3 System with actuator fault

Under the assumption of the occurrence of an abrupt actuator fault, equation (6) becomes

$$a_n y^{(n)} + \dots + a_0 y = \phi_0 + b u + b l_a H(t - \tau_a) \quad (10)$$

From this model structure, a fault diagnosis algorithm is designed by assuming known only order n , control $u(t)$ and output $y(t)$. Thus a_i , b and ϕ_0 are supposed to be unknown. This algorithm is composed of two stages, the generation of redundancy relations and the analysis of these relations.

In that follows, steps needed to generate redundancy equations are described.

First, let us derive the two members of equation (10) to obtain

$$a_n y^{(n+1)} + \dots + a_0 y^{(1)} - b u^{(1)} = \phi_0^{(1)} + b l_a \delta_{\tau_a} \quad (11)$$

where δ_{τ_a} and $\phi_0^{(1)}$ (singularities) have the following properties:

- (1) $Supp(\phi_0^{(1)}) = \{0\}$ and $Ord(\phi_0^{(1)}) = n$
- (2) $Supp(\delta_{\tau_a}) = \{\tau_a\}$ and $Ord(\delta_{\tau_a}) = 0$

To eliminate these singularities, let us just multiply equation (11) by a suitable function (theorem 2.1). Set

$$\alpha_1(t) = t^{n+1}(t - \tau_a) \quad (12)$$

then

$$\alpha_1(a_n y^{(n+1)} + \dots + a_0 y^{(1)} - b u^{(1)}) = 0 \quad (13)$$

because $t^{n+1}\phi_0^{(1)} = 0$ and $(t - \tau_a)\delta_{\tau_a} = 0$. It follows, from (13) that for $t \geq 0$

$$\begin{aligned} t^{n+2}[a_n y^{(n+1)} + \dots + a_0 y^{(1)} - b u] - \\ \tau_a t^{n+1}[a_n y^{(n+1)} + \dots + a_0 y^{(1)} - b u] = 0 \end{aligned} \quad (14)$$

or, by setting for $i = 1, 2$ and $j = 0, \dots, n$

$$W_{i,j} = t^{n+i} y^{(j+1)} \text{ and } V_i = t^{n+i} u^{(1)}$$

$$\left[\sum_{j=0}^n a_j W_{2,j} - b V_2 \right] - \tau_a \left[\sum_{j=0}^n a_j W_{1,j} - b V_1 \right] = 0 \quad (15)$$

$W_{i,j}$ and V_i can be expressed, according to (1) and (2), like a linear finite combination of the derivatives of products $t^k y$ and $t^k u$ respectively. Indeed, we have

$$\begin{aligned} W_{i,j} &= \sum_{k=0}^{\inf(n+i,j+1)} (-1)^k C_k^{n+i} \frac{(j+1)!}{(j+1-k)!} \delta^{(j+1-k)} * (t^{n+i-k} y) \\ V_i &= \delta^{(1)} * (t^{n+i} u) - (n+i)(t^{n+i-1} u) \end{aligned}$$

Equality (15) is satisfied in spite of τ_a (τ_a is not identifiable before fault occurrence) because, for $t < \tau_a$

$$t^n \left[\sum_{i=0}^n a_i y^{(i+1)} - bu^{(1)} \right] = 0$$

However, for $t \geq \tau_a$, the redundancy relations obtained by applying successive convolutions on equation (15) by H lead to the following spectral formulation

$$[A_2 - \tau_a A_1]X = 0 \quad (16)$$

by setting, for $i = 1, 2$

$$A_i = \begin{pmatrix} H^{*n+1} * W_{i,n} & \dots & H^{*n+1} * V_i \\ H^{*n+2} * W_{i,n} & \dots & H^{*n+2} * V_i \\ \vdots & \vdots & \vdots \\ H^{*n+m} * W_{i,n} & \dots & H^{*n+m} * V_i \end{pmatrix}$$

$$X = (a_n \ a_{n-1} \ \dots \ a_{n-p} \ \dots \ a_0 \ -b)^T \quad (17)$$

$m > 0$ (number of rows), $H^{*p} = \underbrace{H * H * \dots * H}_{p \text{ times}}$.

The initial successive convolution (H^{*n+1}) ensures the elimination of all derivatives (numerically less robust than integrations).

In general, for any structured fault, the generation of redundancy relations is the same than the previous one and consists of three steps

- (1) Derivation (apparition of singularities)
- (2) Multiplication (elimination of singularities)
- (3) Integration (spectral formulation)

From expression (16), the estimation of τ_a consists in the computation of generalized eigenvalues of A_2 and A_1 . In practice, signals u and y being corrupted by noises, we rather have $\tau_a \in \Lambda_\epsilon(A_2, A_1)$, $\epsilon > 0$ (see section 2.2). Matrices A_2 and A_1 being time-varying, thus the estimation of τ_a is also. At each time t , the value of τ_a estimated (chosen among the possible values) consists of the generalized eigenvalue associated with vector X wich is the closest possible to the vector associated with the earlier value.

The decision rule is based on the time behaviour of τ_a and l_a estimations (characteristics of the fault).

For an actuator fault (here delayed step), the estimation of τ_a becomes stationary just after the time of the fault occurrence.

After having estimated τ_a , a similar reasoning leads to the estimation of l_a . Indeed, setting $\alpha_2 = t^{n+1}$ yields

$$\alpha_2(a_n y^{(n+1)} + \dots + a_0 y^{(1)} - bu^{(1)}) = bl_a \alpha_2 \delta_{\tau_a}$$

$$\Rightarrow \forall t \geq \tau_a \text{ and } k \geq n+1:$$

$$l_a = \frac{H^{*k} * [\alpha_2(q_{n+2}y^{(n+1)} + \dots + q_2y^{(1)} + q_1u^{(1)})]}{H^{*k} * [-q_1\alpha_2\delta_{\tau_a}]}$$

where $q = (q_{n+2}, \dots, q_1)^T$ is the "smallest singular vector"² of matrix $A_2 - \tau_a A_1$.

² associated to the smallest singular value

The successive convolutions H^{*p} (successive integrations) proceed like a low-pass filter and contribute to attenuate high frequency noises.

For the generation of matrices A_i , we have taken the polynomial function α_1 in order to simplify computations. Damped functions, like $\alpha = e^{-at}t^{n+1}(t - \tau_a)$, $a > 0$, can improve the approach robustness to measurement noises. Indeed, these functions contribute to attenuate noises, in opposition to polynomial functions which accentuate the noise effects.

3.4 System with sensor faults

In this part, it is provided the algorithm ensuring system fault diagnosis when (Σ) is corrupted by a sensor fault. This fault is modelled as a delayed step. We keep the same assumptions as previously.

So, under the assumption of the occurrence of an abrupt sensor fault at time τ_c with a magnitude l_c , the faulty system equation becomes

$$a_n y^{(n)} + \dots + a_0 y = \phi_0 + bu$$

$$+ l_c(a_n \delta_{\tau_c}^{(n-1)} + \dots + a_0 H(t - \tau_c)) \quad (18)$$

The derivation yields to

$$a_n y^{(n+1)} + \dots + a_0 y^{(1)} - bu^{(1)} =$$

$$\phi_0^{(1)} + l_c(a_1 \delta_{\tau_c}^{(n)} + \dots + a_0 \delta_{\tau_c}) \quad (19)$$

To eliminate the singularities in the second member of equation (19), we can consider the function

$$\alpha_3(t) = t^{n+1}(t - \tau_c)^{n+1} \quad (20)$$

which yields (according to theorem (2.1))

$$\alpha_3(a_n y^{(n+1)} + \dots + a_0 y^{(1)} - bu^{(1)}) = 0 \quad (21)$$

By setting for $i = 1, \dots, n+2$ and $j = 0, \dots, n$

$$W_{i,j} = t^{n+i} y^{(j+1)} \text{ and } V_i = t^{n+i} u^{(1)}$$

we obtain the spectral formulation

$$[A_{n+2} - \lambda_{n+1} A_{n+1} - \dots - \lambda_2 A_2 - \lambda_1 A_1]X = 0 \quad (22)$$

where A_i , for $i = 1, \dots, n+1$ and X are defined like in (17) and $\lambda_i = (-1)^{n-i+1} C_{i-1}^{n+1} \tau_c^{n+2-i}$, $1 \leq i \leq n+1$ is the coefficient associated to term t^{i-1} in the polynomial function $-(t - \tau_c)^{n+1}$.

From (22), τ_c can be estimated with the estimation of any λ_i . Since only λ_{n+1} gives a linear estimation of τ_c ($\lambda_{n+1} = (n+1)\tau_c$), then the estimation is obtained through the computation of $\Lambda_\epsilon(PA_{n+2}, (n+1)PA_{n+1})$ where

$$P = \text{coker}(\text{coker}(\dots \text{coker}(A_1)A_2 \dots)A_{n-1})A_n$$

The decision rule is made like in the previous subsection.

As well, the amplitude l_c is estimated after the estimation of τ_c by setting $\alpha_4 = t^{n+1}(t - \tau_c)^n$:

$\forall t \geq \tau_c$ and $k \geq n+1$

$$l_c = (-1)^n \frac{H^{*k} * [\alpha_4(q_{n+2}y^{(n+1)} + \dots + q_2y^{(1)} + q_1u^{(1)})]}{n! H^{*k} * \delta_{\tau_c}}$$

where $q = (q_{n+2}, \dots, q_1)^T$ is the smallest singular vector of matrix $PA_{n+2} - (n+1)\tau_c PA_{n+1}$.

3.5 System with both actuator and sensor faults

In the case of multiple faults (actuator and sensor faults), the use of the previous approaches do not ensure the system fault diagnosis.

Assuming for example the presence of an actuator fault $f_a(t) = l_a H(t - \tau_a)$ and a sensor fault $f_c(t) = l_c H(t - \tau_c)$, the corresponding model is

$$a_n y^{(n)} + \dots + a_0 y = \phi_0 + bu + bl_a H(t - \tau_a) + l_c (a_n \delta_{\tau_c}^{(n-1)} + \dots + a_0 H(t - \tau_c)) \quad (23)$$

The multiplication of the previous equation by

$$\alpha_1 = t^{n+1}(t - \tau_a) \text{ or } \alpha_3(t) = t^{n+1}(t - \tau_c)^{n+1}$$

does not eliminate all singularities obtained after derivation. For this, we can consider the function

$$\alpha_5 = t^{n+1}(t - \tau_a)(t - \tau_c)^{n+1}$$

The spectral expression obtained thanks to α_5 is

$$[A_{n+2} - \beta_{n+1}A_{n+1} - \dots - \beta_1 A_1 - \beta_1 A_1]X = 0 \quad (24)$$

where A_i and X are obtained according to (17) and β_i is the coefficient associated to term t^{i-1} in the polynomial function $-(t - \tau_a)(t - \tau_c)^{n+1}$.

From (24), τ_a and τ_c can be estimated, as for sensor fault case, by the means of the estimation of any β_i . The first fault can be detected like in subsections 3.3 and 3.4. The detection of the second fault can be made through the estimation of $\beta_{n+1} = \tau_a + (n + 1)\tau_c$ which becomes stationary after time $t \geq \max(\tau_a, \tau_c)$.

Finally, the second fault magnitude can be also estimated through τ_a and τ_c estimations.

4. APPLICATION

Consider the first order system ($n = 1$) modelled by the input-output equation

$$a_1 \dot{y} + a_0 y = bu$$

This system is controlled with a PI-controller. The proposed approach is applied to diagnose the system above, simulated in first with a sensor drift and then with actuator and sensor bias. Recall that the proposed approach assumes that parameters a_1 , a_0 and b are unknown and will not be explicitly estimated.

Figure (2) shows input u and output y simulated with a drift (of slope $l_c = 0.3$ at instant $\tau_c = 1$) of the output sensor.

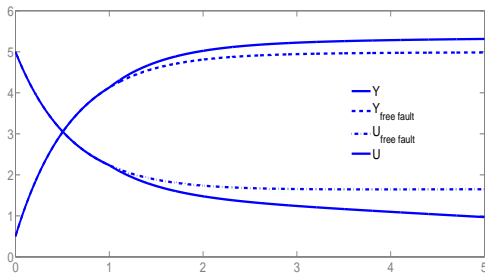


Figure 2. Input and output signals with a sensor fault

Figures (3)-(4) show the graphical results of the estimations (of τ_c and l_c) given by the proposed algorithms. According to these observations, we can conclude to the occurrence of a sensor drift, modelled as $f_c(t) = 0.3(t - 1)H(t - 1)$.

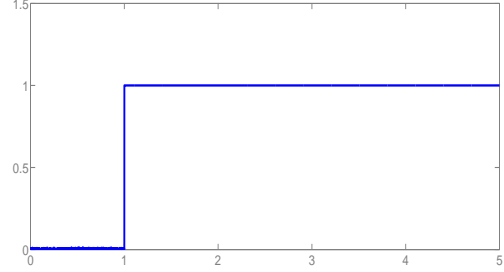


Figure 3. evolution of τ_c estimation

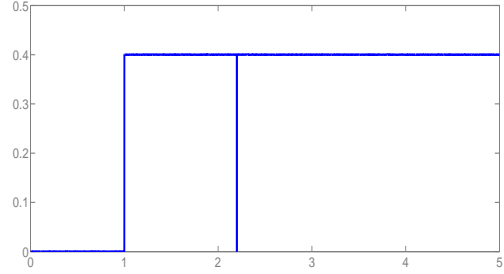


Figure 4. evolution of l_c estimation

In the next, the system is simulated with an actuator and sensor faults of type delayed step (bias). Input u and output y are represented on figure (5).

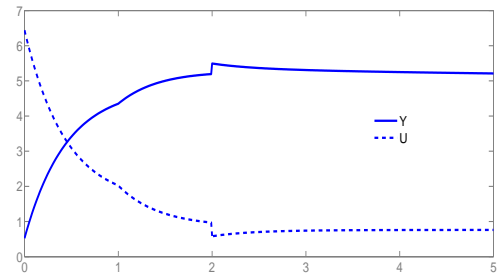


Figure 5. Input and output signals in the case of multiple faults

Graphical results of the estimations given by the proposed algorithms are represented on figures (6)-(10). On figure (6), we can easily see the stationary value of τ_a estimation around value $\bar{\tau}_a = 1$ in the time interval $[1, 2]$. The associated magnitude estimation is equal to 0 before time $t = 1$ and takes a value around 0.8 in the time interval $[1, 2]$ (figure 7).

According to these two observations, the detection of an actuator fault, modelled by $f_a(t) = 0.8H(t - 1)$, can be accomplished in the time interval $[1, 2]$. From $t > 2$, the estimation of τ_a and l_a fails due to the occurrence of a sensor fault. It is important to notice that the later cannot

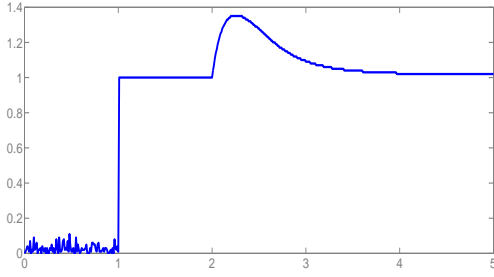


Figure 6. time evolution of τ_a estimation

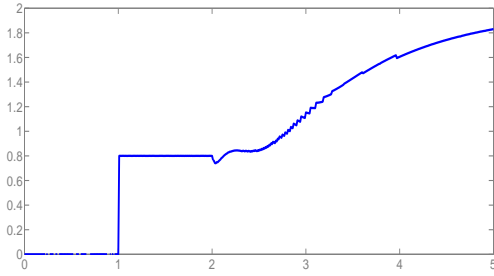


Figure 7. time evolution of l_a estimation

be detected by means of the time evolution of τ_c estimation according to the fact that the actuator fault affects the sensor fault detection.

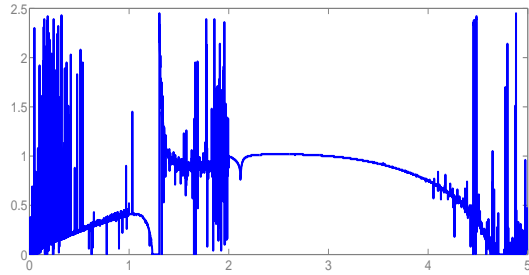


Figure 8. time evolution of τ_c estimation

However, the stationary value of $\beta_2 = \tau_a + 2\tau_c$ from $t > 2$ around 5 (figure 9) allows the detection of the sensor fault. Figure (10) shows the time evolution of the estimation of the sensor fault magnitude obtained by means of β_2 and τ_a estimations. Thanks to τ_a and β_2 estimations, the detection of a sensor bias can be accomplished, it is about $f_c(t) = 0.3H(t - 2)$.

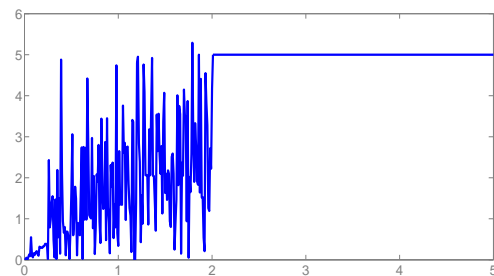


Figure 9. time evolution of β_2 estimation

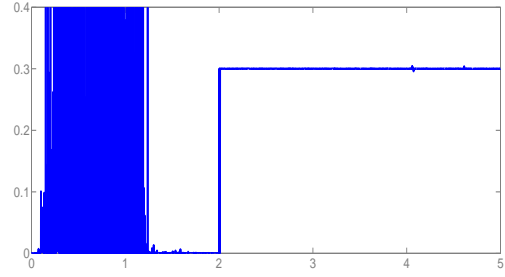


Figure 10. time evolution of l_c estimation

5. CONCLUSION

The algebraic approach for fault diagnosis presented in this paper is only based on the structural knowledge of the system and the faults. On the one hand, the parameters of the system can be unknown and we do not need to estimate them explicitly. On the other hand, the decision rule is not based on the outputs of some residual generators, but rather on the time behaviour of some fault characteristic estimations. Simulation results show that the proposed method gives good results for detecting and isolating actuator and sensor faults even in the case of multiple faults. In future works, robustness of the proposed method with respect to noise signals will be investigated.

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