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Imperfect repair models with planned preventive maintenance

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Abstract—The aim of this paper is to present a general framework for the simultaneous modelling and assessment of the efficiency of corrective maintenance actions and planned preventive maintenance actions for complex repairable systems. The framework proposed generalizes classical Kijima's virtual age models and involves a large number of existing models where preventive maintenance is done at fixed time, age or failure intensity. The main features of this general modelling are derived: distribution of the next failure time, marginal and joint distribution of time and type of next maintenance, likelihood function. Several particular cases of maintenance effects are proposed, based on a virtual age idea. Finally, an application to real data sets issued from electricity production systems is presented.

Index Terms—repairable systems reliability, preventive maintenance, repair efficiency, ageing, virtual age, point processes.

ACRONYMS

ABAO	As Bad As Old
AGAN	As Good As New
ARA	Arithmetic Reduction of Age
ARI	Arithmetic Reduction of Intensity
CM	Corrective Maintenance
NHPP	Non Homogeneous Poisson Process
PM	Preventive Maintenance
QR	Quasi-Renewal

I. INTRODUCTION

The dependability of complex repairable systems depends strongly on the efficiency of preventive and corrective maintenance actions. Corrective maintenance (CM), also called repair, is carried out after a failure and intends to put the system into a state in which it can perform its function again. Preventive maintenance (PM) is carried out when the system is operating and intends to slow down the wear process and reduce the frequency of occurrence of system failures. PM can be planned or condition-based. In this study, we focus on planned PM.

Planned PM are often considered to be done at fixed deterministic times. However, the planning of PM does not necessarily imply deterministic times. For instance, the next PM can be planned as a function of previous failure times. Since failure times are random, the next PM time will then also be done at a random time. Another example is the case of

periodical PM when the periodicity has to be changed because of a reliability centered maintenance analysis. The analysis can conclude that the system is not reliable enough. Then, the PM periodicity, fixed before the initial operation time, has to be reduced. Even if PM times are still periodical, they must be considered as random since this change in the periodicity has not been foreknown at the beginning.

The basic assumptions on maintenance efficiency are known as minimal repair or As Bad As Old (ABAO) and perfect repair or As Good As New (AGAN). In the ABAO case, each maintenance leaves the system in the state it was before maintenance. In the AGAN case, each maintenance is perfect and leaves the system as if it were new. Obviously, reality is between these two extreme cases: standard maintenance reduces failure intensity but does not leave the system as good as new. This is known as imperfect maintenance.

Many imperfect maintenance models have been proposed (see for example a review in Pham and Wang [1]). Most of them, such as Kijima's [2] virtual age models, consider only one kind of maintenance. Rather few paper deal with the joint modelling of PM and CM [3]. In [4], we have proposed a general framework for the simultaneous modelling and assessment of the efficiency of corrective maintenance actions and condition-based preventive maintenance actions. The present paper proposes the same kind of results in the case of planned PM.

The paper is organised as follows. In section II, the notations and mathematical definition of planned PM are introduced. Section III presents the properties of the maintenance process. In section IV, stochastic models for the efficiency of imperfect maintenance actions are developed, based on a virtual age idea. The usual PM policies are replaced in the general framework in section V. Finally, section VI presents an analysis of two real data sets issued from electricity production systems, leading to the assessment of maintenance efficiencies using several particular cases of the general model.

II. NOTATIONS AND DEFINITIONS

The PM-CM process is the sequence of PM times and CM times. Maintenance durations are assumed to be negligible or not taken into account. Then, observations are composed of three point processes, for which we introduce the following notations.

For the global maintenance process (PM and CM):

- $\{C_k\}_{k \geq 1}$ the maintenance times ($C_0 = 0$),
- $\{W_k\}_{k \geq 1}$ the times between maintenances, $W_k = C_k - C_{k-1}$,
- $K = \{K_t\}_{t \geq 0}$ the counting maintenance process,
- $\{U_k\}_{k > 1}$ the indicators of maintenance types,

$$U_k = \begin{cases} 0 & \text{if the } k\text{th maintenance is a CM} \\ 1 & \text{if the } k\text{th maintenance is a PM} \end{cases}$$

For the failure or CM process:

- $\{T_i\}_{i \geq 1}$ the CM times ($T_0 = 0$),
- $\{X_i\}_{i \geq 1}$ the times between CM, $X_i = T_i - T_{i-1}$,
- $N = \{N_t\}_{t \geq 0}$ the CM counting process.

Finally for the PM process:

- $\{\tau_j\}_{j \geq 1}$ the PM times ($\tau_0 = 0$),
- $\{\chi_j\}_{j \geq 1}$ the times between PM, $\chi_j = \tau_j - \tau_{j-1}$,
- $M = \{M_t\}_{t \geq 0}$ the PM counting process.

Figure 1 presents an example of a trajectory of the PM-CM process.

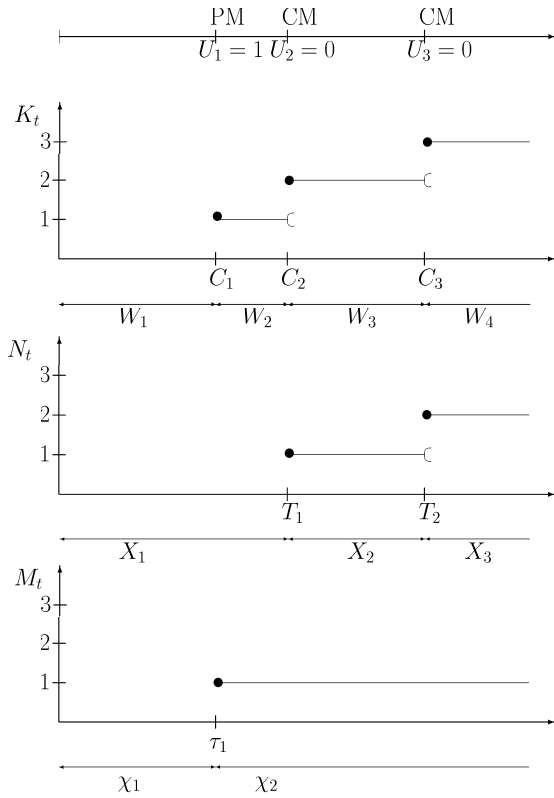


Fig. 1. A trajectory of the PM-CM process

In the following, bold characters denote vectors, for instance $\mathbf{T}_n = (T_1, \dots, T_n)$ and t^- denotes the left hand limit of t .

A PM is said to be *planned* if and only if, at all time $t \geq 0$, the next potential PM time is a deterministic function of the

past of the PM-CM process :

$$U_{K_{t^-}+1} = 1 \Rightarrow \tau_{M_{t^-}+1} = C_{K_{t^-}} + h_{K_{t^-}}(\mathbf{W}_{K_{t^-}}, \mathbf{U}_{K_{t^-}})$$

where for all $k \geq 0$, $h_k(\cdot)$ is a deterministic function. A *PM policy* is a way of choosing the next potential PM time as a function of all available informations at each time. The PM policy is then the set of all the functions $h_k(\cdot)$ for $k \geq 0$. For the sake of simplicity, $h_k(\mathbf{W}_k, \mathbf{U}_k)$ will be simply denoted by H_k .

The next PM will not be done at the planned time if a failure, and then a CM, occurs before. Then, the current time between maintenances is :

$$W_{K_{t^-}+1} = \min(T_{N_{t^-}+1} - C_{K_{t^-}}, H_{K_{t^-}}) \quad (1)$$

and the next maintenance type is :

$$U_{K_{t^-}+1} = I_{\{W_{K_{t^-}+1} = H_{K_{t^-}}\}} = I_{\{H_{K_{t^-}} < T_{N_{t^-}+1} - C_{K_{t^-}}\}} \quad (2)$$

PM at fixed predetermined times are planned PM for which PM times $\{\tau_j\}_{j \geq 1}$ and PM process $\{M_t\}_{t \geq 0}$ are deterministic. The PM policy is then defined by :

$$\forall k \geq 1, \quad H_k = \tau_{M_{C_k}+1} - C_k$$

In section V, other PM policies corresponding to random PM times will be defined: PM at fixed age or at fixed intensity.

Condition-based PM occur at times which are determined according to the results of inspections and degradation or operation controls. Then, they are done at random times. The main difference between planned and condition-based PM is that, in the latter case, the next potential PM time is not a deterministic function of the past of the PM-CM process.

III. PROPERTIES OF THE PM-CM PROCESS

For the CM process or failure process, the failure intensity [5] is defined by :

$$\forall t \geq 0, \quad \lambda_t = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} Pr\{N_{t+\Delta t} - N_{t^-} = 1 | \mathcal{H}_{t^-}\} \quad (3)$$

where \mathcal{H}_{t^-} is the history of the PM-CM process that is to say the natural filtration generated by the past of the PM-CM process at time t :

$$\mathcal{H}_t = \sigma(\{N_s, M_s\}_{0 \leq s \leq t}) = \sigma(\{K_s, U_{K_s}\}_{0 \leq s \leq t})$$

Thanks to (1) and (2), the PM-CM process is completely characterized by the CM process and the PM policy. The PM-CM process history is then simply the failure process history : $\mathcal{H}_t = \sigma(\{N_s\}_{0 < s < t})$. Then, the failure intensity can be equivalently written $\lambda_t = \lambda_t(K_t, \mathbf{W}_{K_t}, \mathbf{U}_{K_t})$ or $\lambda_t = \lambda_t(N_t, \mathbf{T}_{N_t})$. In this situation, the CM process is a self-excited point process [6] and it is completely characterized by its failure intensity. For instance, the conditional survival function of failures times is given by :

$$\begin{aligned} & Pr\{T_{n+1} > t | \mathbf{T}_n\} \\ &= \begin{cases} 1 & \text{if } t < T_n \\ \exp\left(-\int_{T_n}^t \lambda_s(n, \mathbf{T}_n) ds\right) & \text{otherwise} \end{cases} \quad (4) \end{aligned}$$

The contribution of PM times between T_n and t in (4) can be highlighted :

$$\begin{aligned} & Pr\{T_{n+1} > t | \mathbf{W}_{K_{T_n}}, \mathbf{U}_{K_{T_n}}\} \\ &= \exp\left(-\sum_{j=K_{T_n}}^{K_t-1} \int_{C_j}^{C_{j+1}} \lambda_s(j, \mathbf{W}_j, \mathbf{U}_j) ds\right. \\ &\quad \left.- \int_{C_{K_t}}^t \lambda_s(K_t, \mathbf{W}_{K_t}, \mathbf{U}_{K_t}) ds\right) \end{aligned} \quad (5)$$

with for all $j \in \{K_{T_n} + 1, \dots, K_t\}$, $C_j = T_n + \sum_{i=K_{T_n}}^{j-1} H_i$ and $U_j = 1$.

The following property generalizes equations (4) and (5) when the past of the PM-CM process is not only known up to the last failure time, but up to the last maintenance time.

Property 1:

$$\begin{aligned} & Pr\{T_{N_{C_k}+1} > t | \mathbf{W}_k, \mathbf{U}_k\} \\ &= \begin{cases} 1 & \text{if } t < C_k \\ \exp\left(-\int_{C_k}^t \lambda_s(N_{C_k}, \mathbf{T}_{N_{C_k}}) ds\right) & \text{otherwise} \end{cases} \end{aligned} \quad (6)$$

Proof:

The property is trivial for $t < C_k$.

If $U_k = 0$, $T_{N_{C_k}} = C_k$, then the PM-CM process history at C_k is given by the set of the N_{C_k} first failure times. Therefore, for all $t \geq C_k$,

$$\begin{aligned} Pr\{T_{N_{C_k}+1} > t | \mathbf{W}_k, \mathbf{U}_k\} &= Pr\{T_{N_{C_k}+1} > t | \mathbf{T}_{N_{C_k}}\} \\ &= \exp\left(-\int_{C_k}^t \lambda_s(N_{C_k}, \mathbf{T}_{N_{C_k}}) ds\right) \end{aligned}$$

If $U_k = 1$, $T_{N_{C_k}} < C_k$ and $T_{N_{C_k}+1} > C_k$. Then the PM-CM process history at C_k is given by the set of the N_{C_k} first failure times and the fact that no CM occurred between $T_{N_{C_k}}$ and C_k . Therefore, C_k is a deterministic function of already observed failure times $\mathbf{T}_{N_{C_k}}$. For $t \geq C_k$,

$$\begin{aligned} & Pr\{T_{N_{C_k}+1} > t | \mathbf{W}_k, \mathbf{U}_k\} \\ &= Pr\{T_{N_{C_k}+1} > t | \mathbf{T}_{N_{C_k}}, T_{N_{C_k}+1} > C_k\} \\ &= \frac{Pr\{T_{N_{C_k}+1} > t \cap T_{N_{C_k}+1} > C_k | \mathbf{T}_{N_{C_k}}\}}{Pr\{T_{N_{C_k}+1} > C_k | \mathbf{T}_{N_{C_k}}\}} \\ &= \frac{Pr\{T_{N_{C_k}+1} > t | \mathbf{T}_{N_{C_k}}\}}{Pr\{T_{N_{C_k}+1} > C_k | \mathbf{T}_{N_{C_k}}\}} \\ &= \frac{\exp\left(-\int_{T_{N_{C_k}}}^t \lambda_s(N_{C_k}, \mathbf{T}_{N_{C_k}}) ds\right)}{\exp\left(-\int_{T_{N_{C_k}}}^{C_k} \lambda_s(N_{C_k}, \mathbf{T}_{N_{C_k}}) ds\right)} \\ &= \exp\left(-\int_{C_k}^t \lambda_s(N_{C_k}, \mathbf{T}_{N_{C_k}}) ds\right) \end{aligned} \quad \blacksquare$$

This result means that the last PM time can be seen as a left deterministic censoring of the failure process. As it has been done for equation (5), the PM times contribution can be highlighted in property 1.

The following property gives the conditional distribution of the maintenance types and times between maintenances.

Property 2:

$$\begin{aligned} & Pr\{W_{k+1} > w, U_{k+1} = 1 | \mathbf{W}_k, \mathbf{U}_k\} \\ &= \begin{cases} 0 & \text{if } w \geq H_k \\ \exp\left(-\int_{C_k}^{C_k+H_k} \lambda_s(k, \mathbf{W}_k, \mathbf{U}_k) ds\right) & \text{otherwise} \end{cases} \end{aligned} \quad (7)$$

$$\begin{aligned} & Pr\{W_{k+1} > w, U_{k+1} = 0 | \mathbf{W}_k, \mathbf{U}_k\} \\ &= \begin{cases} 0 & \text{if } w \geq H_k \\ \exp\left(-\int_{C_k}^{C_k+w} \lambda_s(k, \mathbf{W}_k, \mathbf{U}_k) ds\right) - \\ \exp\left(-\int_{C_k}^{C_k+H_k} \lambda_s(k, \mathbf{W}_k, \mathbf{U}_k) ds\right) & \text{otherwise} \end{cases} \end{aligned} \quad (8)$$

Proof: If $w \geq H_k$ equation (7) is obviously true. If $w < H_k$, it is proved by applying property 1 since :

$$\begin{aligned} & Pr\{W_{k+1} > w, U_{k+1} = 1 | \mathbf{W}_k, \mathbf{U}_k\} \\ &= Pr\{T_{N_{C_k}+1} > C_k + H_k | \mathbf{W}_k, \mathbf{U}_k\} \end{aligned}$$

If $w \geq H_k$ equation (8) is also obviously true. If $w < H_k$, one has just to notice that :

$$\begin{aligned} & Pr\{W_{k+1} > w, U_{k+1} = 0 | \mathbf{W}_k, \mathbf{U}_k\} \\ &= Pr\{C_k + w < T_{N_{C_k}+1} \leq C_k + H_k | \mathbf{W}_k, \mathbf{U}_k\} \end{aligned} \quad \blacksquare$$

The marginal conditional distributions of the times between maintenances and of the maintenance types can then easily be derived.

Property 3:

$$\begin{aligned} & Pr\{W_{k+1} > w | \mathbf{W}_k, \mathbf{U}_k\} \\ &= \begin{cases} 0 & \text{if } w \geq H_k \\ \exp\left(-\int_{C_k}^{C_k+w} \lambda_s(k, \mathbf{W}_k, \mathbf{U}_k) ds\right) & \text{otherwise} \end{cases} \end{aligned} \quad (9)$$

$$\begin{aligned} & Pr\{U_{k+1} = 1 | \mathbf{W}_k, \mathbf{U}_k\} \\ &= \exp\left(-\int_{C_k}^{C_k+H_k} \lambda_s(k, \mathbf{W}_k, \mathbf{U}_k) ds\right) \end{aligned} \quad (10)$$

Proof: The first equation is proved by adding up equations (7) and (8). The second equation is just an obvious consequence of equation (7). \blacksquare

Using the self-excited point process theory [6], the likelihood of the PM-CM process can be easily derived.

Property 4: The likelihood function associated to a single observation of the PM-CM process over $[0, t]$ is :

$$\begin{aligned} L_t(\theta) &= \left[\prod_{i=1}^{K_t} \lambda_{C_i}(i-1, \mathbf{W}_{i-1}, \mathbf{U}_{i-1})^{1-U_i} \right] \\ &\quad \exp\left(-\sum_{j=1}^{K_t-1} \int_{C_{j-1}}^{C_j} \lambda_s(j-1, \mathbf{W}_{j-1}, \mathbf{U}_{j-1}) ds\right) \end{aligned} \quad (11)$$

where C_{K_t-1} is set equal to t .

In the likelihood, PM times can be seen as right deterministic censorings of the failure process.

It is interesting to compare the cases of planned and condition-based PM through the likelihood function. For condition-based PM, the failure (or CM) intensity is defined as in (3) and is denoted λ_t^N . A similar definition holds for the PM intensity λ_t^M and the global intensity is $\lambda_t^K = \lambda_t^N + \lambda_t^M$. Then, the likelihood function associated to a single observation of the (condition-based) PM-CM process over $[0, t[$ is [4] :

$$L_t(\theta) = \left[\prod_{i=1}^{K_t} \lambda_{C_i}^N(i-1, \mathbf{W}_{i-1}, \mathbf{U}_{i-1})^{1-U_i} \lambda_{C_i}^M(i-1, \mathbf{W}_{i-1}, \mathbf{U}_{i-1})^{U_i} \right] \exp\left(-\sum_{j=1}^{K_t+1} \int_{C_{j-1}}^{C_j} \lambda_s^K(j-1, \mathbf{W}_{j-1}, \mathbf{U}_{j-1}) ds\right) \quad (12)$$

It can be seen that the main difference between both cases lies in the fact that a PM intensity exists only for condition-based PM.

The complete building of a maintenance model needs two steps :

- The definition of the effects of PM and CM. We have chosen to use a generalization of virtual age models, presented in section IV.
- The definition of the link between PM and CM. It is done by a choice of a PM policy, expressed by the values of H_k . Usual PM policies are presented in section V.

IV. MAINTENANCE EFFECT MODELLING

A. Generalized virtual age models

Kijima's [2] virtual age models have been built for CM without PM. They have been generalized in [4] for CM and condition-based PM. In fact, it happens that the ideas presented in that paper can be applied directly to CM and planned PM. The difference is that the virtual age assumptions made on CM and PM intensities in [4] are made here only on failure intensity.

Assume that a new system, not subjected to PM actions, fails at a random time Y . Its failure rate $\lambda(t)$ is called the *initial intensity*. Then, for a system subjected to planned PM, we have :

$$\forall w < H_0, \quad Pr\{W_1 \leq w\} = Pr\{Y \leq w\}$$

The generalized virtual age model assumes that, after k th maintenance, the system is equivalent to a new system never maintained before $A_k = a_k(\mathbf{W}_k, \mathbf{U}_k)$, where $a_k(\cdot)$ is a deterministic function such that $a_0 = 0$. Mathematically this assumption can be written :

$$Pr\{W_{k+1} > w | \mathbf{W}_k, \mathbf{U}_k\} = Pr\{Y > A_k + w | Y > A_k\}$$

for all $w < H_k$. Therefore, thanks to equation (9), the failure

intensity of the model is :

$$\begin{aligned} \lambda_t &= -\frac{d}{dt} \ln Pr\{W_{K_{t-}+1} > t - C_{K_{t-}} | \mathbf{W}_{K_{t-}}, \mathbf{U}_{K_{t-}}\} \\ &= -\frac{d}{dt} \ln \frac{Pr\{Y > A_{K_{t-}} + t - C_{K_{t-}}\}}{Pr\{Y > A_{K_{t-}}\}} \\ &= -\frac{d}{dt} \ln \frac{\exp\left(-\int_0^{A_{K_{t-}} + t - C_{K_{t-}}} \lambda(s) ds\right)}{\exp\left(-\int_0^{A_{K_{t-}}} \lambda(s) ds\right)} \end{aligned}$$

And finally :

$$\lambda_t = \lambda\left(A_{K_{t-}} + t - C_{K_{t-}}\right) \quad (13)$$

$A_{K_{t-}}$ is called the *effective age* at time t and $A_{K_{t-}} + t - C_{K_{t-}}$ the *virtual age* at time t . The effective age is the virtual age of the system just after the last maintenance action.

In the following, we study several basic assumptions on virtual ages, similar to those made in [4]. Note that even if the expressions of failure intensities are the same for planned and condition-based PM, the applications will not lead to the same results since likelihoods (11) and (12) are different.

B. Basic models

1) *ABAO PM-ABAO CM*: This assumption means that both maintenance effects are minimal. Each maintenance restores the system to the state it was just before maintenance action. The effective age is then equal to the last maintenance time, i.e. $A_k = C_k$, for all $k \geq 1$. The failure intensity is then only a function of time, and the failure process is a Non-Homogeneous Poisson Process (NHPP) [7].

$$\lambda_t = \lambda(t) \quad (14)$$

2) *ABAO PM-AGAN CM*: PM is minimal and CM renews the system. The effective age is then equal to the time elapsed between the last maintenance and the last perfect maintenance, $A_k = C_k - T_{N_{C_k}}$. The failure process is then a renewal process with failure intensity

$$\lambda_t = \lambda(t - T_{N_{t-}}) \quad (15)$$

3) *ABAO PM-any CM effect*: It is equivalent to say that PM has no effect on the system. The effective age is then equal to the effective age at the time of the last CM plus the time elapsed since this last CM, $A_k = A_{K_{T_{N_{C_k}}}} + C_k - T_{N_{C_k}}$. Then, the failure intensity is

$$\lambda_t = \lambda\left(A_{K_{T_{N_{t-}}}} + t - T_{N_{t-}}\right) \quad (16)$$

The failure process is a classical virtual age model.

4) *AGAN PM-AGAN CM*: Each maintenance renews the system. Then, the effective age is nil, $A_k = 0$, for all $k \geq 1$. The failure intensity is

$$\lambda_t = \lambda\left(t - C_{K_{t-}}\right) \quad (17)$$

One can notice that the failure process is not a renewal process since the failure intensity is not a function of $t - T_{N_{t-}}$.

5) *AGAN PM-ABAO CM*: It is the dual situation of IV-B2. Then the effective age is $A_k = C_k - \tau_{M_{C_k}}$ and the failure intensity is :

$$\lambda_t = \lambda(t - \tau_{M_{t^-}}) \quad (18)$$

C. Several generalizations of the ARA_1 model

The model first introduced by Malik [8] corresponds to the Kijima type I model [2] with deterministic constant effects. It has been called the Arithmetic Reduction of Age model with memory 1 (ARA_1) in [9]. Its idea is that maintenance effect is supposed to reduce the virtual age of an amount proportional to the supplement of age accumulated since the last maintenance.

The same idea applied to CM and planned PM when both maintenance actions have the same effect, leads to :

$$A_{k+1} = (A_k + W_k) - \rho W_k = A_k + (1 - \rho)W_k \quad (19)$$

Then, the effective age is $A_{k+1} = (1 - \rho)C_{k+1}$ and the failure intensity is :

$$\lambda_t = \lambda(t - \rho C_{K_{t^-}}) \quad (20)$$

Of course, it is likely that, in practice, PM and CM have not the same effect. A first way to take into account different PM and CM effects is to make an assumption similar to (19) with different efficiency parameters, ρ_p for PM and ρ_c for CM :

$$A_{k+1} = \begin{cases} A_k + (1 - \rho_p)W_{k+1} & \text{if } U_{k+1} = 1 \\ A_k + (1 - \rho_c)W_{k+1} & \text{if } U_{k+1} = 0 \end{cases}$$

Then the effective age is :

$$A_{k+1} = C_{k+1} - \sum_{i=0}^k \rho_p^{U_{i+1}} \rho_c^{1-U_{i+1}} W_{i+1}$$

and the first ARA_1 PM- ARA_1 CM model is defined by its failure intensity :

$$\lambda_t = \lambda\left(t - \sum_{i=0}^{K_{t^-}} \rho_p^{U_{i+1}} \rho_c^{1-U_{i+1}} W_{i+1}\right) \quad (21)$$

When $\rho_p = \rho_c = \rho$, the equation is the same as (20). Unlike the usual ARA_1 model for only CM actions, the case $\rho_c = 1$ does not correspond to AGAN CM. In fact, in this case, the CM does not renew the system, but restores it in the state it was just after previous PM. CM is said to be "as good as previous". Similarly, $\rho_p = 1$ does not correspond to AGAN PM. $\rho_c = 0$ and $\rho_p = 0$ respectively imply ABAO CM and ABAO PM.

Another way to generalize the ARA_1 model is to suppose that the virtual age after a PM is equal to the virtual age just after the previous PM plus $(1 - \rho_p)$ times the time elapsed between these two PM. CM effect remains the same as in the previous model. Then, the effective age verifies :

$$A_{k+1} = \begin{cases} (1 - \rho_c)W_{k+1} + A_k & \text{if } U_{k+1} = 0 \\ (1 - \rho_p)\chi_{M_{C_{k+1}}} + A_{K_{\tau_{M_{C_{k+1}}-1}}} & \text{if } U_{k+1} = 1 \end{cases}$$

As in [4], the effective age is shown to be :

$$A_{k+1} = (1 - \rho_c)[C_{k+1} - \tau_{M_{C_{k+1}}}] + (1 - \rho_p)\tau_{M_{C_{k+1}}}$$

and the second ARA_1 PM- ARA_1 CM model is defined by its failure intensity :

$$\lambda_t = \lambda\left(t - \rho_c(C_{K_{t^-}} - \tau_{M_{t^-}}) - \rho_p\tau_{M_{t^-}}\right) \quad (22)$$

The failure intensity of this model is more simple than the previous one. In addition, CM are less efficient than PM, since CM influence the failure intensity (and then the failure process) only up to the next PM.

When $\rho_p = \rho_c = \rho$, equation (22) is the same as (20). Since CM effect is the same as in the previous model, $\rho_c = 1$ does not imply AGAN CM. But the case $\rho_p = 1$ corresponds now to AGAN PM. $\rho_c = 0$ implies ABAO CM, but $\rho_p = 0$ does not necessarily imply ABAO PM.

Finally, Jack [10] has proposed a model that is very similar to the previous one, but that is not an ARA_1 model. Virtual age after a PM is supposed to be equal to the virtual age just after the previous PM plus $(1 - \rho_p)$ times the virtual age accumulated between these two PM. CM effects remain the same as in previous model. If C_{k+1} is a PM time, the virtual age accumulated since the last PM is $W_{k+1} + A_k - A_{K_{\tau_{M_{C_{k+1}}-1}}}$. Then the effective age verifies :

$$A_{k+1} = \begin{cases} (1 - \rho_c)W_{k+1} + A_k & \text{if } U_{k+1} = 0 \\ (1 - \rho_p)[W_{k+1} + A_k - A_{K_{\tau_{M_{C_{k+1}}-1}}} \\ + A_{K_{\tau_{M_{C_{k+1}}-1}}} & \text{if } U_{k+1} = 1 \end{cases}$$

As in [4], it can be proved that :

$$A_{k+1} = (1 - \rho_c)C_{k+1} + (\rho_c - \rho_p)\tau_{M_{C_{k+1}}} - (1 - \rho_p)\rho_c \sum_{j=0}^{M_{C_{k+1}}-1} [C_{K_{\tau_{j+1}}-1} - \tau_j]$$

and the failure intensity is :

$$\lambda_t = \lambda\left(t - \rho_c C_{k+1} + (\rho_c - \rho_p)\tau_{M_{C_{k+1}}} - (1 - \rho_p)\rho_c \sum_{j=0}^{M_{C_{k+1}}-1} [C_{K_{\tau_{j+1}}-1} - \tau_j]\right) \quad (23)$$

When $\rho_c = \rho_p = \rho$, equation (23) is not equivalent to (20), then this model is not exactly a generalization of the ARA_1 model. Since CM effect is the same as in the previous model, the case $\rho_c = 1$ still not corresponds to AGAN CM, but $\rho_p = 1$ implies AGAN PM. $\rho_c = 0$ and $\rho_p = 0$ respectively imply ABAO CM and ABAO PM.

D. The ARA_∞ PM- ARA_∞ CM model

The model proposed by Brown, Mahoney and Sivazlian [11] corresponds to the Kijima [2] type II model with deterministic constant effects. It has been called the Arithmetic Reduction of Age model with infinite memory (ARA_∞) in [9]. Its idea is that maintenance effect is supposed to reduce the virtual age of an amount proportional to its value just before maintenance :

$$A_{k+1} = (1 - \rho)[W_{k+1} + A_k]$$

When both PM and CM have the same effects, it is easy to obtain that the failure intensity is :

$$\lambda_t = \lambda \left(t - \rho \sum_{j=0}^{K_{t^-}-1} (1-\rho)^j C_{K_{t^-}-j} \right) \quad (24)$$

The model can be generalized to the case where PM and CM have not the same effects by using different efficiency parameters ρ_p and ρ_c . The effective age then verifies :

$$A_{k+1} = (1-\rho_p)^{U_{k+1}} (1-\rho_c)^{1-U_{k+1}} [W_{k+1} + A_k]$$

It can be shown that the failure intensity is :

$$\lambda_t = \lambda \left(t - C_{K_{t^-}} + \sum_{j=1}^{K_{t^-}} (1-\rho_p)^{M_t - M_{C_{j-1}}} (1-\rho_c)^{N_t - N_{C_{j-1}}} W_j \right) \quad (25)$$

When $\rho_p = \rho_c = \rho$, equation (25) is equivalent to (24). The cases $\rho_c = 1$ and $\rho_p = 1$ respectively correspond to AGAN CM and AGAN PM. Similarly, $\rho_c = 0$ and $\rho_p = 0$ respectively imply ABAO CM and ABAO PM.

E. The Last and Szekli model

Last and Szekli [12] have proposed a model that is similar to the planned PM model proposed here. In this paper, the history of the PM-CM process and the PM policy depend of external random variables denoted here by $B = \{B_i\}_{i \geq 1}$:

$$\mathcal{H}_t = \sigma(\{N_s, M_s, B_{K_s}\}_{s \geq 0})$$

Then,

$$\begin{aligned} U_{K_{t^-}+1} &= 1 \\ \Rightarrow \tau_{M_{t^-}+1} &= C_{K_{t^-}} + h_{K_{t^-}}(\mathbf{W}_{K_{t^-}}, \mathbf{U}_{K_{t^-}}, \mathbf{B}_{K_{t^-}}) \end{aligned}$$

The authors have also assumed a Kijima type II model for maintenance efficiency :

$$\lambda_t = \lambda(A_{K_{t^-}} + t - C_{K_{t^-}})$$

where for all $k \geq 0$, $A_{k+1} = (1 - B_{k+1})[A_k + W_{k+1}]$.

Last and Szekli have investigated stationnarity properties of the maintenance process and have compared several PM policies. But they did not attempt to estimate maintenance efficiency.

In this framework, models similar to the usual Brown-Proschan [13] model, in which maintenance is AGAN with probability p and ABAO with probability $1 - p$, can be considered. The drawback of these models is that usually the external process B characterizes the random effect of maintenances and then it is not observed.

F. Non virtual age models

Many imperfect repair models have been proposed for only CM. Some of them are not virtual age models. In this section, we present possible generalizations of three of these models in our PM-CM framework.

Doyen and Gaudoin [9] have proposed the Arithmetic Reduction of Intensity (ARI) class of models. These models are

similar to the virtual age models except that maintenance effect does not affect virtual age but failure intensity. For example, the ARI_1 model can be generalized to CM and planned PM as it has been done for failure intensity (22) :

$$\lambda_t = \lambda(t) - \rho_c[\lambda(C_{K_{t^-}}) - \lambda(\tau_{M_{t^-}})] - \rho_p \lambda(\tau_{M_{t^-}}) \quad (26)$$

Wang and Pham [14] have introduced the Quasi-Renewal (QR) model, which has been developed until recently [15]. After the i th repair, the time to next failure is distributed as $\alpha^i Y$, where Y has the same distribution as the first failure time and α is a positive parameter. A way of generalizing this model is as follows. After the k th maintenance, the time to next potential CM, if there is no PM before, is distributed as $\alpha_p^{M_{C_k}} \alpha_c^{N_{C_k}} Y$. Y has the same distribution as the first failure time of a new system not subjected to PM actions. α_p and α_c are positive parameters that respectively characterize PM and CM efficiency. Then, it can be easily proved that :

$$\lambda_t = \frac{1}{\alpha_p^{N_{t^-}} \alpha_c^{M_{t^-}}} \lambda \left(\frac{t - C_{K_{t^-}}}{\alpha_p^{N_{t^-}} \alpha_c^{M_{t^-}}} \right) \quad (27)$$

Dorado, Hollander and Sethuraman [16] have proposed a model that includes both virtual age and quasi-renewal models. We can generalize this model by assuming that the distribution of W_k depends on an effective age $A_k = a_k(W_k, U_k)$ and also of a life supplement $\Gamma_k = \gamma_k(W_k, U_k)$:

$$Pr\{W_{k+1} > w | \mathbf{W}_k, \mathbf{U}_k\} = Pr\{Y > A_k + \Gamma_k w | Y > A_k\}$$

for all $w < H_k$. Then, the failure intensity of the model is :

$$\lambda_t = \Gamma_{K_{t^-}} \lambda(A_{K_{t^-}} + \Gamma_{K_{t^-}} [t - C_{K_{t^-}}]) \quad (28)$$

V. CLASSICAL PREVENTIVE MAINTENANCE POLICIES

The last step for the complete definition of a model is to define a PM policy. A review of maintenance policies is presented in [17]. In this section, we briefly recall the most usual PM policy and replace them in the general modelling framework.

A. PM at fixed time

The most basic PM policy is to make PM at deterministic predetermined times τ_k for $k \geq 1$. The associated counting process $M = \{M_t\}_{t > 0}$ is also deterministic. The PM policy is then defined by :

$$\forall k \geq 0, \quad H_k = \tau_{M_{C_k}+1} - C_k \quad (29)$$

CM are done after an unplanned failure, so the aim of this type of maintenance is to quickly restore the system into a state in which it can perform its function again. As opposed to PM, carefully prepared in advance, CM are often supposed to be minimal, that is to say ABAO. In this case, for PM at fixed time, the failure intensity is a deterministic function. This implies that the failure process is simply a NHPP. For example, with an ARA_∞ PM efficiency, the failure intensity is :

$$\lambda_t = \lambda \left(t - \rho \sum_{j=0}^{M_{t^-}-1} (1-\rho)^j \tau_{M_{t^-}-j} \right) \quad (30)$$

Yun and Choung [18] have partially studied this model. They have estimated the PM efficiency parameter ρ by the maximum likelihood method, on simulated data sets.

Canfield [19] has also proposed a model that can be viewed as a particular case of our framework. Its assumptions are :

- CM are minimal (ABAO).
- PM are done at fixed and periodic times with period Δt .
- Failure intensity is continuous.
- After the k th PM, the failure intensity is increasing as that of a new system of age $t - ks$. PM efficiency is characterized by s , which must be less than Δt in order to have efficient PM.

The failure intensity of the Canfield model is :

$$\lambda_t = \lambda\left(t - \left\lfloor \frac{t}{\Delta t} \right\rfloor s\right) + \sum_{i=1}^{\left\lfloor \frac{t}{\Delta t} \right\rfloor} [\lambda(i(\Delta t - s) + s) - \lambda(i(\Delta t - s))]$$

where $\lfloor x \rfloor$ is the integer part of x .

This model can be generalized to non periodic PM. Then, the failure intensity is :

$$\lambda_t = \lambda(t - sM_t) + \sum_{i=1}^{M_t} [\lambda(\tau_i - (i-1)s) - \lambda(\tau_i - is)]$$

In this model, PM efficiency is not characterized by a rejuvenation of the system, but by a modification of the failure intensity slope, that is to say a modification of the wear out speed. Figure 2 represents the failure intensity of the model for $\Delta t = 1.5$ and $s = 1$. PM times are symbolized by squares and CM times by circles.

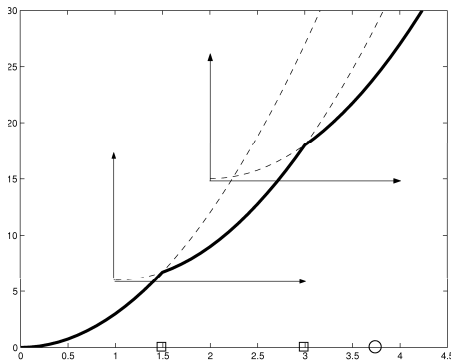


Fig. 2. Failure intensity for the Canfield model.

Some authors among which Nakagawa [20], Chan and Shaw [21], Wang and Pham [14], Sheu, Lin and Liao [22], have proposed particular models of planned PM. Their aim was generally to optimize maintenance costs. Very few authors have tried to estimate maintenance efficiency with this kind of models. Jack [23] has estimated maintenance efficiencies of medical equipments (syringe driver infusion pumps). He has used a model with deterministic periodical PM with ARA_1 or ARA_∞ efficiency, and ABAO CM. In another paper [10], he has analyzed the same data set with his model with intensity (23) and also with ARA_∞ PM and CM, with intensity (25).

B. PM at fixed age

The system is preventively maintained as soon as its virtual age $t - C_{K_{t-}} + A_{K_{t-}}$ exceeds a predetermined threshold $b_{K_{t-}}$. Then the PM policy is defined by :

$$\forall k \geq 0, \quad H_k = b_k - A_k \quad (31)$$

For example, the threshold can be a constant b and maintenance effect can be supposed AGAN. Then, PM that take place between T_i and T_{i+1} will be done at times $T_i + kb$ where k is an integer such that $T_i + kb < T_{i+1}$. The failure intensity is :

$$\lambda_t = \lambda\left(t - T_{N_{t-}} - \left\lfloor \frac{t - T_{N_{t-}}}{b} \right\rfloor b\right) \quad (32)$$

In this case, the failure process is a renewal process that has been studied by Shaked and Szekli [24].

The PM policy at fixed age has been used for instance by Brown, Mahoney and Sivazlian [11], Dagpunar and Jack [25], Lin, Zuo and Yam [26] and Chien [27]. When PM replaces the failed system by a new one, this policy is also known as the age-replacement policy.

C. PM at fixed intensity

The system is preventively maintained as soon as its failure intensity exceeds a predetermined threshold $d_{K_{t-}}$. Then the PM policy is defined by :

$$\forall k \geq 0, \quad H_k = \lambda^{-1}(d_k) - A_k \quad (33)$$

where $\lambda^{-1}(\cdot)$ is the inverse function of $\lambda(\cdot)$. For a constant threshold d , and AGAN PM and CM, the failure intensity is the same as (32), with $d = \lambda(b)$. Figure 3 represents the failure intensity in this case with $b = 1$ or $d = 3$ and an initial intensity $\lambda(t) = 3t^2$.

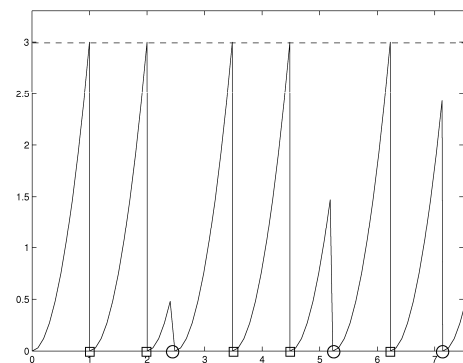


Fig. 3. PM policy at fixed age or intensity, case AGAN.

If maintenance efficiencies are not AGAN, the PM policy at fixed intensity is not equivalent to the PM policy at fixed age. PM policies of that type have been used for instance by Lie and Chun [28], Jayabalan and Chaudhuri [29], and Lin, Zuo and Yam [26].

VI. ANALYSIS OF REAL DATA SETS

The aim of this section is to assess the efficiency of preventive and corrective maintenance actions for two kinds of systems used in electricity production plants of the main French utility EDF. For confidentiality reasons, the time scale is not given.

The first data set given in table I gives the PM and CM times of stubs of the inlet header of the heat exchanger that warms up the feeding water of the boiler of a fossil-fired thermal plant. The second data set presented in table II gives the PM times, CM times and censoring times (Cens) of a fleet of 17 similar units that are parts of the steam turbine of a plant. In both cases, the critical event is an external leak that leads to the (immediate or delayed) break-down of the plant.

TABLE I
FIRST DATA SET.

25	50	93	109	114	141	163	164	195	225	264
PM	CM	CM	CM	PM	CM	CM	CM	CM	PM	PM

TABLE II
SECOND DATA SET.

UNIT 1	2145	4582		
	PM	Cens		
UNIT 2	4368			
	Cens			
UNIT 3	1942	5009		
	PM	Cens		
UNIT 4	3273			
	Cens			
UNIT 5	4120	5040		
	CM	Cens		
UNIT 6	2689	4643		
	PM	Cens		
UNIT 7	4394	5739		
	PM	Cens		
UNIT 8	2602	5130		
	CM	Cens		
UNIT 9	1462	2259	5040	
	PM	CM	Cens	
UNIT 10	1667	2073	3607	
	CM	PM	Cens	
UNIT 11	1090	1207	1705	2483
	CM	CM	PM	Cens
UNIT 12	2921	4454	4703	
	PM	CM	Cens	
UNIT 13	3011	3559	4248	
	CM	PM	Cens	
UNIT 14	3528	5433		
	PM	Cens		
UNIT 15	2008	2968		
	CM	Cens		
UNIT 16	5344			
	Cens			
UNIT 17	6348			
	Cens			

For both data sets, the following assumptions are made :

- PM are done at planned times.
- The initial intensity is a power function :

$$\lambda(t) = \alpha\beta t^{\beta-1}, \quad \alpha > 0, \beta > 1$$

- PM effects follow a virtual age model characterized by a PM efficiency parameter ρ_p .
- CM effects follow a virtual age model characterized by a CM efficiency parameter ρ_c .

For the second data set, the different units are supposed to be independent and similar. Then their failure processes have the same initial intensity and the same maintenance efficiency parameters.

In the following, we apply several imperfect maintenance models to these data sets. We estimate the model parameters by the maximum likelihood method, thanks to property 4. Note that, in order to estimate the model parameters, we have to know the PM times τ_j and the fact that PM are planned. But it is not necessary to know precisely the PM policy, that is to say the functions $h_k(\cdot)$.

A. Analysis of data set 1

Figure 4 is a plot of the observed cumulative number of failures. Vertical dotted lines represent PM times. Globally, this curve is neither concave nor convex. So neither improvement nor degradation of the observed system can be detected during this period. A possible explanation is that the maintenances have managed to stabilize the system reliability.

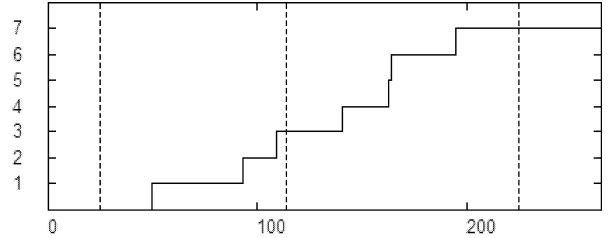


Fig. 4. First data set - Cumulative number of failures

1) ARA_∞ PM- ARA_∞ CM:

PM and CM effects are supposed to be ARA_∞ with different efficiency parameters. The model intensity is given in (25). The maximum likelihood estimates are :

$$\hat{\alpha} = 1.16 \cdot 10^{-5}, \quad \hat{\beta} = 3.05, \quad \hat{\rho}_c = 0.565, \quad \hat{\rho}_p = 1 \quad (34)$$

The value of $\hat{\beta}$ close to 3 indicates a strong intrinsic ageing. But $\hat{\rho}_p = 1$ means that PM is optimal. $\hat{\rho}_c$ close to 0.5 means that CM reduces by a half the system virtual age. The corresponding failure intensity is given in figure 5. This figure shows clearly that maintenance efficiency compensates the effect of wear.

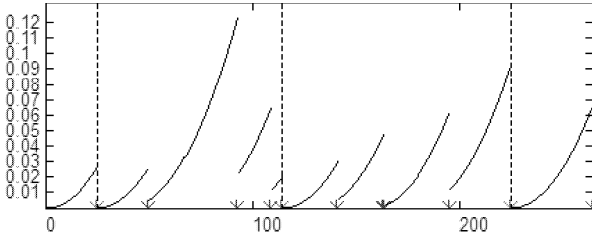


Fig. 5. First data set - Failure intensity for the ARA_{∞} model

2) ARA_1 PM- ARA_1 CM:

We assume now that PM and CM effects are ARA_1 with different efficiency parameters. The chosen version of the ARA_1 model is that with intensity given in (21). The maximum likelihood estimates are :

$$\hat{\alpha} = 1.76 \cdot 10^{-4}, \quad \hat{\beta} = 2.36, \quad \hat{\rho}_c = 0.94, \quad \hat{\rho}_p = 1 \quad (35)$$

As before, the value of $\hat{\beta}$ indicates intrinsic ageing, weaker than the previous one. PM is assessed to be perfect and CM to be nearly perfect. Then, this very efficient maintenances have managed to compensate wear. The corresponding failure intensity is given in figure 6.

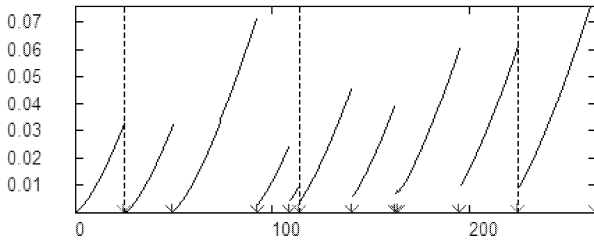


Fig. 6. First data set - Failure intensity for the ARA_1 model

Both ARA_{∞} and ARA_1 models lead to similar conclusions. In both cases, $\hat{\rho}_p = 1$, so PM efficiency is optimal. It can be seen on figures 5 and 6 that for the ARA_{∞} model, it means that PM renews the system (the intensity restarts from 0) whereas for the ARA_1 model, PM is only as good as previous: it restores the system in the state it was just before previous CM.

The MTTF, mean time to the next failure after the last maintenance, can be computed by simulation. For ARA_{∞} , $MTTF = 37.0$ unities of time, while for ARA_1 , $MTTF = 25.8$. This difference is probably linked to the fact that the risk of failure just after last maintenance is greater for ARA_1 than for ARA_{∞} , because the intensity does not restart from 0.

The maximum value of the log-likelihood is -29.5 for ARA_{∞} and -30.7 for ARA_1 , so the ARA_{∞} model can be chosen as the best model for these data.

3) ARA_{∞} PM-ABAO CM:

It is usual to assume that CM are ABAO. Indeed, the aim of CM is to quickly restore a failed system into a state in which it can perform its function again. So in practice, it is often expected that CM effect is minimal. On the other hand, planned PM are meticulously prepared so they are supposed

to be very efficient. That is why it is interesting to apply a model for which CM effects are ABAO and PM effects are ARA_{∞} .

In this case, the maximum likelihood estimates are :

$$\hat{\alpha} = 1.02 \cdot 10^{-3}, \quad \hat{\beta} = 1.74, \quad \hat{\rho}_p = 1 \quad (36)$$

As before, PM is estimated to be perfect. The value of $\hat{\beta}$ lies between 1 and 2 so the system is judged to be wearing but the wear-out speed is decreasing. Then, the estimated intrinsic ageing of the system is less than in (34), because in this case it cannot be compensated by the CM effect.

The failure intensity is given in figure 7.

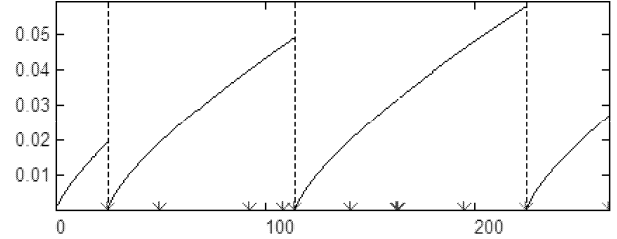


Fig. 7. First data set - Failure intensity for the ABAO CM - ARA_{∞} PM model

The maximum log-likelihood is -31.2 . It is naturally less than -29.5 in the case of (34) since ABAO CM are a particular case of ARA_{∞} CM. The validity of the ABAO assumption should be tested by means of methods like the AIC criterion.

B. Analysis of data set 2

Since the 17 units are supposed to be independent, similar and identically maintained, the likelihood is obtained by multiplying the likelihoods associated to each unit.

For ARA_{∞} PM and CM, the maximum likelihood estimates are :

$$\hat{\alpha} = 3.36 \cdot 10^{-6}, \quad \hat{\beta} = 1.44, \quad \hat{\rho}_c = 0.15, \quad \hat{\rho}_p = 1 \quad (37)$$

For ARA_1 PM and CM, we obtain :

$$\hat{\alpha} = 4.69 \cdot 10^{-6}, \quad \hat{\beta} = 1.40, \quad \hat{\rho}_c = 0.54, \quad \hat{\rho}_p = 1 \quad (38)$$

In both cases, PM is assessed to be optimal and $\hat{\beta}$ lies between 1 and 2. CM are estimated quite inefficient for the ARA_{∞} model and rather efficient for the ARA_1 model.

The corresponding failure intensities for unit 11 are given in figures 8 and 9.

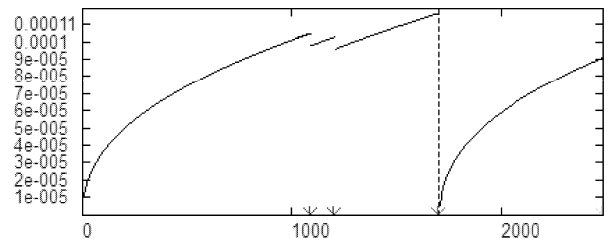


Fig. 8. Unit 11 - Failure intensity for the ARA_{∞} model

Here again, the difference between AGAN and As good as previous optimal PM appears clearly in these figures.

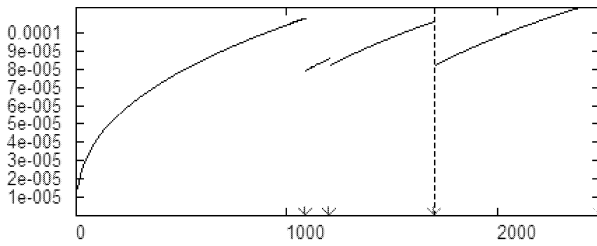


Fig. 9. Unit 11 - Failure intensity for the ARA_1 model

VII. CONCLUSION

The paper has developed a general framework for the joint modelling of system ageing and efficiency of corrective and planned preventive maintenance actions. Most of the usual imperfect repair models can be generalized in this framework. The model also involves the classical PM policies such as PM at fixed time, age and intensity. Knowing the likelihood function allows to estimate model parameters, that is to say to assess simultaneously the system ageing and maintenance efficiency. So it is possible to forecast the future behavior of the PM-CM process and then to update the Reliability Centered Maintenance (RCM) process through PM planning. It is of great interest as regards the problem of extending the operating lifetime of complex systems such as electricity plants.

For all the examples of real data treated in this paper, the parameter ρ_p has been estimated by 1. In our context, it means that preventive maintenance is always estimated to be optimal. In fact, it could mean that the system after PM is better than new. Indeed, this is the case for PM which are done after a technology improvement. Usual imperfect maintenance models cannot take this possibility into account, so there is a need for the building of new models for this situation.

All the models and figures presented in this paper are implemented in the software tool MARS (Maintenance Assessment of Repairable Systems), developed jointly by Grenoble University and EDF R&D. MARS is available upon request at <http://www-ljk.imag.fr/MARS>.

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