The boundary integral equation method for the extrapolation of field measurement

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THE BOUNDARY INTEGRAL EQUATION METHOD FOR THE EXTRAPOLATION OF FIELD MEASUREMENT

F.M. Duthoit*, L. Krahenbuhl** and A. Nicolas**

ABSTRACT

How to extrapolate measured field values in respect to the physical equations? The authors expose the concrete problem they had to solve and the solution they propose, by using the Boundary Integral Equation Method (BIEM).

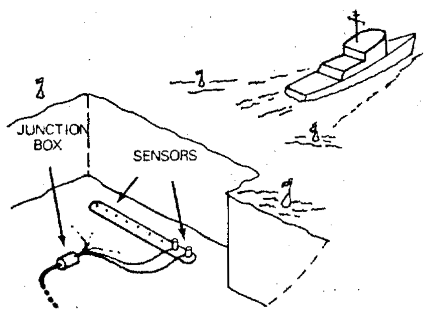
First, fictitious sources are constructed giving exactly the measured values of the field; then we compute the field in other points as for the field of these fictitious sources. In association with l.s. techniques, this method permits also in part the correction of the measurement errors.

1. INTRODUCTION

1.1. The concrete problem to be solved.

A ship is a mass of iron magnetized by the action of the earths' magnetic field. This factor is used by the military to provoke the explosion of "magnetic mines": navigational security therefore demands the demagnetization of ships, achieved with the aid of current loops on the boat.

To regulate the currents it's necessary to measure the boats' magnetic field (Fig. 1): the boat passes over the measuring device, which is made up of triaxial sensors. The measurements are taken at regular intervals, as if there were many rows of sensors on the surface to be measured.



- **Fig. 1:** General cross section of an array -

These two questions are related to this method of measurement:

- With each passage, the water level varies making comparisons impossible without bringing them to a reference level.
- Once the currents are regulated, there rests a residual field surrounding the boat, which has to be estimated to define the boats' zone of protection.

In both cases, it's a matter of extrapolating the measurements in a highly intelligent manner.

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1.2. The method we propose.

We will proceed in two stages:

- The measured field calculation of fictitious sources, which provide for the measuring device (i.e. field of the actual source).
- Using these fictitious sources, now known, calculation of the field outside of the measuring device (intelligent extrapolation).

When the extrapolation principle from source building is assumed, it seems that the most simple method is to associate a magnetic dipole to each important iron part of the ship. This method cannot be applied unless one has a particular model for each boat and its exact position during the measurement (a high order multipole must be used to simulate an off-centered dipole [1]).

Then we propose to build fictitious sources on a standard surface surrounding the boat. This surface must be far enough to consider the incertitude of the boats position, and close enough to give good values also near the boat (the surface will be the same for all the boats, except the scale factor).

The theoretical point of view would be discussed in section 2.. Computation methods and results are presented in section 3..

2. THEORETICAL CONSIDERATIONS

2.1. Introduction.

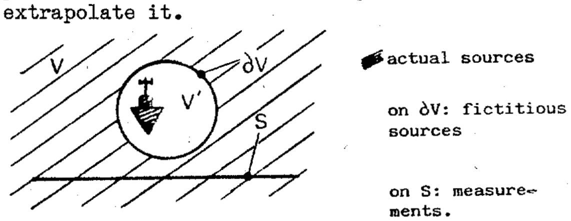
We first prove the existence of surface field sources which are equivalent to the actual sources, and derive some of their properties. We then prove that the field which is given by its values on a surface S is unique (in our example S is the plane on which the measurements are made). Finally we derive an equation which relates directly the repartition of the fictitious sources to the values of the field on S.

Let us define now the physical quantities which we shall use.

2.1.1. The field H and the location of its actual sources.

The magnetic field **Htot** which surrounds the ship results of the combined effects of the earths field, the ships permanent magnetisation, the currents of the compensation loops and the induced magnetisation of the parts of the ship which are made of iron.

From now on we define H as **Htot** minus the earths magnetic field. It follows that H has all its sources within a volume **V'**; this volume (Fig. 2) contains the ship but doesn't contain the region in which we want to extrapolate it.



- **Fig. 2: The model of the ship** -

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2.1.2. The scalar potential ϕ.

Let us define V as the volume which is exterior to V' (i.e. the whole space except V'). Because there is no current flowing, H is in V the derivative of a scalar potential ϕ:

$$H = -\,\mathbf{grad}\,\phi \quad \text{in V} \qquad (1)$$

Flux conservation makes the potential ϕ obey the Laplace equation:

$$\Delta\phi \equiv 0 \quad \text{in V} \qquad (2)$$

2.2. Equivalent surface sources.

2.2.1. Existence and properties.

From Greens identity for the scalar functions ϕ and G_P (the Laplacians Greens function centered on a point P) we get:

$$-\int_{\partial V} (\phi.\partial_n G_P - \partial_n\phi.G_P).ds = \begin{cases} \phi(P) & \text{P in V} \\ \phi(P)/2 & \text{P on } \partial\mathbf{V} \\ 0 & \text{P in V'} \end{cases} \qquad (3)$$

The quantities:

$$\int_S \rho.G_P ds \qquad \text{and} \qquad \int_S \tau_n \partial_n G_P ds \qquad (4)$$

represent the potentials at P of respectively a **monopole surface distribution** (magnetic masses) and a **distribution of dipoles** which are perpendicular to the surface (superficial currents). It follows that we can rewrite (3) in the following form:

$$\phi(P) = -\int_{\partial V} (\tau_n.\partial_n G_P - \rho.G_P).ds \qquad (5)$$

This proves the existence of a system of sources (ρ, τ_n) which is equivalent to the actual sources; these actual sources have to be localised into the volume V' (delimited by the surface ∂V), and this is the only restriction.

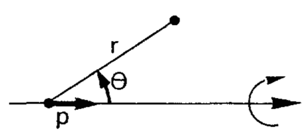
The equivalent, fictitious sources have to be localised on a surface which encloses the actual sources. This equivalence is only valid in V because for any point of V' the l.h.s. of (5) equals zero. The concrete choice of the surface ∂V is absolutly free. Because of the shape ships usually possess, we suggest the choice of an ellipsoid.

2.2.2. Example and other systems of sources.

As an illustration, let us consider the case of a ponctual dipole. In this very simple case, calculations are carried out analytically.

The potential of the dipole of moment **p** is:

$$\phi(r,\theta) = p.\cos\theta/4\pi r^2 \qquad (6)$$



- **Fig. 3:** Potential of a dipole -

Let us choose ∂**V** as being a sphere of radius **a** whose centre is the dipole; we get:

$$\begin{cases} \tau_n(\theta) = \phi(a,\theta) = p.\cos\theta/4\pi a^2 & (7) \\ \rho(\theta) = -\partial_n\phi(r,\theta)\big|_a = p.\cos\theta/2\pi a^3 & (8) \end{cases}$$

Of course there exists other distributions on ∂**V** which produce the same potential outside of the sphere. For example, in the case of a distribution of monopoles only:

$$\rho^*(\theta) = 3p.\cos\theta/4\pi a = 3.\rho(\theta)/2 \qquad (9)$$

From these results, one can see that there are infinitely many solutions which are linear combinations of mono- and di-poles:

$$(3s/2)\,.\,\rho(\theta) + 3(1-s)\,.\,\tau_n(\theta) \quad s\in R \qquad (10)$$

2.2.3. Commentary.

From our considerations it follows that ficticious equivalent sources exist but they are not unique. One has to be careful not to generalise the example of section 2.2.2: we don't pretend it is generally possible to construct equivalent sources made of monopoles only or of dipoles only: for example it is impossible to simulate a ponctual monopole using dipoles only! This case however never appears within the frame of magnetostatics.

2.3. The field on the measurement surface and the uniqueness of the extrapolation.

The measurements correspond to the knowledge of the field on a finite surface S (section 1.1). It is reasonable to think that it is possible to deduce correct values of the field in a close neighbourhood of this surface, particularly just below it.

In the following we show that it is not impossible to do much better.

2.3.1. Theorem of the uniqueness of the extrapolation.

Let H1 and H2 be two fields which (in V) derive from potentials ϕ1 and ϕ2.

Let ϕ1 and ϕ2 obey Laplaces equation in V, and S a finite surface included in V.

One has:

$$\text{H1} \equiv \text{H2 on S} \implies \text{H1} \equiv \text{H2 in V} \qquad (11)$$

2.3.2. Commentary.

Giving H on a surface S (even if S is "small") permits a unique determination of it in the whole space (outside the sources); therefore it is theoretically no obstacle for solving our extrapolation problem.

One can remark that complex functions exhibit similar properties in the complex plane.

2.3.3. Proof.

Let us consider two fields which obey the hypothesis of our theorem. Their difference H derives (in V) from a potential ϕ and equals zero on S. One has:

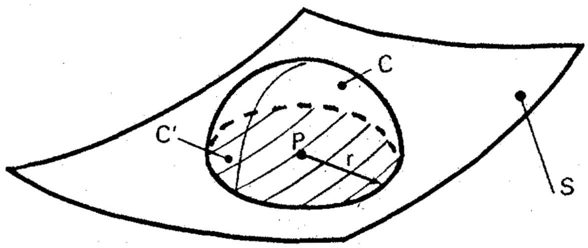
$$\phi \equiv \phi_o \ (\text{Cte}) \quad \text{on S} \qquad (12)$$
$$\partial_n\phi \equiv 0 \quad \text{on S} \qquad (13)$$
$$\Delta\phi \equiv 0 \quad \text{in V} \qquad (14)$$

Before presenting a rigourous proof of the theorem, let us intuitively show its plausibility:

- The condition (12) gives equipotential surfaces which are "parallel" to S;
- The condition (13) gives equipotential surfaces which are "perpendicular" to S;

Both conditions together lead necessarily to a constant potential around S: the property then propagates at every place where $\Delta\phi \equiv 0$.

Let us now prove the theorem rigourously, using a method which is analogous to the proof of Gauss' mean value theorem:



- **Fig. 4:** The uniqueness theorem -

Let P be a point of S (Fig. 4)
C an hemisphere of radius r centered in P
C' the part of S which is delimited by C.

Greens identity gives:

$$1/2\,\phi o(P) = -\int_{C\cup C'} (\phi.\partial_n G - \partial_n \phi.G)\,.\,ds$$

$$= -\partial_n \bar{G}_C \int_C \phi.ds + \bar{G}_C \int_C \partial_n\phi.ds - \phi o \underbrace{\int_{C'} \partial_n G.ds}_{\frac{1}{4\pi}\Omega} \qquad (15)$$

Ω : solid angle from P to C.

Condition (13) on C' and flux conservation give:

$$\int_C \partial_n \phi\,.\,ds = 0 \qquad (16)$$

it follows:

$$1/2\;\phi o = \frac{\Omega}{4\pi}\bar{\phi}c + \phi o.\left(\frac{\Omega}{4\pi}+1/2\right) \qquad (17)$$

and

$$\bar{\phi}c = \phi o \qquad (18)$$

where $\bar{\phi}c$ is the mean value of the potential calculated separately on each of both parts of the sphere.

(18) being verified, there exists at least one line on C which has the potential ϕo. This property is independant of the radius of C, therefore an equipotential surface (of potential ϕo) reaches the point P, and this surface is distinct of S.

The position of P on S being arbitrary, one has:

$$\phi \equiv \phi o \quad \text{in} \quad V \qquad (19)$$

2.3.4. Commentary.

This theorem does not provide any practical method to extrapolate; at any rate one has to fear difficulties due to the precision of the measurements: two fields which are nearly the same on S could differ tremendously far away from S.

For practical purposes one should be advised to choose S in an intelligent manner, so that to minimise the sensitivity of the extrapolation to the measured values.

2.4. Determination of the fictitious sources and calculation of the extrapolation.

2.4.1. Equation of the sources.

Taking the gradient of (5) one gets:

$$H(P) = \int_{\partial V} (\mathcal{T}_n \mathbf{grad}\partial_n G_P - \rho.\mathbf{grad}G_P).ds \qquad (20)$$

If P travels on S where H is known, one gets a system of integral equations with $\mathcal{T}_n$ and ρ as unknown quantities. As we have already shown in section 2.2. the solution is not unique and one has to impose supplementary conditions.

For example one can assume: $\mathcal{T}_n = 0$ or $\rho = 0$ (21) which means using only monopoles or only normal dipoles (see warning 2.2.3.). One can also impose the integral equation (3) onto **∂V** or into **V'** and this leads to:

$$(\rho, \mathcal{T}_n) = (\partial V/\partial n, V) \qquad (22)$$

and we have the conditions of existence of section 2.2.1..

2.4.2. Conclusion.

The sources $(\rho, \mathcal{T}_n)$ being determined, the extrapolation consists of using (20) for each point P on which we want to know the field. This reduces to the mechanical numerical application of a formula.

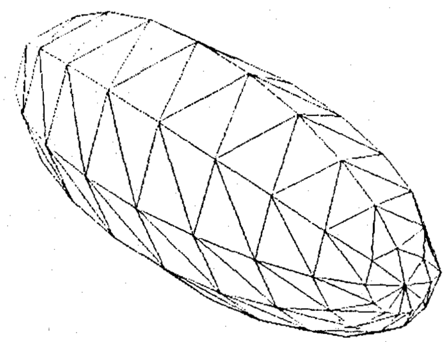
A method to solve equation (20) remains to be found. In practice one has only a finite number of measurement points: we are going to cut the surface **∂V** in a sufficient number of finite elements and represent the sources with the help of these elements.

This classical method [2] is presented in section 3. as well as an example of the results obtained.

3. COMPUTATION METHODS AND RESULTS.

3.1. Discretisation of the B.I.E..

The surface of the equivalent sources is an ellipsoid which one discretises in finite elements:



- **Fig. 5:** Discretisation of the ellipsoidal surface -

A value of the unknown surface densities $\mathcal{T}_n$ or ρ is attributed to each mesh node; the variations of these functions on each element are defined a priori by weighted functions of order one or two. This permits the numerical integration: equation (20) is written in a form of a simple linear combination.

3.2. Pretreatment of the measures.

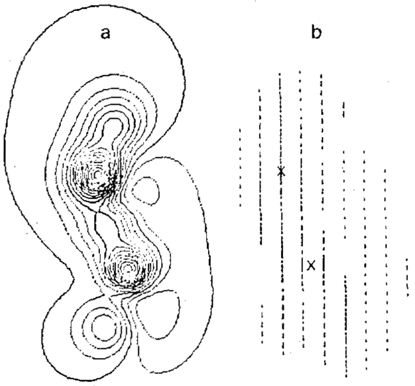
Experience shows us that the available measures are overabondant, taking into consideration the number of nodes which are necessary to discretise the ellipsoidal surface surrounding the boat.

Two questions are thus imposed:

- which measures to keep, which to discard;
- how many measures are really necessary to attain a given accuracy, i.e. how many finite elements are necessary on the surface.

The most interesting points of a characteristic (i.e. the points which make the interpolation more difficult, see Fig. 6-a) are neither these where the field values are high, nor these where the field value varies quickly, but these where the characteristics curvature is high (for example the top of a peak). Also, it would be desirable to promote these points by using more measurement points in their neighbourhood.

The algorithm we have selected uses a threshold criterion with the values of the second derivative of each three components of the field. One example is shown on Fig. 6-b.



- Fig. 6 -

a: measured boats characteristic (z-values);
b: chosen points.

3.3. Linear system and resolution.

The equation (20) is written in its discretised form on the chosen measurement points; it gives a linear system:

$$M \cdot X = B \quad (23)$$

where M is a full matrix, X the vector of the unknown sources and B the vector of the measured values. This system is willfully overdetermined to permit the correction of the measurement errors. It can be solved by a "least squares" method, for example:

$$X = (M^T M)^{-1} \cdot M^T \cdot B \quad (24)$$

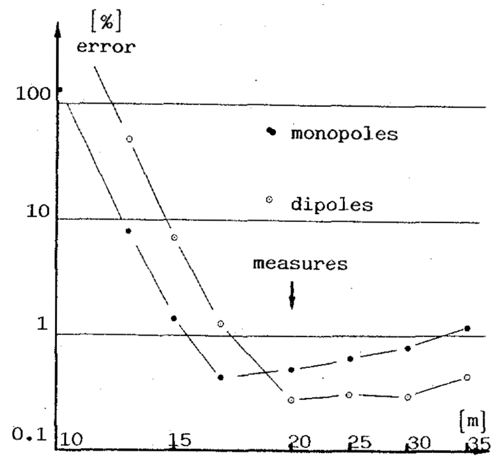
3.4. Example.

To test this method, it's necessary to compare the extrapolated values of the field with its actual values: to do that, we have defined analytic test fields.

The error due to the extrapolation process is conventionally defined for each depth (under the water level) by:

$$\text{error} = \max(Ha - Hex)/\max(Ha) \quad (25)$$

where Ha and Hex are respectively the analytic and the extrapolated values of each field's component. Fig. 7 shows the result for monopolar and dipolar sources:



- **Fig. 7**: Extrapolations error as function of depth -

The monopoles are better near the boat, while their field varies in space more slowly; at a great distance, result degrades and dipoles improve. We think it is possible to improve monopoles by fixing their resultant monopolar moment to equal zero:

$$\int_{\partial V} \varrho \cdot ds = 0 \quad (26)$$

it is also a boundary integral equation.

At the time being, we are testing systematically all the parameters to optimise our method:

- choice of the sources: monopoles, dipoles, mixture (section 2.4.1.);
- ratio between the numbers of the measured values and the values finally used (section 3.2.);
- ratio between the number of B.I.E. and the number of unknowns (3.2.) and method for solving the resulting linear system;
- ratio between the numbers of finite elements over and under the ellipsoidal surface (Fig. 5);
- order of the finite elements, and so on.

CONCLUSION

Built on a strong theoretical base, the extrapolation method that we propose is a very original application of the B.I.E method.

The first numerical results obtained are of a very high quality and the study will be continued to optimise the final solution.

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