The Absolute Relativity Theory

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Abstract

The purpose of this paper is to give a first presentation of a new approach of physics, that we propose to refer to as the Absolute Relativity Theory.

This approach is founded on the refutation of the old idea that our universe can be seen as a space-time, whatever structure it is equipped with, that contains or supports "observers" and "observables".

Instead, the theory begins by exploring what should be, from an algebraic point of view, a consistent theory able to represent the "observation" processes and, in some sense, as complete as possible. Expressed in the general framework of categories, those conditions lead to take as a basis of the theory the category $\aleph$ of all the finite dimensional representations of complex quasi-Hopf algebras, and to explore its morphisms that appear as natural transformations. In this framework, the relation between observables and observers is naturally represented by the usual Hom bifunctor that associates with any couple of objects the set of arrows between them. Its contravariant part is called the point of view functor since its describes the arrows that arrive to this object.

Two principles are then stated in order to link the theory with physics, that we propose to refer to as the Absolute Relativity Principle and the Absolute Equivalence Principle. They express that

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any natural transformation in $\aleph$ corresponds to a true physical phenomenon, and reciprocally. Taken together, those principles lead to think that physics should be the description of the structures in $\aleph$ as seen through the point of view functor.

This is the aim of our first section that process in twelve successive steps that can be summarized as follows.

Since Hopf algebras can be classified according to their primitive part, the well known classification of real Lie algebras leads to determine a set of compatible ones, the representations of which could correspond to usual particles. The characterization of each type of these real Lie algebras through its Cartan subalgebra leads to associate with each type a specific space-time that is linked to the others by the usual embedding of algebras.

Two mathematical facts play then a key role. The first one is a specific property of the endomorphisms of $\mathbf{sl}_n$-algebras, seen as acting on those Cartan subalgebras that define different space-time, that have an order two specific symmetric representation that can be identified with the Lagrangians of physics. The second one is the fact that the point of view functor, being contravariant, induces the change to dual Hopf algebras that have infinite dimensional polynomial-type representations. Any object is represented in those algebras as inducing a creation/annihilation operator that generates the passing of his time. This mathematical fact corresponds globally to the so-called "second quantization" introduced by Quantum Field Theory. The use of Yangians representations gives a useful tool for representing this operator as its infinite dimensional generator J.

The capacity to algebraically represent Lagrangians and the "passing of the time" leads naturally to define physical observers as objects in $\aleph$ together with a well defined Lagrangian and passing of time. The identification with usual physics can now be made.

Euler-Lagrange equations first appear as the expression in our quantized context of the monodromy of the Knizhnik-Zamolodchikov equations. The well-known relations between Lie algebra together with the representation of the passing of time gives a computation process, that should permit to calculate from only one measured data, the fine structure constant α, the characteristics of the particles (like their mass) as identified by contemporary physics.

The theory also predict the existence of non identified (as of today) other particles that could give new interpretations of the "dark matter" and the "dark energy". All those computations may be seen as a new theory that explains why and how the matter does appear in a well defined quantized way : We propose to call this new branch of physics the Mass Quantification Theory.

The first section finally mentions that if one wants to represent space-time from the point of view of a physical observer as a Lorentzian manifold, the identification of local monodromies on this manifold with the above KZ-monodromy applied to our algebraic Lagrangians gives exactly the Einstein equation, but now its right-hand side has also found a mathematical interpretation. Since it comes from the quantized part of physics, this interpretation could be seen as what introduces General Relativity in the realm of quantum physics, i.e. as the unification of the two.

The second section of this paper is dedicated to a more mathematical presentation of the foundations of the Absolute Relativity Theory.

The third section contains the first computations that can be obtained from the Mass Quantification Theory. The almost perfect concordance of the results obtained by pure calculations with the best experimental values can be seen as promising for future progresses.

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"This does not seem to be in accordance with a continuum theory, and must lead to an attempt to find a purely algebraic theory for the description of reality. But nobody knows how to obtain the basis of such a theory."[1]

1 Introduction to the Absolute Relativity Theory

1.1 The status of space-time in physics

By refuting the old Aristotelian distinction between to be "at rest" or "in motion", the classical relativity principle asserted that "motion" can only be defined relatively to Galilean "observers", which presupposes the existence of a Newtonian "space" that defines those observers and contains observable "objects".

Three centuries later, Einstein's special relativity theory replaced Galilean observers by Lorentzian ones and extended space to space-time, without altering the universal character of the latter.

Only the General Relativity theory began to inverse the factors by stating that space-time does not contain the observable "objects" but *is defined by them in an interactive process encoded by Einstein's equation that links Ricci curvature tensors of space-time seen as a Lorentzian manifold with the distribution of energy (or masses) induced by those objects.* But the price paid for this reversal was quite high since the use of Ricci curvature tensors prevented the theory from describing internally asymmetric phenomena. So, until recently, general relativity theory has remained an isolated branch of contemporary physics, mainly dedicated to the study of gravitational phenomena.

The second branch of contemporary physics, born with Schrödinger, Heisenberg and Dirac, kept asserting the intrinsic "existence" of a space-time, thought as "the referential of the laboratory of the observer" on which Hilbert spaces of $\mathbb{C}$-valued functions are defined. Only later, with the emergence of quantum electrodynamics (QED) and quantum fields theory (QFT), systematic efforts were made in order to reverse the factors by assuming for instance that "observable" could also be defined as elements of abstract algebras, and that "states of nature" may be seen as $\mathbb{R}$-valued operators on such algebraically predefined "observable".

Finally, the status of space-time in string theories is even more astonish-

[1] Albert Einstein, The Meaning of Relativity, sixth edition, Routledge Classics, London, 2003, p.170.

ing since its existence is a prerequisite for each of them, but its dimension and folded structure differ from one theory to the other. So, despite many crucial discussions about space-time, string theories do not really challenge its intrinsic existence either[2].

Thus, the modern and contemporary idea of the intrinsic existence of some (generalized) space-time could well be the Gordian knot that has been blocking the path to the unification of physics for one century.

Since this idea amounts to representing the space-time perceived by the set of observers we belong to as universal, we propose to refer it as **the egocentric postulate**.

Unfortunately, since this egocentric postulate always stands at the very beginning of physical theories, refuting it immediately leads to the question of the foundations of the construction we intend to build.

1.2 Observers, observables, and physical phenomena

The theory presented here answers this question *by noticing that the first step in the construction of a physical theory should be to define a representation of the observation process itself, which means a definition of observers and observables together with a description of their relations.*

The algebraic approach suggests *to treat observers and observables on the same footing by seeing them as objects of an appropriate category*, that we propose to call **the preuniverse** $\aleph$, and defining the **point of view of an object** a **(observer) on an other object** b **(observable)** *as the usual set of arrows from* b *to* a*:* $\hom(b, a)$. *Any object will thus be both an observer and an observable.*

We will call the usual contravariant functor $\hom(., a)$ **the point of view functor of** a, and any functor F equivalent in $\aleph$ to $\hom(., a)$ will be said to be **represented by** a. The consequences of the contravariance of the point of view function cannot be overstated since any object, as an observer, belongs to the preuniverse $\aleph$, but, as an observable, it belongs to the opposite category $\aleph^{opp}$ defined by "reversing the arrows". In other words, from the very first algebraic foundations of the theory, physical objects have to be described with a "double nature", first as observers in $\aleph$, second, as they are seen from other objects, namely objects in $\aleph^{opp}$.

By considering then some internal consistency conditions (mainly requiring appropriate "diagrams of arrows" to be commutative) and completeness

[2]In the same way, the recent loop quantum gravity theory (LQG) assigns a specific "loop" structure to space-time, but does not challenge either its a priori existence. Its first successes demonstrate how crucial for physics is the problem of space-time.

conditions (mainly requiring the preuniverse to have "enough" objects to represent "composed" objects and point of view functors), we will precise the structure of the preuniverse $\aleph$ that will appear as a well defined category of **representation functors** $\rho_{A,V}$ (or A **-bimodules**) from various complex Hopf algebras A to finite dimensional bimodules V.

$\aleph^{opp}$ is then the dual category $\aleph^*$, that is the category of (non necessary finite dimensional) bimodules over various dual algebra A^* (in the sense of the restricted Hopf duality). Both algebras A and A^* fundamentally differ : for instance, non trivial A-modules we will consider have a greater than one finite dimension, while A^* -modules are either one- or infinite- dimensional. Therefore, the above "double nature" of objects will have tremendous consequences[3].

Hopf algebras are generally not cocommutative. This means that their dual cannot be seen as a space of functions, and thus that our categorial approach leads directly to so-called "quantized" (or "deformed") situations.

We will get in fact *two type of quantizations*, the first one associated with the non cocommutativity of A will globally correspond to the one of quantum mechanics, the second, more subtle, associated with the non cocommutativity of A^*, will generate creation/annihilation operators that correspond to the "second quantization" introduced by the QFT.

Let us emphasize that *those quantizations do not appear in cocommutative algebras that therefore cannot be seen as basic cases, but as degenerate ones, which explains the devastating consequences of the egocentric postulate.*

By working indeed on operators on an Hilbert space of functions, QFT is compelled to use "perturbational methods" in order to generate quantized situations, instead of representing them directly, as we will do below, *by noticing that such quantized situations intrinsically induce the existence of creation/annihilation operators that, in some mathematical sense we describe precisely, produce at each present instant the immediate future of particles and associated space-time.*

Finally, *a physical theory has to be able to be confronted with experimental results.* This implies that the theory includes a representation of what the universe we perceive is, which we propose to refer to as made of **true physical objects** and **true physical phenomena**, and that it describes

[3]Representations of Hopf algebras are bimodules that reflect both the algebra and coalgebra structures of the Hopf algebra (including their compatibility). Nevertheless, the usual way to deal with coalgebraic structures is to see them from the dual point of view, i.e. as an algebraic structure on the dual Hopf algebra. We will proceed so, and correlatively use the word "module" on one of those algebras, instead of speaking of "bimodule" and precising which of the algebraic or coalgebraic structures we refer to.

the impact of the perception process from the point of view of the observers we are. Considering our purpose, we will not view them as embedded in any given space-time, nor as having any a priori given proper time.

To this aim, *we will postulate that all the true physical objects are representation functors* $\rho_{A,V}$ *for appropriate algebras* A *and modules* V *that we will precise.*

We will then use two (and only two) principles to characterize true physical phenomena, namely *any natural transformation between representation functors induces a true physical phenomenon, and, reciprocally, any physical phenomenon can be seen in the theory as induced by a natural transformation.*

Since those principles will appear as the ultimate extensions of Einstein's relativity and equivalence principles, we propose to call this new theory **the Absolute Relativity Theory (ART)**, and to refer to these two principles as the **ART principles**.

Finally, as predicted by contemporary theories, we will represent ourselves as made of objects we will identify with electrons, and nucleons (made of quarks linked by strong interaction, which is not a trivial fact since it implies, among others, that *there are objects and interactions that arise from algebras bigger than the ones we belong to : we necessarily perceive them not as they are, but only through "ghost effects" as representations of the particles we are made of*). The weak interaction will appear as belonging to this category, which explains why its bosons do *appear* to us as massive, and induces the existence of an energy that we do not perceive as it is, and that therefore could be part of so-called "dark energy".

In the ART framework, the way the observables do appear to observers does not only depend on their relative positions and motions, but also on the type of each observer.

Here is the way we will follow.

1.3 A short description of the Absolute Relativity Theory

The next subsection of this paper gives a tentative twelve steps presentation of the main foundations and results of ART.

We will first precise the definition and characteristics of the above preuniverse $\aleph$, and see how a first classification of its "objects" and "morphisms" leads to a first classification of particles and interactions as associated with some specific *real* form of simple finite dimensional complex algebras. Ultimately, "our perceived" space-time itself will arise in a very natural way from those algebraic foundations.

A specific emphasis has to be made on the algebraic meaning of the well known Lagrangians, and on the origin of the "passing of time" phenomenon.

The first will appear as the consequence of a very specific property of the representations of the algebras $\mathbf{sl}_n \otimes \mathbf{sl}_n$ (with $n > 2$) that admit, as a representation of $\mathbf{sl}_n$, *a trivial component that we will call* ***the free part of the Lagrangian*** *and a non trivial symmetric component that we will refer to as* ***its interactive part****.*

The free part defines an invariant bilinear symmetric form on the dual of the representation space on sl_n^*. Since we will apply $\mathbf{sl}_n$ to the Cartan subalgebras $\mathbf{h}$ of the Lie algebras $\mathbf{g}$ (of rank n) we will work on, this free part will correspond to a (complex) scaling to be applied to the standard restriction to $\mathbf{h}$ of the dual of $\mathbf{g}$ Killing form. It will appear as directly connected to mass and charge of particles.

The interactive part is more subtle, as defined by the following projection operator π (viewing elements of $\mathbf{sl}_n$ as matrices) :

$$\pi(x \otimes y) = xy + yx - \frac{2}{n} trace(xy).Id.$$

It associates in a quadratic way with any element of $\mathbf{sl}_n$ a bilinear symmetric form on $\mathbf{sl}_n^*$. Identifying $\mathbf{sl}_n$ with $\mathbf{sl}_n^*$ through its own Killing form, we get so a bilinear symmetric form on $\mathbf{sl}_n \otimes \mathbf{sl}_n$ itself. Identifying it, again canonically through the Killing form, with $\mathbf{sl}_n \otimes \mathbf{sl}_n^*$, we get a bilinear symmetric form defined on endomorphisms of $\mathbf{sl}_n$. This property is easily transferred to $\mathbf{su}_n$-algebras when working on maximal tori of real compact Lie algebras.

Taken together, the free and interactive parts of Lagrangians define therefore a bilinear symmetric form on the endomorphisms of $\mathbf{gl}_n$. The importance of Lagrangians comes from the fact that scalings and outer automorphisms of a Lie algebra $\mathbf{g}$ have a non trivial action on the endomorphisms of its Cartan subalgebra, while all the inner automorphisms by definition do not have any action.

On another hand, the "passing of time" will appear as directly coming from the contravariance of the point of view functor that induces for any simple Lie algebra $\mathbf{g}$ the passing from the Drinfeld-Jimbo algebra $U_h(\mathbf{g})$ to its dual, the quantized algebra $F_h(\mathbf{G})$ that has a family of infinite dimensional representations in the formal algebra of one variable polynomials. This family is in fact induced through the principal embedding of $\mathbf{sl}_2$ into $\mathbf{g}$ by only one case that arises in $F_h(\mathbf{SL}_2)$ by applying the Weyl symmetry to $U_h(\mathbf{sl}_2)$ before taking the dual.

By considering the point of view of an object on itself and thus going from $U_h(\mathbf{g})$ to $F_h(\mathbf{G})$ due to the contravariance of the point of view functor, we have first to notice that from the uniqueness of principal embeddings of

$\mathbf{sl}_2$ in the Weyl chamber of any complex algebra $\mathbf{g}$, we get a family, indexed by the maximal torus and the Weyl group, of surjection from $F_h(\mathbf{SL}_2)$ to $F_h(\mathbf{G})$. Focusing thus on the situation in $F_h(\mathbf{SL}_2)$, we see the apparition of a new type of *creation/annihilation operators that recall those of the Quantum Field Theory (QFT), but applied to the considered object itself, creating so a new instant after each instant, inducing therefore the "passing of time"of this object.* Time itself (and thus space-time) so appears in the ART as defined from objects in a quantized way : in that sense the "second quantization" of QFT should be thought as the first one.

Furthermore, since those representations are indexed by the maximal torus of the compact form $\mathbf{g}^c$ of $\mathbf{g}$, and by elements of the Weyl group, we will pull back the main characteristics of the real algebra $\mathbf{g}$ on the maximal torus T of $\mathbf{g}^c$ by specifying its dimension, its Weyl invariance group, the induced action of the outer automorphisms of $\mathbf{g}$, and the involution that defines the considered real form[4]. Furthermore, the action of the above creation/annihilation operator that generates the passing of time can also be represented on the dual of the Cartan subalgebra (or maximal torus) of $\mathbf{sl}_2$ (or $\mathbf{su}_2$) as inducing the increase/decrease of two units on the line diagram that corresponds to making the tensor product by the adjoint representation.

This leads to carry on with the story by working on $\mathbf{su}_n$*-type algebras seen as acting on the maximal torus of* $\mathbf{g}^c$.

Now, $\mathbf{su}_n$*-type algebras are precisely those on which Lagrangians can be defined.* They furthermore correspond to a common framework where everything concerning the different algebras $\mathbf{g}$ can be encoded, and, since Lagrangian as above defined, have the aforesaid wonderful property to be invariant under $\mathbf{su}_n$ adjoint action, they also give rise to something that will be independent of the chosen point of view, and that will correlatively be thought as independently existing : *here is the explanation of the feeling we have that "things" do actually exist, ...and the origin of the egocentric postulate.*

The above connection between creation/annihilation operators and polynomial algebra then leads to work on **Yangians algebras** defined from those $\mathbf{su}_n$ algebras. *In this framework, the above creation operator corresponds to the endomorphism J of* $\mathbf{su}_n$ *used to define Yangians*[5]. *Successive applications of this operator become successive applications of J*[6].

[4]All those elements come from their equivalent in the complex Cartan subalgebra, and we will use systematically the equivalence between working on complex Lie algebras or on their compact real form.

[5]See for instance [V.C.-A.P.], pp.374-391 for all notations and results concerning Yangians.

[6]In order to represent also the annihilation operator, one should work on affine

A key point is that, by identifying $\mathbf{su}_n$ *with* $\mathbf{su}_n^*$ *through the Killing form,* J *itself may be seen as an element of* $\mathbf{su}_n \otimes \mathbf{su}_n$ *and has thus a Lagrangian projection, which defines a relative scaling of* $\mathbf{su}_n$ *that is very small (we will compute it as being of order of* 10^{-37}*), but exists. We will show that this fact makes the gravitation appearing as a (quantized) consequence of the passing of time that is also quantized, as we just have seen.*

More generally, one can study the evolution of the Lagrangian with successive applications of the creation/annihilation operator : this corresponds to the passing from a weight N *to the* $N+2$ *one, and therefore, as transposed in a smooth context, to twice the left-hand side of QFT Euler-Lagrange equation, namely* $\frac{\partial \mathcal{L}}{\partial \phi}$.

But there is a consistent way to go from one representation of $U_h(\mathbf{sl}_2)$ *to another one that corresponds to the above passing of time, namely to use Knizhnik-Zamolodchikov equations (KZ-equations) in order to define the parallel transport of Lagrangians and get a monodromy above the fixed point of an appropriate configuration space.* Since in the $\mathbf{sl}_2$ case, Lagrangians reduce to their scalar part (the so-called "free part") defined by a projector analog to the QFT quantity $\frac{\partial}{\partial [D_i \phi]}$, the monodromy of the KZ-parallel transport of this 2-tensor has precisely to be equal to the above right-hand side of the QFT Euler-Lagrange equations. Computing this monodromy from the canonical invariant 2-tensor associated with $\mathbf{g}$ gives therefore the true value of the variation of Lagrangian between the two successive positions, that corresponds to twice the right-hand side of Euler-Lagrange equations. *Equalizing both sides and considering any possible direction in space-time gives Euler-Lagrange equations.*

Those equations therefore appear as describing in the usual continuous and smooth context of differential geometry, the above well defined discrete algebraic creation process that generates the passing of time. Euler-Lagrange equations can therefore no more be seen as some mathematical expression of the old philosophical Maupertuis's principle of least action.

Furthermore, KZ parallel transport implies successive applications of the flip and antipode operators. Now, two successive applications of the antipode in a Yangians algebra are equivalent to a Yangian-translation of $\frac{c}{2}$, with c the Casimir coefficient of the adjoint representation : in the $\mathbf{sl}_2$-case, this translation is by $\frac{8}{2} = 4$, which confirms that KZ-parallel transport has to be made with the above two by two steps of the adjoint representation. KZ-parallel transport therefore defines a motion on the so called "evaluation representations" of the algebra $Y(\mathbf{su}_n)$ that sends it back to $U(\mathbf{su}_n)$ (in a

algebras instead of Yangians. Since both are closely related, we will not explore this more general way in this first presentation of ART.

consistent way with the inclusion map $U(\mathbf{su}_n) \to Y(\mathbf{su}_n)$).

These algebraic views can be used to come back to Einstein's equation and the unification of physics. Indeed, if it is *a priori* postulated that space-time *is* a Lorentzian manifold, its local monodromy algebra is encoded in the space of Riemann curvature tensors of which the symmetric part is the Ricci curvature 2 -tensor. This tensor may thus be seen as the equivalent of the above right-hand term of the Euler-Lagrange equation that arises, in our context, from KZ monodromy.

Writing the equality of both monodromies gives exactly the Einstein equation. Instead of being seen as a physical principle, Einstein equation therefore appears as a mathematical consequence of the way the "passing of the time" do arise. As a by-product we get simultaneously an answer to the question of the origin of the "arrow of time" in General Relativity (GR) : namely, only the "past part" of GR Lorentzian manifold that represents the universe does exist from the point of view of an observer, and this is precisely the "passing of time" of this observer that contributes locally to extend it.

All this leads to the theoretical possibility to compute the Newton constant G from the value of the fine structure constant α and the consistency of the theoretical result with the measured value of this constant could be seen as the symbol of Einstein's come back in the quantized universe, giving to General Relativity Theory its right place in quantized physics.

It makes thus sense to explore the consequences of the above interpretation of the weak interaction as a "ghost effect" from a higher order four dimensional today unknown interaction. *The introduction of the corresponding Lagrangian gives a new interactive term that could correspond to the mysterious "dark energy"*, but we did not try to evaluate this term by introducing it in Einstein's equation, nor to test this conjecture. We also did not try to explore systematically the cosmological implications of ART.

T*he last step is to use ART to explain the nomenclature and compute the characteristics of particles and interactions. This is the purpose of a new branch of physics that we propose to refer as the Mass Quantification Theory.* With the exception of the above new interpretation of weak interaction as a "ghost effect" from an higher order one, it should have as an objective to compute and precise the nomenclature and characteristics of the particles as recognized by contemporary physics. *MQT also leads to conjecture the existence of new types of particles that appear as good candidates to be constituents of the "dark matter"*, but we did not try to test this conjecture.

Although the MQT theoretically leads to describe and compute any phenomenon it predicts, only some computations and predictions are introduced in this first paper.

We summarize in the next subsection, as twelve successive steps, the con-

struction we have just sketched.

The second section is a more mathematical one dedicated to a more special presentation of the theoretical foundations of the ART. It does not contains any physical result, but we thought it was necessary to ensure the global consistency of ART. This second section is completed by an Appendix that outlines the abstract theoretical conditions that a physical theory has in any case to comply with and that stand behind those foundations.

The third section is dedicated to the first computations that result from the MQT, giving first the aforesaid relation between Newton constant G and the fine structure constant α. As illustrative instances some precise results concerning among other the ratios between well-known masses of some particles are also presented here.

Far from being meant to give an achieved view of the Absolute Relativity Theory, this paper has to be considered as a first proposal to the scientific community to explore new ways that could lead contemporary physics to some new results. It will certainly also lead to new questions and difficulties that will have to be answered by a lot of complementary work.

1.4 The Absolute Relativity Theory in twelve steps

Since the new approach we propose will lead to many constructions and results that differ from those of contemporary theories, but are linked to them, a short review may be useful for a first reading of this paper. This is the purpose of this subsection. Precisions and justifications will be given in the following sections.

1. **The algebraic preuniverse $\aleph$**

   The preuniverse $\aleph$ may be first chosen as the category of complex finite dimensional A-modules with A any deformation of any complex quasi-Hopf algebra. We will restrict ourselves to Hopf algebras and to the quasi-Hopf algebras that they generates by a gauge transformation. Those quasi-Hopf algebras may be classified according to their primitive part $\mathbf{g}$. We will furthermore restrict ourselves to Hopf-algebras with $\mathbf{g}$ a finite dimensional semi-simple complex Lie algebra, beginning with simple ones. As aforesaid, those Hopf algebras are generally not cocommutative : thus instead of beginning as usual with the universal enveloping algebras, we will consider modules ever Jimbo-Drinfeld Quantum Universal Enveloping algebras (QUE), $U_h(\mathbf{g})$ as a starting point. We will restrict ourselves to finite dimensional ones in order to apply the restricted duality to go from $U_h(\mathbf{g})$ to its dual $F_h(\mathbf{G})$.

2. **The intrinsic breach of symmetry in complex Lie algebras**

Since the ART is founded on a categorial framework that is defined up to an equivalence, it is invariant under isomorphisms of Lie algebras. We may thus choose any Cartan subalgebra $\mathbf{h}_{\max}$ of the maximal (in the sense of inclusion) simple algebra $\mathbf{g}_{\max}$ (of rank $r_{\max}$) we will work on (we will precise it below). We may then choose a Cartan subalgebra and an ordering of its roots to define the corresponding elements of any $\mathbf{g}_i$ of its subalgebras (with $i \in I$, the finite set of Lie algebras we will work on, $\mathbf{h}_i$ the corresponding Cartan subalgebra and r_i its rank). *Any irreducible object V of $\aleph$, as an $U_h(\mathbf{g}_i)$-module or $U_h(\mathbf{g}_i)$ -representation functor $\rho_{U_h(\mathbf{g}_i),V}$ (or in short $\rho_{\mathbf{g}_i,V}$), is characterized by a maximal weight $\lambda_{\mathbf{g}_i,V}$ and will be said to be at* ***the position*** *V (or $\lambda_{\mathbf{g}_i,V}$).*

So, like tangent spaces in General Relativity, our main objects will be linear representation, but instead of using only the four dimensional representations of the Lorentz group and seeing them as linked by a Lorentzian connection, we will work on different QUE and their different finite dimensional representations linked together by their algebraic intrinsic relations that simultaneously define their relative characteristics and (discrete) positions.

The set of all the $\lambda_{\mathbf{g}_i,V}$ defines a set of lattices all included in the dual of the Cartan subalgebra $(\mathbf{h}_{\max})^*$. We will call the lattice of weights Λ_i associated with the algebra $\mathbf{g}_i$ the **$\mathbf{g}_i$-type space-time** (in short **$\mathbf{g}_i$-space-time**). *It is not the usual space-time, but it corresponds to the first step of our algebraic construction of space-time from observables. As it is easy to check in any Serre-Chevalley basis, although $\mathbf{g}_i$ is complex algebra, each Λ_i is a copy of $(\mathbb{Z})^{r_i}$, and any* **$\mathbf{g}_i$-space-time** *is thus an essentially real object.*

So, although all the possible choices of a Cartan subalgebra $\mathbf{h}_{\max}$ are linked by inner automorphisms that are complex ones, the set of finite dimensional modules defines in any isomorphic copy of $\mathbf{h}^*_{\max}$, the same real lattice. Since a multiplicative coefficient $e^{i\phi}$ does not preserve this lattice, we have here an algebraically defined breach of symmetry in the complex plan.

This breach of symmetry is intrinsic in the sense that his existence does not result from any postulated field such as, for instance, Higg's field, but from the fact that *finite dimensional representations of a simple complex algebra are defined by a $\mathbb{Z}$-lattice.*

3. **Working with both complex and real algebras**

We have thus to focus on real algebras, and we will proceed very carefully by begining with the compact form canonically associated with each complex algebra. Since any real form of any $\mathbf{g}_i$ is defined by a *conjugate linear* involutive automorphism, two real forms σ and τ are said to be compatible if and only if they commute, which means that $\theta = \sigma\tau = \tau\sigma$ is a *linear* involutive automorphism of $\mathbf{g}_i$. Furthermore, since any real form is compatible with the compact one σ^c, the study of real forms comes down to the study the compact form and of its linear involutive automorphisms.

Since any linear involutive automorphism of $\mathbf{g}_{\max}$ *that respects a subalgebra* $\mathbf{g}_i$, induces on $\mathbf{g}_i$ a linear involution, any real form on $\mathbf{g}_{\max}$ induces a real form on this subalgebra $\mathbf{g}_i$; reciprocally, if we identify a specific real form τ_i associated with some linear involution θ_i of an algebra $\mathbf{g}_i$ as associated with a family of true physical objects, θ_i will have to be compatible (i.e. commute) with the restriction to $\mathbf{g}_i$ of $\theta_{\max}$. The same reasoning applies to other inclusions of algebras, and will determine a lot of necessary theoretical conditions on compatible real forms of algebras. Since the compatibility of real forms is a "commutative diagram"-type consistency condition for the theory, this way of reasoning will be the key of the determination of the real algebras that define the particles that can coexist in our universe.

Finally, the only real algebras we will be interested in, are the compact ones, and only one family of other ones. This leads to distinguish in the Weyl chamber associated with the lattice of weights Λ_i those that are compact and the other ones : the first characterizes what we propose to call ***folded dimensions****, the other ones to* ***unfolded dimensions****. Any (different from $\mathbf{sl}_2 \approx \mathbf{su}_{1,1}$) non compact real $\mathbf{g}_i^{\theta_i}$-space-time has unfolded dimensions that we will identify with usual space-time, but also folded dimensions (that correspond to its maximal compact subalgebra).*

Therefore, any real form $\mathbf{g}_i^{\theta_i}$ of the complex Lie algebra $\mathbf{g}_i$ of complex dimension n_i and rank r_i, has d_i unfolded dimensions, $r_i - d_i$ folded dimensions, $n_i - r_i$ dimensions that characterize the Lie algebra structure, but do not correspond to any dimension of space-time : that is the reason why we propose to call them **structural dimensions**.

4. **A first algebraic classification of particles**

We are now ready to give the basic correspondence with usual particles

- **the neutrino** corresponds to the algebra $\mathbf{su}_2$. It has no unfolded dimension, and thus no intrinsic proper time (which implies that they appear as going at the speed of light). Nevertheless, we will see that neutrinos appear to us as having a mass that comes in fact from the asymmetry of our proper time that we will describe below. This mass could therefore be seen as a "dynamical ghost" mass.

- **the electron** corresponds to the algebra $\mathbf{so}_{1,3}$, the realified algebra of $\mathbf{sl}_2(\mathbb{C})$. It has one folded and one unfolded dimensions that supports its proper time. We will see how the unique unfolded dimension is linked to the gauge invariance group $\mathbf{SU(1)}$ that characterizes electromagnetic interaction. The splitting $\mathbf{so}_{1,3} \approx \mathbf{su}_2 \oplus \mathbf{su}_{1,1}$ has a very special importance since it generates a one dimensional non rigidity that we will characterize by the fine structure constant α seen as the ratio between the inertial and the electromagnetic energies of the electron.

  Another important fact is that the smallest representation of $\mathbf{so}_{1,3}$ is four dimensional, which is the dimension of our usual space-time. As an advantage, this allows us to have a complete view on its structure, and thus to deduce its equations from experimental processes. As a drawback, this has reinforced the idea that this space-time *in some sense includes all the physical phenomena, which is a completely wrong idea as far as other interactions (excepting gravitation) are concerned.*

  Finally, the outer automorphism that corresponds to the symmetry between the two (isolated) nodes of the $\mathbf{so}_4$ Dynkin diagram, induces at each position the fugitive coexistence of a representation of $\mathbf{so}_{1,3}$ with the direct sum of two copies of $\mathbf{su}_2$ that corresponds to two copies of the above algebra. Since the corresponding neutrinos, as aforesaid, leave the electron almost at the speed of light, we will say that the true physical phenomenon associated with this symmetry of the $\mathbf{so}_4$ Dynkin diagram is an **anti-confinement phenomenon**.

- **the usual quarks** correspond to the algebra $\mathbf{so}_{3,5}$ (or $\mathbf{so}_{5,3}$). It has one folded and three unfolded dimensions. We will see how those three unfolded dimensions are linked to the gauge invariance group $\mathbf{SU(3)}$ that characterizes the strong interaction. *The well known triality, that comes from the group of automorphisms of D_4 Dynkin's diagram which is S_3, ensures the coexistence of three copies of the different representations of the algebra at each position* : this very special mathematical property explains the usual confinement of three quarks by giving a precise mathematical sense to their usual three "colors", namely the three isomorphic forms (standard, spin + and spin -) of the complex algebra $\mathbf{so}_8$ that necessarily do appear together on any position V independently of any complex involution that defines the considered real form of this algebra. The folding of those three in the algebra $\mathbf{g}_2$ leads to protons and neutrons at the corresponding position, as one could have guessed.

The important fact is that one can compute the characteristics of all those particles from the well known characteristics of those algebras. As aforesaid, we will give some instances of such computations in our last section.

5. **The algebraic interpretation of the four known interactions**

From ARP, natural transformations between representation functors in the theory should correspond to interactions, and reciprocally. Here is the basic correspondence with usual interactions :

- *outer automorphisms of any complex algebra, that are associates with the automorphisms of its Dynkin's diagram, correspond to interactions* : the one of D_2 applied to $\mathbf{so}_{1,3}$ corresponds **to electromagnetism**, the triality of $\mathbf{so}_{3,5}$ to **strong interaction**.

  The weak interaction will appear as corresponding to the outer involutive automorphism that comes from the compact form the real form $E(II)$ of the algebra E_6. We do not perceive it as such, but only through "ghost effects", i.e. as representations of the particles we are made of, which induces a splitting of the corresponding E_6-bosons (that have to be massless) into different parts. Let us emphasize that such parts can individually appear as massive, exactly in the same way a photon would be perceived as a massive particle by one-dimensional observers with our proper time. Bosons W and Z correspond to this situation. E_6 is also the "largest" exceptional Lie algebra that has outer automorphims, which is one reason why we can choose it as our g_{max}.
- *Weyl symmetries* applied to the space-time defined by an observer correspond to changes of the Weyl chamber in the corresponding weights diagram : we propose to call **changes of "flavors"** the corresponding relativistic effects, since, in the case of the algebra $\mathbf{g}_2$ (that corresponds to our nucleons), the six "flavors" of leptons and quarks, associated with the corresponding antiparticles, will appear as related to the twelve copies of the Weyl chamber in

this algebra. *The important fact is, as above, that this view of the "flavor" of particles gives another example of relativistic effects that modify the characteristics of the observables according to their position (in the general sense of ART) relative to the observer.*

- Since any element of any simple compact group $\mathbf{G}^c$ of rank r is conjugate to exactly one element of each of the $|W|$ sheets that cover its maximal torus, there is exactly one element t in a given copy of the Weyl chamber associated with any element of $\mathbf{G}^c$. This allows us to see any element of $\mathbf{SU}(r)$ as acting on any element of any specific sheet of the Weyl chamber, by keeping this element if its image remains in the same sheet, or combining with an appropriate Weyl symmetry, that appears as a change of "flavor", if it is not the case. This allows us to encode everything in the maximal torus as explained in subsection 1.3., and then to restrict ourselves to one of its sheets. We propose to call **generalized Lorentz transformations** the pull back of the so defined transformations through any specific involution that defines any non compact real form.

  There is also a possibility of working on any complex Cartan subalgebras of the algebra $\mathbf{g}$ instead of the maximal torus of $\mathbf{G}^c$ by substituting $\mathbf{sl}_r(\mathbb{C})$ to $\mathbf{SU}(r)$ and replacing exchanges of sheets by exchanges of roots[7]. A key fact is that those two ways are equivalent since we will need both of them : the compact real form in order to access to any other real form by an appropriate complex involution, and the complex algebra in order to give sense to this involution.

- **the gravitation** will appear as a direct consequence of the passing of time as we will explain below.

6. **Some algebraic conjectures on particles and interactions**

   Our algebraic approach will also lead us to conjecture the existence of some unknown particles and interactions :

[7]The way ART is built legitimates the use of "Weyl's unitary trick" : we will use it mainly to pull back any configuration in a Cartan subalgebra on the maximal torus of the compact real form. Beginning with a postulated space-time prohibits from applying the necessary involutions to go from the compact real form to any other one. By contrast, in ART, this gives in particular a way to make probabilist computations without having to deal with infinites by some renormalization methods. Nevertheless, we will not use those possibities in this first paper.

- The compact form of the $\mathbf{so}_8$ algebra should define particles that are, as those associated with $\mathbf{so}_{3,5}$, linked by the strong interaction induced by the mathematical triality. They should be also folded into compact copies of representations of $\mathbf{g}_2$. We thus propose to call them **dark quarks** that have to be folded into what we suggest to call **dark neutrons**[8]. Since they correspond to a compact form, the space time associated with those particles should not have any unfolded dimension, and they should be related to usual quarks by an as above defined anticonfinement phenomenon. Dark neutrons should also get a mass from the same relativistic effects as neutrinos. Therefore, they could be part of the "**dark matter**" contemporary physics is looking for.
- The representations of the algebra $\mathbf{F}_4$ comes itself from the folding of the outer automorphism that links the compact form of $\mathbf{E}_6^c$ and the afore mentioned $\mathbf{E}(II)$ real form of $\mathbf{E}_6$. Those three algebras should define particles and phenomena we do not perceive as they truly are. As aforesaid, the weak interaction should be one of them, but it is only a ghost effect.

  To access to the true effect, it is necessary to represent the above outer automorphism, and compute the corresponding Lagrangian as explained below. This Lagrangian should define a new interaction that encompasses the weak interaction. Furthermore, since both algebras $\mathbf{F}_4$ and $\mathbf{E}(II)$ have real rank four, which explains the dimension of our usual space-time, this new interaction should correspond to high valued Lagrangians, which makes it a good candidate to be the main part of the mysterious "**dark energy**".

  On another hand, let us notice that, as made of smaller particles, we perceive those particles not as they are but through **splitting effects** that transform the rigid structure of any irreducible representation of $\mathbf{F}_4$ into a direct sum of representations of smaller algebras. Since "branching formulas" on those special algebras are growing very fast with the maximal weights of their representations, this splitting process will induce the appearance of huge numbers of splitted particles. Since, furthermore, direct sums do not induce any rigidity, this splitting process also explains the incredible freedom of their possible arrangements.

7. **The existence and meaning of Lagrangians**

[8]We will understand later on why there should not be white dark protons.

The next step is to consider the impact of the point of view functor by considering the algebra $F_h(\mathbf{G})$, the (restricted Hopf) dual of the corresponding $U_h(\mathbf{g})$, that should define what we perceive of all physical objects and phenomena.

Beginning as above with compact algebras, *the first key point is that, as we said before, all the representations of all the Hopf algebras $F_h(\mathbf{G}^c)$ are indexed by the maximal torus $T_{\mathbf{G}^c}$ that is a copy of $(T_1)^{r_\mathbf{G}}$* (with T_1 the trigonometric circle).

Let us then notice that the group $\mathbf{SU}(r_\mathbf{G})$, that corresponsal form of $\mathbf{SL}(r_\mathbf{G})$, define the group of all the transformations of the maximal torus.

But only those that correspond to unfolded dimensions can generate transformations that we can perceive. With above notations, we have thus also to consider the smaller torus $(T_1)^{d_\mathbf{G}}$ and the smaller group $\mathbf{SU}(d_\mathbf{G})$. With the above definitions of electromagnetism and strong interaction, we find here the usual gauge invariance groups of the QFT, namely $\mathbf{SU}(1)$ and $\mathbf{SU}(3)$.

Now, any Lie algebra $\mathbf{sl}_n(\mathbb{C})$ with $n > 2$, which compact real form is $\mathbf{su}_n$, has a very specific property : by identifying it through the Killing form with its dual, one sees that the tensor product $\mathbf{su_n} \otimes \mathbf{su}_n$ contains, as for any simple Lie algebra, a one dimensional representation of $\mathbf{su}_n$ (the image of the identity map, i.e. the Casimir operator of $\mathbf{su}_n$), and the image of the adjoint representation of $\mathbf{su}_n$ (that is antisymmetric). But, in the $\mathbf{su}_n$ (or $\mathbf{sl}_n(\mathbb{C})$) cases, and only in them, *there is in $\mathbf{su_n} \otimes \mathbf{su}_n$ a second irreducible representation of $\mathbf{su}_n$ that is symmetric and thus defines a bilinear symmetric form $\mathcal{L}'$ on $\mathbf{su}_n^* \otimes \mathbf{su}_n^*$* that may be identified with the help of the Killing form with an endomorphism of $\mathbf{su}_n$. Let us emphasize that Lagrangians do not act directly on the Cartan subalgebra, but on the algebra of the endomorphisms of this algebra seen as a vector space. Therefore, although the action of any element a of $\mathbf{GL}_n$ on the Cartan algebra $\mathbf{h}$ of any Lie algebra $\mathbf{g}$ respects $\mathbf{h}$ only globally - this corresponds to a simple change of observer in usual physics -, Lagrangians as corresponding through the Killing form of $\mathbf{su}_n$ to *endomorphisms* of $\mathbf{su}_n$ itself, will remain invariant : the action of a for them is the one of a simple change of base in $\mathbf{su}_n$.

Coming back to any simple algebra $\mathbf{g}$, an outer automorphism ϕ of $\mathbf{g}$ induces a change of the primitive roots, and thus of the fundamental roots and Weyl chamber, that does not result from the action of the Weyl group. It sends correlatively the connected component of the identity of $Aut(\mathbf{g})$ to another one. This change in the roots diagram induces an automorphism $\widetilde{\Phi}$ belonging to $Aut(\mathbf{su}_r)$; $\widetilde{\Phi}$ may be seen as above through the Killing form of $\mathbf{su}_r$ as an element Φ of $\mathbf{su}_r \otimes \mathbf{su}_r$. It splits therefore into irreducible representations of $\mathbf{su}_r$ with a one dimensional projection on the Casimir operator, which we will call **the free part of the Lagrangian $\mathcal{L}_\Phi$**, and the above $\mathcal{L}'_\Phi$ that we will call **its interactive part**. The sum of both parts will be the **Lagrangian of the interaction corresponding to Φ**.

Since it neutralizes the component that comes from the adjoint representation of $\mathbf{su}_r$ that generates the above connected component of the identity, *the Lagrangian can be seen as characterizing a scaling on* $\mathbf{su}_r$ *and a change of connected component of the identity in* $Aut(\mathbf{g})$ *independently of any basis chosen for the Cartan subalgebra* $\mathbf{h}$ *. This explains its additivity and its independence of the basis chosen for representing* $\mathbf{g}$*, and also of the choice of the Cartan subalgebra (since they are all conjugate) and finally of* $\mathbf{g}$ *itself (except through r and d). and the Lagrangians it defines*

Therefore, the choice of coherent Cartan subalgebras we made in the beginning appears now only as a commodity for computations.

Furthermore, if an algebra is included in another one, the smallest defines a degenerate bilinear form on the space-time associated with the first, but the construction keeps making sense. *This gives its main interest to the Lagrangian.*

This unfortunately does not mean that it is easy to compute. Indeed, since it is invariant under the action of the adjoint group and thus of the one of the Weyl group, the computation of the Lagrangian, in the cases where it is not reduced to its free part, has to include at least as many elements than the cardinal of the Weyl group (which is 192 for $\mathbf{so}_8$, 51840 for E_6 and 1152 for F_4 !). Although there are many symmetries to be used, we will not try to make those computations here.

Since the algebra $\mathbf{so}_{1,3}$ has a two-dimensional Cartan subalgebra, electromagnetism has only a free part easy to identify with the usual one. The same is true for the gravitation that is, as we will see, directly connected to the Casimir operator.

Another historically important, but singular aspect of electro-magnetism has to be quoted here : it admits a true four dimensional representation, which means that it can fully be represented in our space-time that is four dimensional, but for other reasons. The successes of corresponding theories have therefore sustained the idea that this space-time should contain everything that concerns physics : this is the egocentric postulate.

The correct way to approach electromagnetism in our context is to see it as associated with a two dimensional Cartan subalgebra (forgetting it is only semi simple), which means a two dimensional space-time. The exchange of perpendicular roots (that defines $\mathbf{so}_{1,3}$ from $\mathbf{so}_4$) defines a rotation on the compact form that expresses the charge in a two dimensional context : it is in fact the two dimensional version of Dirac equation. Let $\lambda_{1,2}$ be the generator of this rotation. It easy to check (for instance by using the complex representation) that the associated (free) Lagrangian is given by $\lambda_{1,2}^2$. But, as we have said, Lagrangians that comes from the Killing form are invariant under the action of the Weyl group, that in our degenerated case corresponds to the group S_4. The true Lagrangian is therefore $\sum_{i,j=1,4_{i\neq j}} \lambda_{i,j}^2$, that corresponds to the usual one with appropriate choices of units.

Finally, let us notice that, essentially for simplicity of notations, we have presented here all the elements in the non quantized situation. They are easy to transpose to the quantized one.

8. **The passing of time and the definition of physical observers**

The change to $F_h(\mathbf{G})$ induced by the point of view functor has a second fundamental consequence.

Beginning with the $\mathbf{SU}_2$ case, $F_h(\mathbf{SU}_2)$ admits, beside one dimensional representations, a family π_t (indexed by its maximal torus T_1) of infinite dimensional representations on the space of formal polynomials with one variable that may be seen, through formal characters, as the space of finite dimensional representations of $\mathbf{SU}_2$ (with highest weight the degree of the polynomial). Let us emphasize that this representation π_t does not appear in the non quantized case, and that, considered as a quantization of a Poisson-Lie group, it is associated with the Weyl symmetry w of $\mathbf{SL}_2(\mathbb{C})$ that exchanges the compact and non compact roots (associated with $\mathbf{SU}_{1,1}$) of its realified.

More precisely, those polynomials that correspond to $\mathbf{SL}_2(\mathbb{C})$ are polynomial in $(x + \frac{1}{x})$, the one that corresponds to the standard two-dimensional representation V ; $(x+\frac{1}{x})^n$ corresponds to $V^{\otimes n}$ and $x^n+\frac{1}{x^n}$ to the representation with maximal weight n ; π_t has thus to be seen as acting on the polynomials in the variable[9] $X = (x + \frac{1}{x})$, an operation that is associated with acting on tensorial powers $V^{\otimes n}$ of V.

It is easy to check[10] that, with the usual notation $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ for the linear forms that define $F_h(\mathbf{G})$, a reduces the degree by 1, while d increases it also by 1, and b and c leave this degree unchanged. Furthermore, from the defining relations of $F_h(\mathbf{G})$ (seen as morphisms of the q-plan with $q = e^{h/2}$), the action of the commutator $[a, d]$ splits into a sum of multiples of bc, cb and the identity, all three that keep the degree of the polynomial unchanged : considered as acting on the space of representations, $[a, d]$ is therefore, up to a scaling, equivalent to the identity.

Since we have seen that maximal weights define the positions of the objects, *the action of the linear form d may be seen as a move to the right on the one-dimensional space-time induced by taking the tensorial product by V, which "creates" a new instant after the last one, and "moves" to the right all the others. The action of the linear form a is reciprocal, inducing a move to the left and annihilating the first instant. We get thus creation/annihilation operators that apply directly to time.*

More precisely, in our case of $\mathbf{su}_2 \oplus \mathbf{su}_{1,1}$ that may be seen as corresponding to "a pure time" (it is in fact the time of an electron), d corresponds to a "creation operator" of new (discrete) instants that induces an increasing move of all the past instants on the time line : this is exactly a (discretely) mobile point on the time line ; a has exactly the opposite effect.

The sum $qa + q^{-1}d$, that is the dual of the quantized identity K_q of the q-plan, induces therefore a double motion : the first one that corresponds to the first coordinate in the q-plan, may be seen as the motion of a fixed frame expressed in a moving one, the second as a motion of a moving frame expressed in a fixed one. Exchanging left and right actions exchanges those two situations as K_q and $K_{q^{-1}}$, which is perfectly consistent. $L_q = \frac{K_q - K_q^{-1}}{q - q^{-1}}$ that quantized the usual generator of the Cartan subalgebra has an analog action.

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[9] We use here the standard representation of the algebra in order to get a one unit progression. The same reasoning applied to the adjoint representation is more physical since it does not exchange at each step bosonic and fermionic representations. To use the square of X instead of X gives the transposition.

[10] We give a complete expression of those representations in section II. References are in [V.C.-A.P.], pp. 435-439..

In that sense, in this simple case, the passing of time may be seen as a relativistic phenomenon induced by the point of view functor applied by an observer belonging to the space generated by K_q and K_q^{-1} on himself. Using Drinfeld-Jimbo QUE, the same reasoning applies to the dual of the usual quantized Cartan generator.

It is interesting to notice that making a tensor product by V is equivalent to applying the point of view functor from the point of view of V^* (seen as a representation of $\mathbf{sl}_2(\mathbb{C})$, thus differing as a $\mathbf{sl}_2(\mathbb{C})$-module,from V by the application of the antipode S_h).

By identifying V and V^* through the symplectic invariant form, one sees that the above operation of tensoring by V is equivalent to applying the usual algebraic flip that exchanges left and right operations, the antipode and the contravariant hom functor : the present instant appears so as a (reversed) point of view from the last past instant on always the same $\mathbf{sl}_2$-type particle at the same position V. One therefore can see the above equivalent of a moving frame as defined by successive applications of the point of view functor on *the same particle at the position V, that is thus seen as non moving from its own point of view.*

Coming now to the general case $F_h(\mathbf{G})$, the existence of a surjective homomorphism (that comes by passing to the dual from the principal embedding[11] *of $U_h(\mathbf{sl}_2(\mathbb{C}))$ into $U_h(\mathbf{g})$) from $F_h(\mathbf{G})$ to $F_h(\mathbf{SL}_2(\mathbb{C}))$ canonically associated with any choice of a Weyl chamber allows us to generalize the above construction after any choice of an element of the maximal torus and an element of a Weyl chamber, which defines an operator $\pi_{t,w}$.*

Any such operator $\pi_{t,w}$ defines as above ***the proper time*** *associated with any $\mathbf{g}$-type particle that takes place in the space of successive powers $V^{\otimes n}$ of the standard representation V. The dependence in this definition on the choice of an element of the Weyl group (and thus a Weyl chamber) justifies what we have said above on the action of this group as changing the "flavor" of particles.*

Furthermore, the indexation of those representations by the maximal torus corresponds to the above gauge invariance groups, the complete one $\mathbf{SU}(r)$ and the observed one $\mathbf{SU}(d)$. The application of this last

[11] The choice of the principal embedding is the only consistent one for a given a Lie algebra after choosing a Cartan subalgebra, a set of roots and an ordering of them (that defines the Weyl chamber). See for instance [A.L.O.-E.B.V.], pp. 193-203.

group may be seen as transferring the time direction in any other one in the Weyl chamber : this corresponds to what we have called above a generalized Lorentz transformation. *This gives the possibility to transpose the above defined proper time direction in any other one in the Weyl chamber extending correlatively the above definition from time to space-time.*

A ***physical observer*** *may be now defined as the combination of :*

- *a real type* $\mathbf{g}^R$, which means a complex simple Lie algebra $\mathbf{g}$ characterized by its compact form $\mathbf{g}^c$ and an involutive transformation
- *a position defined by a maximal weight* λ_0 *in* $\mathbf{h}^*$ the dual of the Cartan subalgebra of $\mathbf{g}$,
- *three elements that characterize the above principal embedding of* $U_h(\mathbf{sl}_2)$ *into* $U_h(\mathbf{g})$, namely :
  - *the choice of a copy of the Weyl chamber* by reference to a first one arbitrary chosen : this is equivalent to the choice of an element of the Weyl group (or to the choice of one of the$|W|$ copies of the maximal torus by pulling back as above explained to the compact form) ;
  - *the choice of an element* ν *in the Weyl chamber* (or t in the above chosen sheet of the maximal torus by pulling back v to the compact form) by reference to a first one arbitrary chosen ; this is equivalent to *the choice of an as above defined generalized Lorentz transformation* that applies to an intially chosen roots diagram in order to identify the direction defined by ν with the one of the principal embedding of $\mathbf{sl}_2$;
  - *the choice of a free Lagrangian* $\mu^2 = (m_0 + iq_0)$. m_0 defines the embedding of the unfolded part of the realified of $\mathbf{sl}_2$, while q_0 the compact one. By convention, we take $m_0 > 0$ for a physical observer, and we will say that $-\mu$ defines the corresponding anti-observer. m_0 will be called **the mass** of the physical observer and q_0 its **charge**.
    Since m_0 *corresponds to a scaling factor applied to the dual of the algebra* $\mathbf{sl}_2$ *that contains the dual* $\mathbf{h}^*$ *of its Cartan subalgebra, it defines a scaling of* $\mathbf{h}^*$ *and thus the density relative to the dual of the Killing form of integer values, which means successive finite dimensional representations. In that sense,* m_0 *defines the speed at which the time creation operator generates new instants. The higher it is, the closer (in the sense of*

the dual Killing form) are the successively generated instants, which is simply the generalization of the Einstein-Planck relations.

The existence of a mass (and a charge) associated with any physical observer is no more a mysterious property of nature, nor something that particles take out from a mysterious ambient field in order to "slow" them : it is the purely logical consequence of two mathematical elementary facts, namely the passing of time induced by the point of view functor as above explained in $F_h(\mathbf{SL}_2)$*, and the existence of a uniquely defined (in a given root ordering) principal embedding of* $\mathbf{sl}_2$ *in any simple Lie algebra* $\mathbf{g}$.

Let us emphasize that, by the main property that led us to their definition, *Lagrangians are invariant under the action of generalized Lorentz transformations that generalized usual changes of point of view of physical observers. This means that all the physical observers that differ only through such transformations will share the same Lagrangians.*

In usual context of physics, this explains the universality of Lagrangians. In our context, this explains first why we perceive an "existing universe" (and therefore why physics has been so trapped by the egocentric postulate). *Indeed, as being (up to "flavors" that appear like generating different particles) associated with the same Weyl chambers of the same algebras, our particles share the same Lagrangians despite their relative moves, and therefore appear as sharing the same universe. This universe is thus seen as independent of each of them and can be easily taken as the general scene where the whole story is played.*

Another very important fact has to be emphasized here : *there is an elegant way to express* μ *in a Lagrangian context : since* μ^2 *is proportional to the Lagrangian and* μ *defines the above "speed"* $\dot{z}$ *of the "passing of time" of this observer,* $\frac{\partial \mathcal{L}}{\partial \dot{z}}$ *is proportional to* μ*. The right–hand side of Euler-Lagrange equations will simply be the expression of that fact in a context taking also into account the principal embedding of* $\mathbf{sl}_2$ *in* $\mathbf{g}$.

Let us notice finally that the creation/annihilation operators of a type of particles seen as physical observers appear as the creation/annihilation operators of their proper time. Since, in ART, space-time is defined from particles, space is naturally expending while time is passing. *Therefore, the so-called second quantization corresponds exactly to the quantization of the time, as the first one to the quantization of energy.*

It induces a quantized structure of time and correlatively of space-time as we will see now, beginning as usual with the one dimensional case.

9. **The structure of the time of a physical observer**

In order to explore the structure of space-time from the point of view of a physical observer, one sees from the above construction of $F_h(\mathbf{g})$ representations, that we have in fact to describe the situation for the pure time of a physical observer of type $\mathbf{su}_{1,1} \oplus \mathbf{su}_{1,1}$ (i.e. an electronic-type observer), then to extend it to a space-time by inserting it in the Weyl chamber of a bigger algebra $\mathbf{g}$ and finally to apply all the possible generalized Lorentz transformations.

Let us thus consider an electronic observer at the position N, which means it is associated with the space $V^{\otimes N}$. This space splits into irreducible representations ; *the biggest one V_N corresponds to the weight N, with multiplicity* 1 *and may be seen as* ***the actual position of the observer***. The other ones have multiplicities given by the Newton coefficients in $(x+\frac{1}{x})^N$. So defined and split, $V^{\otimes N}$ may be seen as the **universe from the point of view of the considered observer at the instant** N.

In this construction, the universe at the instant $N_1 < N$ was isomorphic to $V^{\otimes N_1}$ that is *not* the restriction to representations of weights less than or equal to N_1 of $V^{\otimes N}$, which corresponds to the past instants *as seen from the present.* This means that, as far as time itself is concerned, an observer cannot access to the past without being influenced by $N - N_1$ successive applications of the above mix of the flip and antipode operators combined with the point of view functor.

Since the situation with $\mathbf{su}_{1,1}$ can, as above, easily be transferred to the quantized case and to any other simple real Lie algebra $\mathbf{g}$, this means that *the past universe as seen from the present is not the past universe as it was when it was the present.*

In other words, when we try to access the past through some experimental processes (for instance by long distance astronomical observations), *we always access to representations of the past as seen from the present ($V^{\otimes N}$) and not as it was from the points of view of successive instants (which leads to work on $V_1 \otimes ... \otimes V_m$).* Let us emphasize that by dualizing to Hilbert spaces of function as in QFT, we get in the first case usual the Fock spaces of QFT while in the second case, we have to represent the fact that past instants were not exactly the same as present ones.

We will call the past as it is seen from the present as **the ghost past**, and the past as it was **the true past**. Since ART defines space-time from observables, the notion of space-time *from the point of view of a physical observer* is defined by some $V^{\otimes N}$ with V the standard representation of the Lie algebra that characterizes the type of the observer. In the above sense, space-time is thus a "ghost space-time", and, since a physical observer "knows" only his own point of view, it is doubtful that the notion of "a true space-time" from his point of view could even make sense.

Nevertheless, both "past" are interesting from an experimental point of view, since the study of the ghost past predicts what we perceive from the past, and the study of the true past gives access to the true story allowing us to compute or predict physical data that can also be compared with measured values, which increases therefore the falsifiability of the theory.

In particular, we will see in our last section that it is theoretically possible, *by representing the true past*, to compute the characteristics of all the particles and interactions from our actual point of view. The same set of algebraic relations implies that all the computations can also be done *in the ghost past that defines what we perceive today*, which means that *all those characteristics paradoxically appear as having been constant, despite their true variations in the true past.*

We propose to call this law, inconceivable with the usual vision of an *existing* space-time, *t**he ghost constancy law of constants** : it means the numerous contemporary observations that seem to show that usual constants of physics are actually constant, are in fact biased by the fact that the models used for those studies do not represent the difference between the true past and the ghost past.*

In other words, as no observer can hope to determine directly his own motion by looking to objects attached to himself, nobody can hope to determine directly if the constants of physics are really constant without taking into account and compensating the fact that all our past is in some sense moving with us.

We are now quite far from the usual vision of a space-time as a smooth preexisting Lorentzian manifold. In particular, the above process of creation in a quantized way of "new instants" for all the physical observers in all the space-time exclude to use any preexisting differential structure. In fact, the only solid tool we have at our disposal in order to represent space-time from the point of view of a physical observer is using the creation operator itself applied to different (and different types of) physical observers.

We will proceed by successive steps beginning with, as usual, the $\mathbf{sl}_2$ case.

We know that the t-indexed representations $\pi_t(F_h(\mathbf{SL}_2(\mathbb{C})))$ induce the move on representations of $U_h(\mathbf{sl}_2(\mathbb{C}))$ that corresponds to the passing of time while the group of rotations $\mathbf{SU(1)}$ generated by the standard Cartan element of $\mathbf{su}_2$ acts on the torus T that indexes the representations π_t.We have to deal with this complex situation in a canonical way that will allow us to generalize our results to other algebras. The idea is to see the free linear space generated by the set of irreducible representations as **a configurations space**[12] with N dimensions and to quotient by the symmetric group S_N in order to have all the representations at the same point P.

It is then possible to define an operation of parallel transport of the 2 -tensors algebra $\mathbf{su}_1 \otimes \mathbf{su}_1$, that contains the above Lagrangian, according to Knizhnik-Zamolodchikov equation (KZ equation) associated with some 2-tensor $\mathbf{r}_{ij}$.

This operation is consistent, i.e. defines an appropriate flat connection allowing us to define a monodromy group above P, if and only if $\mathbf{r}$ satisfies the classical Yang-Baxter (YB) equation with spectral parameters (which corresponds to a "commutative diagram"-type consistency condition for the dodecagon diagram associated with an operator of this type).

We have now to determine the 2-tensor $\mathbf{r}_{ij}$ that corresponds to the passing of time of a physical observer. Since we have seen that the above defined creation/annihilation operator that induces the passing of time is the (quantized) identity which corresponds through the Killing form to the Casimir operator, this 2-tensor has to be build from the canonical polarization of this operator, namely the canonical 2-tensor $\mathbf{t}_{ij}$ defined by :

$$\mathbf{t}_{V,W} = \tfrac{1}{2}(C_{V\otimes W} - C_V \otimes Id_V - Id_V \otimes C_W).$$

This tensor is invariant by the gauge transformations, therefore its use is compatible with the categorial fondations of the ART.

[12]See for instance [C.K.] pp.451-455 for exact definitions, and then pp. 455-479 for the KZ equation and Kohno-Drinfeld theorem. We refer also to [V.C.-A.P.] pp. 537-550 for the connection with Yang-Baxter equation.

In order to satisfy YB consistency condition, we have then to set :

$$r(z_1 - z_2) = \frac{\mathbf{t}}{z_1 - z_2}.$$

KZ equation may then be written for a function f with values *in the true past* $V_1 \otimes ... \otimes V_N$:

$$\frac{\partial f}{\partial z_j} = \frac{h}{2i\pi} \sum_{k=1_{k \neq j}}^{N} \frac{\mathbf{t}_{jk}}{z_j - z_k} f,$$

where the formal coefficient is introduced only in order to facilitate the below formulation of Drinfeld-Kohno theorem.

Let us emphasize that, since we are in fact working on the realified of $\mathbf{sl}_2(\mathbb{C})$, the choice of the identity map that corresponds to four applications of the rotation of roots induced by the outer automorphism of $\mathbf{so}_4(\mathbb{C})$, and therefore of the Casimir operator, is perfectly coherent with the (free) Lagrangian associated with the electromagnetic interaction that we have above defined. This justifies our choice of the definition of the electron and will ensure that in the ART, like in the QFT, it will be possible to consider the charge as an imaginary mass justifying the above given definitions of those quantities.

On another hand, we know that the Hopf algebra structure of $U_h(\mathbf{su}_1)$ defines on each representation V a canonical bilinear element of $\mathbf{su}_1^* \otimes \mathbf{su}_1^*$ that corresponds to the action of the Casimir operator on this representation, which is a scaling by the coefficient $C_2(V)$. It is given for the representation of $\mathbf{sl}_2(\mathbb{C})$ with maximal weight N by the formula $C_2(N) = (N+1)^2 - 1$. This coefficient is also well known for other algebras in such a way that the transposition to other algebras of our construction will be possible.

We can thus attach to any irreducible representation V an element of $\mathbf{su}_1 \otimes \mathbf{su}_1$ defined by $\frac{1}{C_2(V)}$.

Therefore, the change from representation N to $N+1$ induces a change of this canonical element of $\mathbf{su}_1 \otimes \mathbf{su}_1$ according to the ratio $\frac{C_2(N)}{C_2(N+1)}$.

The key point is the following : we may apply to this element the closed parallel transport that defines the KZ monodromy getting a coefficient given by $e^{h\mathbf{t}/2}$ *with* $\mathbf{t}$ *the above canonical* 2*-tensor and* h *the formal* *coefficient in KZ equation. We can therefore choose*[13] *a specialization of the coefficient h in Drinfeld-Jimbo algebra such that this coefficient corresponds to the ratio* $\frac{C_2(N)}{C_2(N+1)}$. *We get a Lagrangian (that in this simple case is reduced to its free part) that by parallel transport in the configuration space is a continuous function from one representation to the next one.*

In an informal way, the "jump" of Lagrangian that is induced by the change of intrinsic form at each change of representation can be "absorbed" by the R-matrix effect of the flip that, as above explained, corresponds to the successive applications of the point of view functor.

This means that when the passing of time induces a move from the position N to $N+1$, the quadratic form that has to be applied to define the corresponding variation of the scalar coefficient in $U_h(\mathbf{sl}_2)$ is$\frac{k^2}{C_2(N)}$ for some fixed positive coefficient k. Therefore, the increase of this scalar coefficient that corresponds to the passage from N to $N+1$ is $u_n = \frac{1}{\sqrt{C_2(N)}}$.

Finally, the scalar coefficient after N steps as seen from the point of view of a physical observer, i.e. *as pulled back from its dual after application of the point of view functor*, is given by the corresponding series $S_N = \sum_{n=1}^{N} u_n$.

Now, in order to be able to represent a physical observer as stable on the maximal torus, we have to normalize the bilinear forms we use in such a way that, on the adjoint representation, where we know that the action on $\mathbf{h}^*$ goes on a 2 by 2 basis and the action on the torus has a π-periodicity. *We have thus to normalize the Killing form in such a way that the image of* π *is* 2 *which gives a coefficient* $\frac{2}{\pi}$ *by reference to a standard orthonormal initial choice. We will refer to this quadratic form as t**he normalized quadratic form**. It is the one that should be used in the computations of the MQT since it induces the stability of any of the t-indexed representations of* $F_h(\mathbf{SU}_2)$ *that define the passing of the time from the point of view of the considered observer.*

Correlatively, we get a well defined sequence h_i of values of h that we will call **the h-sequence**. *Since we have seen that Lagrangians*

[13]To be precise, we need to work with h a transcendental nmber in order to respect the h-adic structure of Drinfeld-Jimbo algebras. We do not adress this question here since we can replace the above choice by arbtrary close transcendental values getting only a function arbitrary close to a continuous function which is enough for the computations we intend to propose.

characterize the outer automorphisms of the simple Lie algebras, using an h-sequence instead of a fixed value of h is fundamental : otherwise, there would be small jumps in the Lagrangians that do not correspond to any true physical phenomenon and that, correlatively, would lead to the introduction of many corrective actions.

Let us emphasize furthermore that the structure of the time of a physical observer arises from this monodromic approach that corresponds to a topological braided quasi-bialgebra usually called $A_{\mathbf{sl}_2,\mathbf{t}}$ *that is not coassociative, but only quasi-coassociative, and has the same product and coproduct than the non deformed universal enveloping algebra.* It has thus the same algebraic left-right symmetry than this latter algebra, which is a huge advantage, but thinking the composition of duration as non associative is quite difficult, and in any case not currently used by physics.

That is the reason why we have chosen to begin by working on Drinfeld-Jimbo algebras that are coassociative. *Furthermore, since by Drinfeld-Kohno theorem*[14]*, for any semi-simple Lie algebra* $\mathbf{g}$ *there exist a gauge transformation* F *and a* $\mathbb{C}[[h]]$*-linear isomorphism* α *that links* $U_h(\mathbf{g})$ *to* $(A_{\mathbf{g},\mathbf{t}})_F$*, the representations of both quasi-bialgebras define the same categorial structure, which ensures the consistency of our whole construction that began precisely by those categorial monoidal structures.*

There is nevertheless a price to pay to go from the one to the other : we transform quasi-associativity in associativity, but we loose the algebraic left-right invariance of $A_{\mathbf{g},\mathbf{t}}$*. We will refer to this asymmetry as* ***the asymmetry of the time*** *of any physical observer.*

From a physical point of view, it is the counterpart of having an associative law for durations, and any precise calculation should be made either on $U_h(\mathbf{g})$ *taking into account this asymmetry of time, or on* $A_{\mathbf{g},\mathbf{t}}$*, but taking into account Drinfeld associator that involves iterated integrals and Riemann* ζ *function.*

Nevertheless, in this introducing paper, we will compute the Anomalous Magnetic Moment of electron in the algebra $U_h(\mathbf{sl}_2)$ *without compensating the asymmetry of its proper time, and use the difference with experimental results to access directly to an evaluation of this asymmetry.*

Let us come back finally to "ghost" space-time. It is easy to understand where does the difference come from with true space-time we have

[14] See for instance [C.K.] page 460.

just described : the variations of the specializations of h defined by the above defined h-sequence that are necessary to have a continuous Lagrangian are not taken into account since all the KZ story is told on the same (initial) representation.

Unfortunately, contemporary physics seems to work exclusively on the "ghost" space-time from our point of view. This has tremendous consequences since it forbids physics from accessing to its unification with the understanding of the nature gravitation and to the Mass Quantification Theory as we will see it below. Let us before come back to Euler-Lagrange equations and their interpretation.

10. **Recovering Euler-Lagrange equations**

    We have already given in our subsection 1.3 above the main ideas in order to understand the origin of Euler-Lagrange equations. So we only give here some complementary elements :

    - We have seen that, if one thinks, in any given direction the operator $\frac{\partial}{\partial \dot{z}}$ applied to the Lagrangian gives its free part that corresponds to a ratio of scaling in the corresponding direction. We are thus always working on multiples of $1 \otimes 1$. Since the Casimir operator, and correlatively the canonical 2-tensor makes successively an action and its dual on anyone of the directions of the considered Lie algebra, those variations are simply added if one uses an orthogonal basis of the Lie algebra. But, since the canonical 2-tensor $\mathbf{t}$ and KZ equations are defined intrinsically, and therefore independently of the choice of any basis, the action of $\mathbf{t}$ makes it possible to extend our way of reasoning in all the directions to be considered.
    - Remembering that any universal enveloping algebra of a Lie algebra $\mathbf{g}$ of simply connected Lie group $\mathbf{G}$ can be seen[15] as the algebra $D(\mathbf{G})$ of operators on $\mathcal{C}^{\infty}(\mathbf{G})$ generated by the identity and all the left invariant vector fields on $\mathbf{G}$. Let us apply this to the group $\mathbf{SU}(d)$ (or $\mathbf{SU}(r)$ on $\mathbb{C}$). The element of the universal enveloping algebra are now represented by partial derivative operators that correspond to the usual expression of the right-hand side of Euler-Lagrange equations.

11. **The unification of physics, the gravitation and a conjecture on dark energy**

[15]See for instance[S.H.], pp. 107-108.

As for the preceding step, we have already given in subsection 1.3 the main aspects of this part, except as gravitation is concerned. Therefore, we give only here some complements and precisions.

- As far the unification of physics is concerned, let us first notice that despite the name we propose, ART is clearly a quantized theory. Let us emphasize that quantizations in ART comes from the contravariance of the point of view functor since it obliges in the beginning to work on bialgebras of which primitive part are well classified Lie algebras, and since it gives rise to the fundamental above creation/annihilation operators that sustain all the construction. In particular, we did not introduce any physical hypothesis to get such a quantization.

  On another hand, we do not present here any formal link with usual branches of Quantum Theories. But, uncertainty principle for instance is easy to deduce from the fact that a physical observer is only spectrally defined despite the continuous view on itself obtained from KZ-parallel transport. In the same way, the fact that exchanging algebraic right and left (i.e. changing to the linear but not algebraic dual in non deformed algebras) exchanges (roughly) time and energy directions should lead to derive easily usual "first" quantization formulations that concern energy-momentum. We have also sketched the way to get Dirac equation from the ART context.

  There are also many connections with QFT we have quoted , but we did not try to achieve formally the construction of a way to derive QFT from ART.

- Far more interesting is the way we sketched in 1.3 in order to derive Einstein equation from ART. The left hand of this equation is purely mathematical, and directly results from the choice made by Einstein to use a Lorentzian manifold in order to represent space-time. One could therefore say that to derive from quantics a tensor that can be identified with usual Lagrangian achieve the unification of physics since the right-hand side of the Einstein equation is considered as given.

  In fact we have had a little more. Since the variations from some given point of our Lagrangians arise as a well defined KZ monodromy of the symmetric part of an algebra of automorphisms of the Cartan subalgebra that represents space-time in our context, and since the Ricci-curvature tensor is its exact equivalent in a

Lorentzian manifold context, both sides of Einstein equation have now exactly the same mathematical status. Using, as Einstein did, Euler-Lagrange equations for the representation of the physical universe in the right side of Einstein equation appears now as an useless detour : according to Einstein's hope, the true nature of his equation is mathematical.

- As far as gravitation is concerned, let us first notice that the KZ monodromy that we have described in order to define space-time from the point of view of an observer has, from any representation V to another one, an action on $\mathbf{su}_n$ and thus a Lagrangian. As it comes from an $\mathbf{sl}_2$-embedding effect, this Lagrangian has only a free part, and his square root may be seen as defining a scaling of the Cartan subalgebra itself. We have already analyzed this effect that corresponds, from the definition of the KZ-parallel transport, to the change of the intrinsic quadratic form associated with the change of the value of he Casimir coefficient. This leads directly to the computation of the Newton constant presented in section III.
- Coming back to Einstein equation, the fact that the gravitation arises from a passing of time phenomenon that in any case defines a Lagrangian, leads to conjecture that the right-hand side of Einstein equation would not be modified by considering particles with an associated space-time of only two- or three- dimension. But, the left-hand side would be modified by the change of the unitary substraction that gives the projection on the pure Ricci component in the space of Riemann curvature tensors. The law of decrease of the gravitation would correlatively be slower (with a logarithmic potential in dimension 3, instead $\frac{1}{r}$ in dimension four). This phenomenon is probably not purely a theoretical one. Indeed, since, as we will see in the MQT part, protons and neutrons because they are of type g_2 that as the normal form of G_2 has a rank of 2, cannot be described by small balls in space but by small bars in their own spacetime. Therefore, a concentration of them in one or two spatial dimensions if they are rotating seems to be conceivable and could generate a gravitational field equivalent to the existence of some “dark matter”. This remains nevertheless for us only a conjecture.
- Finally, let us remind the conjecture in 1.3 about “dark energy”. If one admits that, for the aforesaid reasons, weak interaction as we perceive it, is only a “ghost effect” induced by some E_6-

type interaction, there is necessarily an (heavy) Lagrangian that corresponds to it : a very good candidate for "dark energy". Furthermore, if such particles and interactions exist their "passing of time" should generate a sequence of increasing representations that split very fast in smaller particles as the one we are made of. This leads to an alternative scenario to the Big-Bang for the cosmological preradiative era... that we did not at all explore.

12. **The Mass Quantification Theory**

The way to compute the mass and the charge of particles is to compute the free part of the associated Lagrangians, and, after taking the square root that has a positive mass (for particles), to compare with corresponding characteristics of the electron that we have chosen as a reference.

In order to compute this Lagrangian, it is first necessary to position rigorously the principal embedding of $\mathbf{sl}_2(\mathbb{C})$ that defines the direction of the proper time of the considered particle : this in fact a simple consequence of the way the creation/annihilation operator that generates passing of time is positioned relatively to $F_h(\mathbf{G})$ as above explained. Positioning the real and imaginary parts of the $\mathbf{sl}_2$ embedding has then to be made, either, if it is non ambiguous, by identification of an unfolded direction, or by making the embedding in the compact form and applying consistently the complex involution that defines the considered real form.

The most difficult part is to compute how the passing of the time of the considered particle has modified its free Lagrangian, since we do know neither this evolution, neither the number N of successive "instants" that correspond to this particle in its present proper time.

Fortunately, there is one exception : the electron that does not correspond to a simple Lie algebra, but only to a semi-simple one. Therefore, the ratio between its electromagnetic energy (that is orthogonal to its passing of time, and thus does not change) and the one that corresponds to its mass, that evolves as above explained, may be used to compute the present value of N. This gives the apparent age of the universe (in its ghost past) and permit, as aforesaid, the computation of Newton constant. As aforesaid we will also use, in this first paper, the anomalous magnetic moment of the electron in order to evaluate the impact of the asymmetry of time that should be taken into account for other computations here. By contrast with α that characterizes our cosmological position, the anomalous moment of electron should be computed from Drinfeld's associator in further developments of the theory.

The idea for the other particles is then to use this value of N to determine the impact of their passing of time. To this aim, one has first, by computing the above principal embedding and projecting on it the root that corresponds to the adjoint representation of the considered algebra (or its image through a Weyl transformation if the flavor is not the standard one), to determine the relative speed of the passing of time of the considered type of particle and compute its relative lag or advance by reference to the speed of the adjoint representation of $\mathbf{sl}_2$, and after a small conversion, relative to the one of the electron.

Once this lag or advance ratio is determined, one has to compute the impact of this lag (for instance) relative to the electron. To this aim, the idea is to consider the point of view of the electron looking to the past, which means passing to the dual and correlatively representing the annihilation operator from his point of view.

Using Yangians gives an easy way to make this computation since motions in time can be represented by translations on the evaluations representations. Now the derivation D canonically associated with the Yangian J (which is obtained by applying successively the cocommutator and the Lie bracket on the non deformed algebra) corresponds simply to $D = \frac{d}{du}$. Applying this twice,since we move in the adjoint algebra, we get the equation that has to be applied to any scaling to describe its evolution, namely :

$$\frac{d^2K}{(du)^2} = K.$$

Since the initial value is 1, and since if one forgets the asymmetry of the time, which implies a symmetric law, we get what we propose to refer as the **cosh law** : the coefficient to be applied in order to compute the actual value of a particle is (in the lag case) equal to the $cosh$ of the delay. Appropriate transformations have to be made in cases of advance.

It is theoretically possible to improve those computations by adding a small part that corresponds to the above explained asymmetry of the time, but we do not present here any such improvement.

Although everything should be representable and thus computable, all those difficulties lead us to present only some instances of such computations in this first paper. This the purpose of our third section.

It is time now to go further into details on more theoretical aspects of the ART. After those twelve steps, we come back to the foundations.

2 The mathematical foundations of the Absolute Relativity Theory

Our objective in the first section was to outline the main physical issues and perspectives of the ART. Therefore we only sketched its mathematical foundations in the first steps of our overview in subsection 1.4.

Our purpose now is to give some complementary mathematical description of those foundations. In order to facilitate an independent reading of this section, we do not refer to elements given in the first section, but present here a complete set of algebraic definitions and justifications.

2.1 The algebraic approach

Since we do not allow ourselves to postulate the existence of any space-time, the first difficulty we have to overcome is to find a way to begin with our theory.

According to Einstein's intuition quoted as epigraph, algebra soon appears as the most natural way to undertake our quest.

We then begin to determine the algebraic conditions physics has to respect to be a consistent and falsifiable science : with Kant's terminology in mind, we propose to call them ***the transcendental conditions****. Those transcendental conditions are algebraic and define a very precise framework, independent of any observer and of any observation, that also allows us to represent the perception process itself as the action of a specific contravariant functor.*

*This general algebraic framework has to be connected with "the reality" we perceive and that we suggest referring to as the set of "****true physical phenomena****".*

With this aim in view, ***we will state two and only two physical principles***

- *The first one,* ***the absolute relativity principle****, will appear as the ultimate extension of the above relativity principle.*
- *The second one,* ***the absolute equivalence principle****, will appear as the ultimate extension of Einstein's equivalence principle, and, in some way, as a reciprocal principle of the first one.*

2.1.1 The preuniverse and the point of view functor

In order to avoid beginning with any specific restriction to the theory we are trying to build, it is natural to base the foundations of our construction on the very general algebraic language of graphs. As explained in Appendix I, Popper falsifiability conditions then immediately lead us to focus on graphs that respect the axioms of *categories :* from now on, we will assume that the general framework of the theory we intend to build is a category that we will call **the preuniverse** $\aleph$. It will appear as a category of **representation functors**, namely functors between two one-object categories, each one of them equipped with morphisms that characterize its own internal structure. The morphisms in $\aleph$ will thus be morphisms of functors, i.e. **natural transformations**.

It is then natural to define in each category **the point of view of an object** a **on an object** b as the set of arrows $Hom(b, a)$; this defines **the point of view (contravariant) functor at** a, $Hom(., a)$. Now, a functor F is said to be **representable** *and* **represented by** a if there exists some object a in the target category of F such that F is equivalent to $Hom(., a)$. Thus, the point of view functor $Hom_a(., a)$ will be the functor that sends a to the set of arrows that defines its algebraic morphisms as a one object category, while $Hom_\aleph(., \rho_a)$ will be the functor that sends the representation functor ρ_a in $\aleph$ to the set of natural transformations with the functor ρ_a as target.

We sketch in Appendix I a way to show that some purely logical conditions on the theory we try to build, such as being consistent in the sense of categories and having "enough" representable functors, lead to specify a little more the preuniverse $\aleph$: *it has first to be a category of A-modules (or "representations of A") for different algebras A. Then, since each of the subcategory $\aleph_A$ associated with an specific algebra A has also to be a monoidal category for some coherent coproduct Δ_A and counit ε_A, each $(A, \Delta_A, \varepsilon)$ has in fact to be a quasi-bialgebra*[16]*. For technical reasons, we will restrict ourselves to those quasi-bialgebras that are equipped with an antipode operator, i.e. quasi-Hopf algebras.*

More precisely, in order to begin with the simplest of those algebras and to use the restricted Hopf duality, we will focus on the category of all the finite dimensional $A_h(\mathbf{g})$-modules for all the formal (h-adic topological) deformations $A_h(\mathbf{g})$ of the universal enveloping algebras $U(\mathbf{g})$ of all the finite dimensional semi-simple complex Lie algebras $\mathbf{g}$. We will mainly use the well known Drinfeld-Jimbo Quantum Universal Enveloping algebras (QUE) $U_h(\mathbf{g})$.

[16]See for instance [C.K.], pp. 368-371.

A first approach on particles and space-time Classically, those modules may be split into a direct sum of irreducible submodules indexed by maximal weights that belong to the dual $\mathbf{h}^*$ of any chosen Cartan subalgebra $\mathbf{h}$ of $\mathbf{g}$. This leads to the idea that *those irreducible modules could correspond to elementary particles, that arise, up to isomorphisms, as indexed by the elements of maximal weights included in* $\mathbf{h}^*$.

This would mean that space-time does not preexist, but is defined as a set of indexes by each type of particles, and in particular that its dimension is defined by the rank r *of the Cartan subalgebra that corresponds to their type.*

Now, since the lattice of maximal weights is linearly isomorphic to $(\mathbb{Z})^r$, *any choice of a Cartan subalgebra induces a breach of symmetry in the complex algebra* $\mathbf{h}^*$ *that defines a specific embedding of* $\mathbb{R}$ *in* $\mathbb{C}$. Since everything in a theory is done up to an isomorphism, we will assume in all what follows that such a choice is made for the biggest algebra we will have to consider, namely, the algebra E_6, and, except when otherwise specified, we will always consider that the Cartan subalgebra associated with any subalgebra of E_6 is chosen as the restriction of the one of E_6. We will apply the same rule for the linear form that defines the ordering of the weights.

Since the above definition of space-time induces a specific embedding of $\mathbb{R}$ in $\mathbb{C}$, we will have to work on real forms of Lie algebras. Each of those algebras has a real rank d that defines what we suggest calling its **unfolded dimension**, and a complex rank r, the difference $r - d$ corresponding therefore to **folded dimensions**.

Therefore making the definition of space-time dependant of each type of particles instead of postulating them in an *a priori* given space-time independant of them should make us able to understand why our space-time does appear as four-dimensional.

Let us define **an elementary particle of type g at the position** V as the irreducible representation functor of $U_h(\mathbf{g})$ on the one object category V, itself generally characterized by a maximal weight λ_V.

For instance, the electron will be defined as the realified of the algebra $\mathbf{sl}_2(\mathbb{C})$ *that has a two dimensional real Cartan subalgebra. Then the space-time of the electron will appear as only one-dimensional since the compact root will correspond to an unfolded dimension. We will see that our usual time is in fact the time of the electron, and that it arises algebraically equipped with a both spectral and continuous structure, far away from the copy of* $\mathbb{R}$ *that usually represents the time in contemporary physics.* Any bigger algebra $\mathbf{g}$ with real rank d should in the same way generate a d-dimensional space-time.

Concerning real forms of Lie algebras, let us remind[17] that each of them is defined as an invariant *conjugate linear* involution of the corresponding complex algebra. The compact real form $\mathbf{g}^c$ is unique up to isomorphisms and may be used to define all the other real forms of $\mathbf{g}$: namely each one can be obtained by applying an invariant *linear complex* involutive isomorphism θ (that can clearly not be a real one) to the compact form. Furthermore, each real form is characterized by the way θ acts on its primitive roots, which can be summarized in *a Satake diagram.*

Finally, let us notice that Algebraically replacing the usual left action by the right one defines a second way to go from a set equipped with an algebraic structure to a one-object category : we will say that those two ways are **left-right conjugates**, and refer to the algebraic operation that exchanges algebraic left and right actions as the **left-right conjugation** that will play a very special role in the absolute relativity theory. Except when otherwise specified, the left convention will always be used.

This conjugation that corresponds to the change to **opposite** algebras (symbol *op*) or coalgebras (symbol *cop*) generally corresponds to the switch from one representation to *the linear dual representation.*

The dual category $\aleph^*$ *The left-right conjugation that leads to the use of the linear duality has to be clearly distinguished from the algebraic operation of "reversing the arrows" that is involved by the contravariance of the point of view functor : in the latter case, everything is replaced by its dual, which means, in the case of the above bialgebras, that coproducts are turned into products and (with the restricted dual notion) products into coproducts.*

As usual, we will note the restricted dual of the algebra $U_h(\mathbf{g})$ by $F_h(\mathbf{G})$. We will work on the dual algebra to describe and explore the properties of the coalgebras and comodule structures of the bialgebras and bimodules structures involved in our construction. $F_h(\mathbf{G})$ is classically a quantization of the space $F(\mathbf{G})$ of complex valued functions on the simply connected Lie group $\mathbf{G}$, and Tanaka-Krein duality allows us to see this group as the dual of $F(\mathbf{G})$ seeing $U_h(\mathbf{g})$ as the quantum group associated with $\mathbf{G}$.

The contravariance of the point of view functor will make $F_h(\mathbf{G})$ play a very central part. *Their representations are indexed by elements of the Cartan subalgebra the duali of which is seen above as a support of the space-time.*

Furthermore, we some of those representations are defined on the space of one-dimension formal polynomials, a space which is isomorphic to the space of the representations of the algebra $\mathbf{su}_{1,1}$*, the non compact component of the*

[17] See for instance [A.L.O.-E.B.V.],, chapter 4, sections 1 to 4, pp.127-162.

realified of $\mathbf{sl}_2(\mathbb{C})$, *the algebra we already guessed characterizing the electron. Now this algebra has a one-dimension non compact Cartan subalgebra, a natural support for a one dimensional space-time, i.e. a time. The change to the dual induced by the contravariance of the point of view functor leads to associate the electron with this polynomial representation that, as we have seen in section I, appears to be the origin of the passing of time. Our time therefore corresponds to the time of electrons (and thus of electromagnetism).*

Finally, having renounced to postulate any specific structure for the space-time leads us to represent it with a far more sophisticated structure than anyone we could have *a priori* postulated.

2.1.2 The ART two physical principles

At this point, *the preuniverse is nevertheless a pure mathematical construction that needs to be linked to physics through the representation of the phenomena we perceive.*

The refutation of the egocentric postulate gives here its first dividend : since the structure of the preuniverse is not breached by any specific point of view, one can guess that *up to the application of the point of view functor, any natural transformation in the preuniverse should correspond to a physical phenomenon, and, reciprocally, that any physical phenomenon should correspond to a natural transformation in this preuniverse.*

The above point of view functor would then associate any **g** type particle at the position V with all the true physical phenomena that have an action on it, which corresponds to a natural representation of the usual concept of "the perceived physical phenomena". Finally, **physical laws** should be defined as families of natural transformations defined in some universal way.

Those two reciprocal ideas are expressed respectively by the absolute relativity principle and by the absolute equivalence principle.

The absolute relativity principle Let us notice first that the contemporary way to express the old idea of "formal invariance of physical laws under a change of observers" is to say that such a change has to be defined by a natural transformation (the adjoint action of Galileo or Poincaré groups for instance).

Now, the (classical) relativity principle defines a family of natural transformations that link together Galilean observers, which effectively induces a true physical phenomenon : the conservation of the energy-momentum. In the same way, Einstein's relativity principle induces the equivalence of Lorentzian observers, which implies also a true physical phenomenon : this is Einstein's famous equivalence between energy and mass.

From those two examples one sees that the first of the above guesses generalizes the idea that *the compatibility of a natural transformation (like a change of observer) with the theory is significant of a true physical phenomenon* : we therefore refer it as **the absolute relativity principle (ARP)** *: any natural transformation in the theory represents a true physical phenomenon.*

The absolute equivalence principle With his equivalence principle, Einstein went in the reciprocal direction by *inducing*, from the numerical identity of the gravitational mass and the inertial one, that the gravitation, *as a true physical phenomenon*, can be cancelled by extending the category of observers to accelerated ones. This means that gravitation may also be represented by natural transformations in this extended category.

Mathematically, this assertion has a direct consequence on the structure of the space-time that has to be a differential manifold equipped with a connection (necessarily Lorentzian in order to respect locally special relativity theory).

This equivalence principle that "cancels" a physical phenomenon by applying a natural transformation that embeds the category of Lorentzian observers in a bigger one (or equivalently the category of affine spaces in the bigger one of affine manifolds), we will be refered to **the absolute equivalence principle (AEP)** : *any true physical phenomenon may be represented by a natural transformation of the theory.*

The structure of the absolute relativity theory *Basically, taken together, the ARP and the AEP splits the representation of the true physical phenomena into two parts :*

1. *the description and classification of the representations functors and natural transformations that are the objects and morphisms of the pre-universe* $\aleph$,

2. *the analysis of the impact of the point of view functor on those objects and morphisms, which leads to work simultaneously on* $\aleph$ *and* $\aleph^*$*.*

By classifying all those representation functors and natural transformations between them, and combining this with the application of the point of view functor, we should thus get a natural classification of the physical particles and physical phenomena, which corresponds to the unification of all particles and interactions. ***.*** *As we have seen in section I, this physical unification comes with the theoretical one of the two main branches of contemporary physics.*

As mentioned above, this also leads to the first Mass Quantification Theory (MQT) that should explain the existence, the nomenclature and the characteristics of the particles already identified by physics, and also predict the characteristics of some new ones.

Cutting the Gordian knot of space–time opens thus a new road to new landscapes we are now ready to explore.

So, let us now begin with the first of the two steps above.

2.2 A first classification of true physical phenomena

2.2.1 The fundamental structure of the preuniverse ℵ

The preuniverse ℵ may be seen as "encapsulating" three levels of "arrows" and universal constructions.

The first level encapsulates the basic transcendental conditions presented in Appendix I and leads to consider R-modules for some ring R.

The second level encapsulated in the different representation functors $\rho_{A,V}$, defines monoidal structures (see also Appendix I) on the category of those modules, and induces the possibility to classify them according to the different (complex and real) Lie algebras $\mathbf{g}$ that correspond to the primitive part of different Hopf algebras $U_h(\mathbf{g})$. This level leads to the representation of particles and correlatively to the MQT. In that sense, the existence of a well defined nomenclature of particles is the consequence of the fact that, as explained in Appendix I, we want the theory to have "enough" objects to ensure that the composed functors defined by successive applications of the point of view functor, is representable.

The third level corresponds to morphisms between those representation functors, i.e. to natural transformations that will define, as aforesaid, true natural phenomena that appear to any particle at any position through the point of view functor.

Any natural transformation between two representation functors may thus be seen as reflecting a (generally partial) formal coincidence between two such structures of successive arrows and universal constructions that "exist" independently of any other point of view functor.

As true physical phenomena are usually characterized by the fact that they do not depend on any point of view, this explains why we have chosen to associate them with natural transformations accordingly to the above relativity and equivalence principles.

2.2.2 Objects and morphisms in the preuniverse $\aleph$

To pursue our road, we have now to go into more details with the description of the preuniverse.

Splitting the category $\aleph$ in subcategories $\aleph_{\mathbf{g}}$ First, if we specify different simple complex Lie algebras $\mathbf{g}$, *we get different and distinct subcategories of functors we will denote* $\aleph_{\mathbf{g}}$. Furthermore, since we have to work on the real forms of those algebras, the same classification in subcategories applies to those real algebras.

In particular, *the classification of elementary particles by their type arises this way : the leptons correspond (up to relativistic effects that define their flavor as we will see later) to the real forms of* $\mathbf{sl}_2(\mathbb{C})$*, the neutrinos to the compact form and the electrons to the split one, while usual quarks will appear as representations of a specific real form of* $\mathbf{so}_8(\mathbb{C})$*.*

Clearly, this way of seeing the particles should enable us to predict the existence of so far undetected particles. For instance, since the compact form of a real algebra is compatible with any other real form, the compact form of the algebra $\mathbf{so}_8$ *should also correspond to physical particles that should go by three as the quarks and that we propose to call* ***dark quarks.*** *We propose correlatively to call* ***dark neutrons*** *the particles associated with their folding in* $\mathbf{g}_2$.

Those particles are indeed good candidates to be elements of ***the unknown dark matter*** *contemporary physics is looking for.*

We will also be able to predict the existence of heavier particles (associated with the algebra $\mathbf{E}_6$ *and its folding in* $\mathbf{F}_4$*) that we see only through the explained splitting and ghost effects induced by the fact that we see them from the point of view of the lighter particles we are made of. One could conjecture that those heavy particles originate* ***the dark energy*** *according to a process we will try to describe at the end of our second section.*

2.2.3 The natural transformations between g-type representation functors : different physical effect

Let us now classify the morphisms in any $\aleph_{\mathbf{g}}$ *of the above subcategories of functors* (with $A = U_h(\mathbf{g})$ in order to simplify notations) :

Since each element of $\aleph_{\mathbf{g}}$ is a functor $\rho_{A,V}$ between the one-object categories A and V, a natural transformation may be :

- either **type I**, induced by **an intertwining operator**. Each one is defined by a **morphism of representation** $\phi : V \to W$ that connects the two representation functors $\rho_{A,V}$ and $\rho_{A,W}$ by the relation : $\phi \circ \rho_{A,V} = \rho_{A,W}$. As a natural transformation, each one is a morphisms of $\aleph_{\mathbf{g}}$.

Furthermore, if V and W are irreducible, Schur's lemma implies $V \approx W$ and that ϕ is a scaling. An important consequence of this fact is that, since any two representations of $\mathbf{su}_{1,1}$ at different positions are never isomorphic, two distinct representations (or **instants**) of $\mathbf{su}_{1,1}$ cannot be related by any natural transformation, which explains why, as it is well known, from any specific point of view, all the past instants do not exist from the point of view of the present one.

Thus, *if we restrict to irreducible representations, type I physical phenomena can only be automorphisms of any given representation.*

From the universality of $U(\mathbf{g})$ in the category of the representations of $\mathbf{g}$, one may deduce that the elements of its center are the only ones (excepting trivial scaling) that induce a natural transformation on each representation functor. We will be mainly concerned by the Casimir operator C_h that generates this center and defines the coefficients of the intrinsic duality on $U_h(\mathbf{g})$-modules, and that will appear as strongly connected to the passing of time and the gravitation.

- or **type II** to **type IV**, induced by automorphisms of the Hopf algebra $A = U_h(\mathbf{g})$. Since they have to preserve the primitive part of this algebra, those automorphisms are associated with automorphisms of the Lie algebra $\mathbf{g}$. They can come from outer automorphisms of $\mathbf{g}$ (**type II physical phenomena**), or by inner automorphisms that, for a fixed choice of the Cartan subalgebra, are the product of a an element of the Weyl group of $\mathbf{g}$ (**type III physical phenomena**), and a transformation that preserves the Weyl chamber (**type IV** physical phenomena or "**generalized Lorentz transformation"**). This classification first applies to complex algebras and has then to be specified by considering real forms of those algebras.

  Let $\mathbf{G} = \mathbf{Aut}(\mathbf{g})$ the group of the automorphisms of $\mathbf{g}$: it is isomorphic to the adjoint Lie group associated with $\mathbf{g}$, and may be obtained as a quotient of its universal covering that is simply connected. Let $\mathbf{G}_0 = \mathbf{Int}(\mathbf{g}) = \exp(\mathbf{g})$ the group of inner automorphisms of $\mathbf{g}$ that is also the connected component of the identity in $\mathbf{G}$. Classically, the quotient group $\mathbf{G}/\mathbf{G}_0$ defines the group of the outer automorphisms of $\mathbf{g}$. It is isomorphic to the group of the isomorphisms of the Dynkin diagram of $\mathbf{g}$, and each of its elements corresponds to a connected component of $\mathbf{Aut}(\mathbf{g})$. By contrast with the Casimir operator that acts differently on each representation functor, automorphisms of $\mathbf{g}$ act in the same way on all its representations and thus *can only define "relativistic effects", namely effects that link a representation to another one : they thus define true physical phenomena that connect the corresponding particles.*

Let us examine the main characteristics of each of those three types of natural transformations :

1. The existence of outer automorphisms (type II) implies that *with each* $\mathbf{g}$*-module* V *are necessarily associated as many non connected copies of itself as there are such automorphisms.* Furthermore, the mathematical operation of "folding" of those copies that defines a specific subalgebra $\mathbf{g}'$ of $\mathbf{g}$, induces on the same space V a $\mathbf{g}'$-module structure.

   The first example of type II physical phenomenon comes from the triality[18] defined on the algebra $\mathbf{so}_8$, that links together through natural transformations on any position V, the three fundamental isomorphic representations of $\mathbf{so}_8$ (standard, spin$^+$ and spin$^-$). *Such representations appear to be good candidates corresponding to the three "colored" quarks*, while the natural transformations that connect those functors should correspond to the strong interaction that ensures the confinement of those quarks into *nucleons that should therefore correspond to representations of the algebra* $\mathbf{g}_2$.

   There should be another couple of type II physical phenomena, that are still unexplored. Namely, S_2*-type automorphisms of Dynkin's diagrams induce another type of confinement, we propose to refer to as* ***the anticonfinement*** *since it links for instance electrons and neutrinos but in a repulsive way.*

   Those automorphisms also imply the existence of new particles such as the afore mentioned dark quarks that come from the compact form of D_4 *-representations folded in the above mentioned dark neutrons.*

   Furthermore, we will see that there is also a type II physical phenomenon that concerns E_6*-particles with an* F_4 *folding we do not perceive directly.*

   Finally, since we will also be concerned with real representations, let us notice that, since it is an involutive automorphism, any S_2-type automorphism of a Dynkin's diagram induces a modification of the real

[18] We refer here to the true mathematical triality that corresponds to outer automorphisms of the so_8 algebra, and not to the one usually quoted in physics and associated to su_3, that refers only to order 3 elements of the Weyl group of this algebra. See the next subsection for an explanation of the role of the algebra su_3 instead of so_8 in the usual interpretation of the strong interaction.

form of the algebra well defined by the corresponding compact form. This is the reason why, for instance, we mentioned above the anticonfinement of electrons and neutrinos and not of same type particles. We will refer to those changes of types of particles as **Type IIr physical phenomena**. In practice, those phenomena correspond to the arrows in the Satake diagram that defines each real form.

2. *Type III physical phenomena, like type II, correspond to automorphisms of the Lie algebra* $\mathbf{g}$*, but instead of outer automorphisms, we consider now elements of the inner automorphisms group* $\mathbf{G}_0$*. By contrast with type II, their action preserves each connected component of the adjoint group* $\mathbf{Aut}(\mathbf{g})$*.*

   It is known that the group of automorphisms of any (complex) Lie algebra $\mathbf{g}$ is generated by the three dimensional algebras $\mathbf{g}_{\alpha_i}$ in the adjoint representation where α_i runs on all the roots of a root system of $\mathbf{g}$, and that the Weyl group W acts simply and transitively on the system of simple roots to get all the root system of $\mathbf{g}$.

   Our theory of the passing of time implies that the definition of an observer is biunivocally associated with the choice of a system of simple roots that defines an $\mathbf{sl}_2$ *-principal embedding.* Thus, *from the point of view of an observer*, another observer associated with the same roots system appears as the image of the first one through an element of W. This group therefore defines a new set of natural transformations that, although non kinematical, depend on the choice of an observer, and thus correspond to purely relativistic effects.

   We will see that *the so called "flavour" of leptons and quarks correspond to type III physical phenomena associated with the group (*G_2 *) that defines the protons and the neutrons which we are made of.*

   As for type III physical phenomena, the change from complex numbers to real ones oblige to be careful since the composition of the conjugate linear form that defines a real Lie algebra with an involutive transformation changes the real form itself. Furthermore, the central Weyl symmetry always corresponds to the change to the dual representation, a fact with many consequences in the computation of the actual characteristics of the concerned particles in the framework of the MQT presented in the section III. When necessary, we will thus specify type III physical phenomena by adding the suffix $\mathbf{r}$ (for real).

3. Finally, since the W-orbit of any element in the dual of the Cartan subalgebra intersects a closed Weyl chamber exactly once, we may al-

ways, by an appropriate type III transformation, send the $\mathbf{sl}_2$-principal embedding that defines, as we will see below, the proper time of an observer into the Weyl chamber associated with another one. This operation eliminates the "flavor" of the second particle relatively to the first.

We may then apply a change of basis that sends the set of primitive roots that defines the first $\mathbf{sl}_2$-principal embedding onto the corresponding one defined by the second principal embedding. Since the two systems are normalized by the Killing form of the algebra $\mathbf{g}$, this transformation has to be an orthogonal one.

We will say that the corresponding physical phenomenon is **a purely kinematical effect**, or **a relative speed effect**, or else, since the first instance of such a phenomenon is clearly the Lorentz group, **a generalized Lorentzian effect**. Those natural transformations that arise in this way will be said to be **type IV natural transformations** or **generalized Lorentz transformations.**

We see here how restrictive the usual notions of relativity were : the kinematical effects that were the only ones concerned by those usual notions are now reduced to ***residual effects*** *that play their part only after all the other ones.*

The natural transformations induced by the inclusion functor between algebras Having listed the nomenclature of natural transformations for a fixed algebra $\mathbf{g}$, i.e. the morphisms of the subcategory $\aleph_{\mathbf{g}}$, we have now to study the morphisms of $\aleph$ that arise from the inclusion of a subalgebra $\mathbf{g}'$ in a bigger algebra $\mathbf{g}$.

Namely, if $\mathbf{g}'$ is a subalgebra of $\mathbf{g}$, the inclusion morphism associates functorially with any representation functor of $\aleph_{\mathbf{g}}$ a representation functor of $\aleph_{\mathbf{g}'}$ that splits by a natural transformation of $\aleph_{\mathbf{g}'}$ into irreducible elements according to well known "branching rules". Furthermore, since this process is functorial, any natural transformation in $\aleph_{\mathbf{g}}$ induces a natural transformation between the corresponding elements of $\aleph_{\mathbf{g}'}$.

This means that a type $\mathbf{g}$ *elementary particle at the position* V *seen at the same position* V *from the point of view functor* $Hom_{V,\mathbf{g}'}(.,V)$ *decomposes into type* $\mathbf{g}'$ *particles, and that physical phenomena relating type* $\mathbf{g}$ *particles are seen through the point of view functor* $Hom_{\aleph_{\mathbf{g}'}}(.,\rho_{\mathbf{g}',V})$ *as physical phenomena relating type* $\mathbf{g}'$ *particles.*

We will refer those natural transformations respectively as **type V** and **type VI**. We will also say that a type V natural transformation induces a **splitting effect**, and a type VI induces a **ghost effect**.

Those two effects express that the elements and morphisms of a category associated with an algebra $\mathbf{g}$ do appear from the point of view of the objects of a category associated with a subalgebra $\mathbf{g}'$, not under their true form but as $\mathbf{g}'$-type objects linked by specific relations that only make sense in $\aleph_{\mathbf{g}'}$. By contrast type $\mathbf{g}'$ objects do not exist from the point of view of a $\mathbf{g}$-type object since $\mathbf{g}$ does not respect any of its subalgebras.

We will thus also refer to the above two effect as **the weakest law**. *It implies, by opposition to all the contemporary theories, that the "nature" and the characteristics of an object as perceived from the point of view of another object depends on the nature and characteristics of this second object.*

So, having cut the Gordian knot of the space-time gives us the freedom to explore the algebraic relations between the objects that are usually simply set down as existing in a preexisting space-time. In other words, by restricting the scope of the relativity principle to kinematics, modern physics has implicitly postulated that the appearance and the nature of the "objects", and correlatively of the space-time itself, are independent of any observation and of any observer.

With the splitting and ghost effects, the absolute relativity theory frees physics from this constraint.

The consequences of this change can not be overestimated. Let us review some of them.

Some consequences of the weakest law We have seen above that the "passing of time" corresponds to a change of position in the dual of the Cartan subalgebra of a copy of the realified of $\mathbf{sl}_2(\mathbb{C})$ that we will associate with the electrons. Thus in each real Lie algebra such that the principal embedding of $\mathbf{sl}_2(\mathbb{C})$ may be done with this real form, we will get a copy of this "passing of time" consistent with the one the electrons.

This remark will help us to determine what are the real forms of the Lie algebras that can correspond to particles we perceive as able to share our proper time.

Another important consequence of the weakest law concerns nucleons.

Protons (associated with G_2) do exist from the point of view of electrons ($D_2 \subset G_2$) that orbit them, but, from the weakest law, no electron does exist from the point of view of a proton ; therefore, the electrons around the nucleus may move from one to another since no one exists from the point of view of any nucleon. By contrast, each of the quarks ($\mathbf{so}_8$ or D_4) included in a proton or a neutron (G_2) does exist from the point of view of this one since G_2 is a subalgebra of D_4 ; therefore, the three quarks belonging to a specific nucleon cannot be mixed up with other ones inside the nucleus.

Another very important consequence of the weakest law concerns the real form[19] E_{II} of the algebra E_6, the highest real non compact dimensional one that contains in its primitive roots a copy of the realified of $\mathbf{sl}_2(\mathbb{C})$ that represents electrons. Its real rank is 4, and, since its Dynkin diagram has an S_2-symmetry, it has a singular subalgebra defined by folding, namely the split form F_I of the algebra F_4, the real rank of which is also 4

Now, the complex algebra F_4 contains the direct sum algebra $A_1 \oplus G_2$. This means that any representation of E_6 (or F_4) may be seen as a representation of $A_1 \oplus G_2$, a direct sum that corresponds to the couple lepton-nucleon. Thus, the atoms we are made of, will appear as representations of this direct sum algebra, and instead of perceiving E_6-bosons as they are, and consequently as massless like any boson, we perceive them as split in an A_1 type (anti-) particle and a G_2 type particle[20].

Considered separately, G_2-bosons, since they are not the true ones for the description of the weak interaction, may naturally appear as having a mass exactly in the same way as a 1-dimensional observer with our time direction would perceive a photon as a very short-lived massive particle. This approach gives a way to have an order of magnitude of the masses of W- and Z-bosons, and therefore to explain their surprisingly high values.

At a smaller scale, the inclusion of algebras $D_4 \subset B_4 \subset F_4$ means that E_6-bosons may also be seen as D_4-representations, which means that one can also interpret their action at the level of quarks.

Let us note furthermore that, if, as above guessed, the weak interaction as a true physical phenomenon corresponds to the type II isomorphism associated with E_6, seen with some ghost effects from our point of view, the space-time from the point of view of the observers we are, made of both neutrons and protons, containing both electrons and neutrinos, should correspond to the Cartan subalgebras of two non connected copies of those E_6 algebras that for some phenomena can be grasped globally through their folding in F_4.

But, since the algebra F_4 is fundamentally asymmetric, the space-time associated with it in the above sense does not preserve the left-right symmetry (symmetry P) respected by the space-time associated with D_4. Thus, the symmetry we assign to the space-time does not come from its true structure, but also from ghost effects induced by the symmetries of the particles we are

[19]See for instance [A.L.O.-E.B.V.], table 4, pp. 229-231 for all those real form discussions.

[20]Let us notice that su_2 (resp. su_1) is the real compact form of the 2-dimensional (resp. 1-dimensional) linear group that acts on any Cartan subalgebra of g_2 (resp. su_2) explains why the gauge invariance group associated with the weak interaction is usually represented by the product $SU_1 \times SU_2$.

made of.

In other words, if space-time were the one associated with D_4 particles, all the physical phenomena would respect the symmetry P. The fact that this is not experimentally the case justifies the above guess (that may also be seen as a mathematical consequence of the absolute equivalence principle applied to E_6 algebras).

Finally, let us notice that *this dependence of the perceived objects on the perceiving observer extends also to kinematics effects.*

For instance, we have already mentioned that neutrinos have a null real rank and thus should have, as we will see, a null mass. But, from the point of view of an electron, they do appear with a "relativistic" mass that is in fact inherited from the non associativity of the structure of the time of the electrons (or nucleons) we are made of. This "relativistic" mass allows us to explain oscillations with other flavored neutrinos, therefore avoiding the contradictions that would arise from the existence of a speed of light massive particles.

All those examples are hopefully sufficient to give a first idea of the various physical phenomena explained by the weakest law : we are thus now ready to go to the next step by examining how those phenomena arise through the point of view functor.

The impact of the point of view functor Any embedding of $\mathbf{g}'$ into $\mathbf{g}$ also induces also a surjective homomorphism from the dual algebra $F_h(\mathbf{G}')$ into the dual algebra $F_h(\mathbf{G})$ that allows us to associate with any representation of the first a representation of the second.

Let us note that in the non quantized case, such dual algebras $F(\mathbf{G})$ are algebras of functions, and the functor is a trivial restriction functor. Furthermore, since all those algebras are commutative, their irreducible representations are one dimensional and thus isomorphic. Only their own dual is non trivial and corresponds to the group $\mathbf{G}$ itself through the Tanaka-Krein duality[21].

[21] Let us emphasize here that the Lie algebra being defined on the tangent space at the identity element of a Lie group, the non quantized figure is not stable under the adjoint action of the group G on itself.

Since the adjoint action defines natural transformations, beginning with the non quantized case would have leaded to global consistency difficulties.

That is the reason why we have always considered the quantized case instead of beginning with the simpler non quantized one. See [V.C.-A.P.] pp.182-183 for a way (due to Reshetikhin) that precisely builds the quantization from the adjoint action of the group through an appropriate use of the Baker-Campbell-Hausdorff formula.

In the quantized case, there is also a family of equivalent one dimensional representations parametrized by the elements of the Cartan subalgebra (or the maximal torus in the real compact case), and the weakest law in that case defines what we propose to call **a projection effect** : any element of the Cartan subalgebra (or maximal torus) of $\mathbf{g}$ can be projected onto the one of $\mathbf{g}'$ (that is included into), and defines so a one dimensional representation of $F_h(\mathbf{G})$. *This means that the process can not be reversed, which justifies the name of weakest law we proposed before.*

But, the most important fact comes from the case $\mathbf{g} = \mathbf{sl}_2$ with $\mathbf{g}$ any Lie algebra, which defines a family of infinite dimensional representations of $F_h(\mathbf{G})$ on $l^2(\mathbb{N})$ with only a finite number of terms different from zero, itself isomorphic to the polynomial algebra $\mathbb{C}[X]$ (which is isomorphic to the ring of representations of $U(\mathbf{sl}_2)$)[22].

In the real compact case, those representations are parametrized by the elements w of the Weyl group $W_\mathbf{g}$ and those t of the maximal torus $T_\mathbf{g}$ of $\mathbf{g}^c$[23]. We will denote by $\rho_{w,t}$ any such representation. Those representations will play a fundamental role to explain "the passing of time" and to describe the structure of space-time, as we will see in the next subsections.

We will call this fundamental consequence of the weakest law the **structural law**. *It will appear as the mathematical foundation of* ***the fundamental theorem of dynamics****.*

2.3 Recovering space-time and Lagrangians

2.3.1 The paradox of space-time

Since we just have seen that space-time appears only as a by-product of the particles on a type by type and relativistic way, *we have now to explain why its use in physics has been so fruitful for more than four centuries.*

This apparent paradox comes from two mathematical facts :

- The first one is well known : we have just seen that space-time associated with $\mathbf{g}$ type particles should be defined from its Cartan subalgebra $\mathbf{h}$ and its dual (the maximal torus $\mathbf{T}$ of the group $\mathbf{G}^c$ and its dual in the real compact case). This Cartan subalgebra is commutative and may thus be seen as the product $\mathbb{C}^r$ of r copies of $\mathbb{C}$ $((\mathbb{R}/\mathbb{Z})^r$ in the real compact case) with $r = \dim(\mathbf{h})$. Since it is a solvable Lie algebra, all its representations are one-dimensional and its only interesting structure is the one of a vector space.

[22] See for instance [V.C.-A.P.] pp. 234-238 and pp. 435-439 for the compact case.
[23] See for instance [V.C.-A.P.] pp. 433-439.

But, *despite its triviality, a Cartan subalgebra* **h** *encodes almost everything that concerns a complex simple Lie algebra* **g**. Namely, the Weyl group that completely characterizes **g** may be defined from its restriction to **h**. The root system, the embeddings of subalgebras can also (up to isomorphisms) be defined on **h** and its dual $\mathbf{h}^*$. Furthermore formal characters that characterize all the representations are also defined on the Cartan subalgebra. In the real compact case, the character functions, defined on the maximal torus **T**, generate a topologically dense subset of the class functions, and every element of $\mathbf{G}^c$ is exactly conjugate to $\mid W \mid$ elements of **T**. The system of primitive and fundamental roots which permutations or embeddings in the roots system define the above type II and type III natural transformations correspond clearly to Cartan subalgebras (or maximal torus) properties.

Turning to the real non compact algebras, it is also well known that all the real forms are characterized by a Satake diagram, itself also connected to the diagram of primitive roots included in $\mathbf{h}^*$. Finally, all the finite dimensional representations of $U(\mathbf{g})$ and its deformations $U_h(\mathbf{g})$ are indexed by the weights lattice that is included in $\mathbf{h}^*$, while, as aforesaid, the representations of the dual quantized algebra $F_h(\mathbf{G})$ are indexed[24] by the product of the Weyl group and the Cartan subalgebra **h** itself (or the maximal torus **T** in the real compact case).

It is thus understandable that, *by putting everything "by hand" on the vector spaces that correspond to Cartan subalgebras, physics has been able to induce directly from experimental facts many properties that characterize Cartan subalgebras of specific simple Lie algebras, although, from a mathematical point of view, those properties could have been directly deduced from the analysis of each Lie algebra.*

The usual inductive method of physics has also been made easier by *the following, more subtle, mathematical fact that stands at the origin of Lagrangians and of their universal application.*

- The Lie algebras $\mathbf{sl}_{n+1}$ of type A_n with $(n > 1)$ that act canonically on vector spaces of dimension $n + 1$ without any specific structure have a very unique property[25]. Namely, from the Jacobi identity, the tensor product $\mathbf{sl}_{n+1} \otimes \mathbf{sl}_{n+1}$ contains, as the product $\mathbf{g} \otimes \mathbf{g}$ for any other Lie algebra **g**, a copy of the adjoint representation. *But in the A_n case, it contains also a second copy of $\mathbf{sl}_{n+1}$ that belongs to the symmetric*

[24]See for instance [A.C.-V.P]. Th.eorem 13.1.9, page 438.

[25]This fact is easy to check directly by using plethysm. See also [A.C.-V.P.], pp.387 for a reference.

part of $\mathbf{sl}_{n+1} \otimes \mathbf{sl_{n+1}}$. The projection π on this second copy is given by*:* $\pi(x \otimes y) = xy + yx - \frac{2}{n+1} trace(xy).Id.$

Any element of $Aut(\mathbf{sl_{n+1}})$ [26] corresponds to an element of $\mathbf{sl_{n+1}} \otimes \mathbf{sl_{n+1}}$ through the Killing form, and, therefore, has a projection through π onto a symmetric representation of $\mathbf{sl}_{n+1}$. This projection is clearly null for the image of the identity map that corresponds to the Casimir operator, but it can be non trivial in other cases. Thus, we may associate a bilinear symmetric form on $\mathbf{sl_r}^*$ that is a representation of $\mathbf{sl}_r$ with any Cartan subalgebra of rank r with $r > 2$. This correspond to the Lagrangian that we have defined and used in our first section.

Let us notice finally that for algebras with $r = 2$, the above component of $\mathbf{sl_r} \otimes \mathbf{sl_r}$ is null, and the Lagrangian is reduced to its free part as we shall see below for the electromagnetic interaction (that will appear as a type II interaction that changes the compact real form $\mathbf{so}_4$ to the realified $\mathbf{so}_{1,3}$ of $\mathbf{sl}_2(\mathbb{C})$).

2.3.2 The structure of space-time

Let us come back before to space-time as it appears now associated with each type of Lie algebra and therefore to each type of particles.

Let us keep the above notations and define **space-time associated with g** (or **g-space-time)** as the r -dimensional vector space as above defined by the dual $\mathbf{h}^*$ of a Cartan subalgebra $\mathbf{h}$ supposed chosen in the beginning of the construction.

The first two properties of this space-time are that it is canonically equipped with the bilinear symmetric form defined by the restriction to $\mathbf{h}$ of the Killing form $kill_{\mathbf{g}}$, and that it contains the lattice of maximal weights that indexes all the finite dimensional representations of $U_h(\mathbf{g})$.

Since this lattice is defined as a copy of $(\mathbb{Z})^r$*, it is clearly not invariant through* $e^{i\phi}$ *changes of phase : space-time arise first as a discrete real mathematical object. This leads us to work on real Lie algebras without any assumption of an "external" breach of symmetry in* $\mathbb{C}$.

Now[27], any real form is defined by an invariant conjugate linear form σ. Two linear forms σ and τ are said to be **compatible** if they commute, which means that the composite map $\theta = \sigma\tau$, that is a linear one, is involutive. Any real form is compatible with a real compact form. We may thus begin by working on the compact real form $\mathbf{g}^c$ of $\mathbf{g}$, and go then to any other real

[26] considered as the tensorial product $sl_{n+1} \otimes sl^*_{n+1}$

[27] We follow here [A.L.O.-EB.B.V.], page 133.

form by applying an invariant involutive *complex* transformation[28].

Let us emphasize that, since such a transformation corresponds to a type II or type III natural transformation, *the particles associated with two compatible real forms do coexist in space-time, and are related by a true physical phenomenon.*

For instance, neutrinos do coexist with electrons and positrons[29] as associated respectively with the compact forms $\mathbf{so}_4$ and $\mathbf{so}_{1,3}$ of the real algebra $\mathbf{sl}_2(\mathbb{C})$. Furthermore, as associated with the same compact form, neutrinos and antineutrinos should be the same particle, but we will see that they differ slightly due to dynamical ghost effects that generate their mass[30] as we will explain in the next subsection. In the same way, dark quarks should coexist with usual ones corresponding with the compact form $\mathbf{so}_8^c$ and be folded in dark neutrons associated with the compact algebra $\mathbf{g}_2^c$.

On the other hand, if two non compact forms are associated respectively to the involution θ and θ' that do not commute, which is the general case, the corresponding τ and τ' do not commute and the two real forms are not compatible : the corresponding particles can not coexist, and therefore, in general only one real form of each type of algebra can coexist with the compact form. This remark has been used extensively in our section II, since we will have to find only one non compact real form for each type of particle[31].

[28]It is interesting to notice that there always exist on the compact form a Haar measure that authorizes convergent integral calculus. The pull back of this calculus on a non compact real form defines canonically a way to make convergent calculus on this non compact real form that is generally itself equipped with a divergent invariant measure. This remark could explain in our context the success of renormalizations methods of QFT, but we did not go deeper into this way.

[29] This fact can be confirmed by a well known fact on Lie groups. The group SO4 splits into two copies of the group SO3. Since the multiplication by i in C acts as an orthogonal rotation on the real plan, the Weyl symmetry may be seen as applying such a rotation on the real plan, which corresponds exactly to the Lie bracket [i,j] in the su2 algebra.

Applied on any two directions, this means that applying the above involution is equivalent to go from the (-) connection on the sphere that is the usual one without torsion, to the (+) connection that has a torsion defined by the Lie bracket on the tangent space at the neutral element.

The compact form then appears as the torsion -free manifold associated with this second one.

There is another possibility to define the (+) connection by left-right conjugation, while the compact form is unique. See for details [S.H.], pp. 102-104.

[30]This mass may be increased by type III transformations that induce mu and tau neutrinos.

[31]This uniqueness of the non compact real form does not implies that the corresponding particle appears only under one form : indeed, as we have seen before, there can be relativistic type III effects that change the "flavor" of the particles,

We have now to analyze how $\mathbf{g}^c$-space-time is seen through the point of view functor. Going to the corresponding analysis for other algebras will then be easy by applying the transpose of the above involution, which is legitimate since, by tensoring by $\mathbb{C}$, we can take everything on the complex field, and then apply the above complex involution.

Now, beginning with the $\mathbf{su}_2$ case, it is known[32] that the representations of $F_\varepsilon(\mathbf{SU}_2)$ are parametrized by the maximal torus t with $t \in \mathbb{C}$, $|t| = 1$, and are either one-dimensional τ_t or infinite dimensional π_t given by, for linear forms a, b, c, d arranged as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ in the usual presentation :

$$\tau_t(a) = t \ ; \ \tau_t(b) = 0 \ ; \tau_t(c) = 0 \ ; \ \tau_t(d) = t^{-1} \ ;$$
$$\pi_t(a)(e_k) = (1 - \varepsilon^{-2k})^{1/2} e_{k-1} \ ; \ \pi_t(b)(e_k) = -\varepsilon^{-k-1} t^{-1} e_k \ ; \ \pi_t(c)(e_k) \varepsilon^{-k} t e_k \ ;$$
$$\pi_t(d)(e_k) = (1 - \varepsilon^{-2k-2})^{1/2} e_{k+1},$$

with the e_k belonging to the formal one variable real polynomial algebra.

The first one may be connected with zero dimensional symplectic leaves of the Poisson Lie group $\mathbf{SU}_2$, while the second corresponds to the two-dimensional ones (associated with the opposite face of the sphere S^2, and therefore to the application of the Weyl symmetry).

Finally we see that, in that case, the representation of $U_\varepsilon(\mathbf{su}_2)$ are parametrized by a lattice in the dual of the maximal torus, and that representations of the dual Hopf algebra $F_\varepsilon(\mathbf{SU}_2)$ are parametrized by the maximal torus itself. Furthermore, if, by putting $X = (x + \frac{1}{x})$, we identify the above polynomial algebra with the algebra of characters, and correlatively of representations, of $U_\varepsilon(\mathbf{su}_2)$, we see that successive applications of the diagonal elements of the dual Hopf algebra induce a move of increasing degrees representations which we will refer to as **a growth of representations** and that may be seen as corresponding to successive applications of the point of view functor : this will be the origin of the passing of time we will describe in the next subsection. Reciprocally, a diagonal element of the universal enveloping algebra generates a one-parameter group that induces a continuous move on the representations of $F_\varepsilon(\mathbf{SU}_2)$ as parametrized by t.

We have thus a perfectly reciprocal situation. It is well-known that particles mysteriously have a double nature, corpuscular and undulatory ; the structure of the time that arises canonically as we just have seen shows that in some sense, time itself has a very precise double structure, spectral and undulatory, but there is nothing mysterious in this situation : the one corresponds to the Quantum Universal Algebra ; the second to its (restricted) dual as it appears after application of the point of view functor.

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and also purely dynamical ones, as we will explain below.

[32]See for instance [V.C.-A..P.], page 437.

Such a structure that arises canonically in our context was probably difficult *a priori* to postulate, although it is not so far from many recent approaches.

As mentioned before, this situation may be generalized to any compact algebra $\mathbf{g}^c$ by using the principal embedding of $\mathbf{su}_2$ in the Weyl chamber of $\mathbf{g}^c$, and then composing with applications of the Weyl group : this gives the aforesaid $\rho_{w,t}$ representations. [33].

We therefore have exactly the same situation for any real compact algebra $\mathbf{g}^c$ *as the one we have just described for* $\mathbf{su}_2$, *except that we have now an* r *-dimensional torus, and that there is no more one copy of the algebra of representations, but as many as there are elements of the Weyl group* W *, each copy being connected to the others by a type III transformation.*

Applied to $\mathbf{g}_2$ *(or* $\mathbf{so}_8$ *), this situation gives rise to the "flavors" of particles: their proper time may stand, up to a generalized Lorentz transformation, in any of the copies of the Weyl chamber of* $\mathbf{g}_2$ *that corresponds to the nucleons we are made of. Furthermore, since generalized Lorentz transformations can partially cover two distinct copies of the Weyl chamber of* $\mathbf{g}$, *oscillations of "flavors" can arise.*

The structure of space-time is finally totally defined by the characteristics of the algebra it is generated by : it nevertheless always has a quite sophisticated fibered continuous structure combined with a discrete one, each one of those two components acting on the second.

3 The Mass Quantification Theory

As announced at the twelfth step of section 1.4, the general methodology that we will use will take as a reference for our time the time of the electron. Then, we will do the calculations in this time frame considered as embedded in the algebra that caracterizes the type of particle chosen, alongside the direction defined by the principal embedding of sl_2, sometimes composed with the appropriate transformation of the Weyl group of the Cartan subalgebra.

The principal embedding and the Weyl group are indeed the two elements that define at each point of the compact representation associated with the particle considered the creation / annihilation operators which generate the time of this particle. The MQT therefore implies a preliminary step that will make more precise our vision of the electron.

It will be the first point of this section, and the second one the calculation of the Newton "constant" and the third point the application of the genreral

[33]See ibidem pp. 437-439.

guidelines defined in the previous sections from a frame tied to an electron as an an bserver to some examples that will illustrate the possibilities offered by the MQT.

The computations given as examples below, by no means extensive, have the main goal to show that the ART provides numerous numerical results that can be checked by measures, and that the MQT can appear as a new branch of physics. The authors recognize that these calculations have to be upgraded and extensively completed for other types of particle

3.1 Calculations for the electron

The electron is defined from the realification of the complex algebra $SL_2(\mathbb{C})$. The tensorial product by the adjoint representation will amount to a displacement of two units in the dual of the Cartan subalgebra. On the other hand, it is also on the adjoint representation of SU_2 (which is like the sphere SO_3) a rotation of π angle on the maximal torus.

A necessary condition of coherence for the fact that the KZ-monodromy that defines the passing of time is not changed by the rotation on the torus on which the representations of $F_h(SL_2(\mathbb{C}))$ are indexed, is that the quadratic form which connects the Cartan subalgebra to its dual be such that to 2 on the dual corresponds π on the torus.

The time unit for the time of the electron corresponds therefore to 4 units on the dual (this corresponds to two successive monodromy in the adjoint algebra, each one of them inverting the algebra orientation because of the flip operator) or to a translation of 4 in terms of the Yangians.

Thus a creation cycle of one unit of time for the electron is like a $2\,\pi$ rotation on the maximal torus.

The involution that generates the real form of $Sl_2(\mathbb{C})$ from the compact form of $SO_4(\mathbb{C})$ being also the transformation that exchanges algebraically the left with the right, one period of time of the electron will then be like $2\,\pi$ on pure imaginary direction.

Now, as we have done before, let us represent the Cartan subalgebra in the 0 and 1 directions (or the real and imaginary ones) associated to the space-time of the electron, which corresponds to $su_{1,1}$. The dimension 2 and 3 will correspond to the compact part, and therefore will be be associated with its charge. The "hyperbolic" rotation in $su_{1,1}$ is the image of the rotation in su_2 before the application of the involution which transforms the compact form into the real one.

The ratio between these two rotations is therefore a characteristic of the relative energies of these two components (electromagnetism and mass). It is thus given by the fine structure constant α. And we just showed that we have to use instead $\alpha' = \frac{\alpha}{2\pi}$, that characterizes the ratio between the energy of the electron that comes form his mass and the electromagnetic energy, because of our normalisation.

This modelisation allows several calculations. The figure before the application of the Weyl transformation contains a right-left symmetry if we read the product $e_0 \wedge e_1$ as the dual of $e_2 \wedge e_3$ (Hodge-$*$ operator).

1. The rotation that happens on directions 0 and 1 before the application of the involution induces an equivalent rotation on directions 2 and 3 because of the Hodge-$*$ operator.

2. The rotation in the 2 and 3 directions that corresponds to the electron charge is thus changed by a precession coming from the previously described rotation that comes from the passing of time of the electron. As a first approximation, it is then

$$p = 1 - \frac{1}{1 + \alpha'}$$

To be more precise, the measure to consider on the maximal torus has to take into account the fact that this torus is in reality a main circle of a sphere of radius 1 that stands for SO_3. The norm to take into account at the second order is therefore $1 - \frac{1}{3}\alpha'^2$ (the limited development at the second order of the Riemannian form in the neighborhood of a point), and thus we obtain

$$p = \alpha' - \alpha'^2 - \frac{1}{3}\alpha'^2$$

If we compare this result with the coefficients that comes from the QFT, usually expressed as a function of $\alpha'' = \frac{\alpha}{\pi}$, and that are $\frac{1}{2}\alpha'' - 0.3285\alpha''^2$, we see that the difference at the second order is, taking into account the measured value $\alpha' = 0.00116140981411$

$$(\frac{1}{3} - 0.3285) \times \alpha''^2 \cong 6.6057 \times 10^{-10}$$

The QFT includes higher components that come either from the electromagnetic part or from potential other particles. We did not try in this first paper to represent this phenomena. Roughly, our first approximation gives

$$p_{calc} \cong 0.001159611317$$

compare to the measured value $p_{exp} \cong 0.001159652$

In our calculation we did not take into account the asymmetry of time which comes from the passage from the KZ-algebra to the DZ-algebra. We can guess that at least a part of the difference comes from this approximation.

3. On the other hand the fact that the left-right conjugation exchanges the directions 0 and 1 with the directions 2 and 3 implies that the precession p is the right equivalent of α, the fine structure constant. Each term is calculated by supposing the other fixed. We will then introduce the coefficient

$$\psi = \frac{\alpha'}{p}$$

to represent the impact of the left right conjugation on the electron representation.

4. We will calculate now the position N of the electron in its own mobile frame. By reasoning in the "ghost time", we consider that the electron in the past is like it is now. So by getting N we will get the age of the universe.

We have seen that a move of two units in the dual of the Cartan subalgebra is equivalent because of the Casimir coefficient to a variation of $\frac{1}{\sqrt{C_2(n)}}$ in the subalgebra itself (cf. the previous sections I and II). This reasoning applies to the electron and its associated bosons (that is to say to all the successive value of n), and therefore we have to consider that the variation of the quadratic form on the Cartan subalgebra is given by the sum

$$\Sigma_N = \sum_{k=1}^{N} \frac{1}{\sqrt{(k+1)^2 - 1}}$$

*This length is measured in the basic algebra, and thus in the dual algebra of the one the time belongs to. Since the energy is given by $\frac{\partial \mathcal{L}}{\partial \dot{z}}$ with $\mathcal{L}$ the above defined Lagrangian, it is merely proportional to the quantity $\dot{z}$ in the dual of time. Therefore, it is thus natural to define its inverse as **the energy associated to the position** N.*

Since even and odd representations correspond to distinct groups representations, and thus to distinct quantum groups, it will be more convenient to apply the above construction separately to even and odd representations that will correspond to distinct particles. One get then two distinct summations :

$S_N = \sum_{k=1}^{N} \frac{2}{\sqrt{(2k-1+1)^2-1}}$ and $T_N = \sum_{k=1}^{N} \frac{2}{\sqrt{(2k+1)^2-1}}$.

A small computation shows that for $N > 10^6$, the sum $\Sigma_N = S_N + T_N$ may be approximated (with a less than 10^{-11} precision) by :

$\Sigma_N = \ln(2N) + 0.08396412352$.

From the modelling of the electron in the MQT previously described we have seen that the fine structure constant α corresponds to the ratio between the electromagnetic energy (that does not varies with time) and the energy corresponding to the inertial mass (that decreases with the above law) of the electron. It can thus be used to compute $2N$ with the above formula.

More precisely, we will get from hyperbolic trigonometric considerations[34] that we must have

$$\Sigma_N = \frac{2}{\pi}\sqrt{\frac{1}{\alpha^2} - 1}$$

and thus

$$2N = \exp(\frac{2}{\pi}\sqrt{\frac{1}{\alpha^2} - 1} - 0.08396412352)$$

We will state again $\alpha' = \frac{\alpha}{2\pi}$ and the measured value of

$$\alpha' \cong 0.00116140981411$$

gives

$$2N = 7.08432804709 \times 10^{37}$$

To come back to the international system of units, we use the numerical value of the speed of light $c = 2997932458\ m/s$ to define the unity of length and Planck constant $h = 6.6260755 \times 10^{-34}\ kg.m^2/s$ to define the unity of mass. We get $\frac{h}{c^2} = 7.37250327 \times 10^{-51} kg.s$.

With the mass of the electron $m_e = 9.109384 \times 10^{-31}\ kg$, this leads to the period of the electron

[34] This comes directly from the exchange of the direction 0 and 1 (that is direction of the real axis and the direction of the imaginary one) after we take into account the invariant quadratic form and the passage from an euclidean form to a lorentzian one

$$T_e = 8.09355159 \times 10^{-21}\ s$$

that defines the apparent age (or "ghost" age) of the universe from our point of view as

$$(2N)T_e = 5.73373745 \times 10^{17} s \approx 18,18 \times 10^9$$

in years[35].

3.2 The Newton constant calculation

The ART makes of the gravitation a consequence of the passing of time. It should then be possible to calculate the value of the Newton "constant" (remember it appears such from the present of an observer looking at his "ghost" past) from the previously calculated value of N.

The variation of the measurement unit in the direction of time corresponds to the component of the trace operator projected on the identity (cf. the coefficient $\frac{\tau}{2}$). A given variation in 2 dimensions will therefore have an spatial impact twice the one in 4 dimensions. This is equivalent to replace N by $\frac{N}{2}$.

So to find the electron we have to change the coupling coefficient by two.

Instead of calculating the Newton constant by itself, we will instead compare two dimensionless terms that should be equal if the intrinsic coupling coefficient is indeed 1 as the ART predicts. Such a checking is obviously the same as the calculation of the constant, depending only on the choice of units.

The right term is given by the effects of the two coefficients which after being multiplied make the volume form changes (here the volume form is a surface form for the space-time of the electron). If we take into account the 2 coefficient afore mentioned, we obtained, with $\alpha \cong 2\pi \times 0.00116140981411$ and the previously computed age of the universe $N \cong 7.08432804709 \times 10^{37}$:

$$\frac{\omega_t}{\omega_0} = 2 \times \frac{\alpha}{\sqrt{1-\alpha^2}} \times \frac{1}{N} = 2.06019465385 \times 10^{40}$$

[35] This age is of the same order of magnitude as the one actually recognized by contemporary physics. It is nevertheless slighty higher, but the absolute relativity theory leads to a view on the cosmological beginning of the universethat goes far more slower scenario than the usual ones.

But the most significant confirmation of this value will be the computation below of the Newton constant that results from the absolute relativity theory that is based on this value and fits perfectly with experimental values.

On the other side, if m_e is the mass of the electron, and if we take for G the value measured as of today, that is 6.67259×10^{-11} in the SI unit system, and if we do not forget that the initial mass to take into account is modified by the duality, that is $(\psi\alpha')^{-1}$ instead of α'^{-1}, with $\alpha' = \frac{\alpha}{2\pi}$, the volume form should be modified by the dimensionless coefficient defined by :

$$\frac{\omega_t}{\omega_0} = \frac{G(\psi\alpha'^{-1}m)^2}{c\,h}$$

which is

$$\frac{6.67259 \times 10^{-11} \times (\frac{1}{1.0015156511 \times 0.00116140981411} \times 9.109384 \times 10^{-31})^2}{299792458 \times 6.626176 \times 10^{-34}}$$

which is

$$\cong 2.06016657202 \times 10^{-40}$$

The ratio between the two coefficients is therefore

$$\frac{2.06019465385 \times 10^{40}}{2.06016657202 \times 10^{-40}} \cong 1.00001363085$$

so the difference is less than 0.002 %, which corresponds to the margin of error on the numerical coefficients used and to some evolutions of the measured physical data, and also comes from the asymmetry of time ...

We have therefore confirmed that

$$G = \frac{\alpha_3\,c\,h}{(2\,\pi\psi m)^2 N\sqrt{1-\alpha^2}}$$

The quality and precision of this result confirms at the same time the generalised Einstein equation we have established (and especially its dependance of the dimension of the space time associated with the particle) and also our calculation of the age of the universe, especially because it involves the exponential of a number around 100 : the precision we have obtained around 0.001 % show that the precision was before the application of this exponential around 10^{-7}.

3.3 Mass calculations of the proton and the neutron

Having done the calculations in the case of the electron, now we only have to follow the guidelines given in the previous parts of this text. The proton and neutron will then be of type G_2, and we will look first at the particle without an electric charge.

We will work in the normal (split) real form g of the the complex lie algebra g_2 of the complex Lie group G_2.

We use the usual euclidean representation of the roots of g_2, with the longest one having a length of $\sqrt{2}$.

In that case, the first simple root is $\alpha_1 = (-1/3, 2/3, -1/3)$ and the second simple root $\alpha_2 = (1, -1, 0)$. The weight of the adjoint representation is the longest fundamental weight

$$\alpha_6 = 3\,\alpha_1 + 2\,\alpha_2 = (1, 0, -1)$$

The principal embedding of $sl_2(\mathbb{C})$ in g_2 has the following representation

$$f = 18\,\alpha_1 + 10\,\alpha 2 = (4, 2, -6)$$

and a norm of 56.

The cosinus of the principal embedding angle a with the adjoint representation root is then

$$\cos a = \frac{(\alpha_6, f)}{\sqrt{(\alpha_6, \alpha_6)(f, f)}} = \frac{5}{2\sqrt{7}}$$

Because we want to compare the time of the electron with a particle with a time lagged, we have to work in the dual of its Cartan subalgebra, that is the one after the weyl symmetry is applied. It is the algebra of the coroots, in which the fundamental embedding has the representation $5\,\alpha_1 + 3\,\alpha_2$ (the half sum of the positive roots) after the appropriate exchange and renormalization of the two simple roots. In that case, with the same convention for the norm (the longest root has a $\sqrt{2}$ length) its norm is $\frac{56}{3}$, that is a third of the norm of the initial principal embedding vector before the transformation.

Therefore, the Dynkin index of the principal embedding being 28, we have to divide it by 3 when we work in the dual because of the time lag. By taking into account the standard representation of the electron we begin with, we also have to correct it by the ratio of the Casimir coefficients determining the passage from the adjoint to the standard representation in $sl_2(\mathbb{C})$, that is $\frac{8}{3}$.

Finally, the ratio of the intrinsic forms we have to consider is

$$i = 28 \times \frac{1}{3} \times \frac{8}{3} = \frac{224}{9}$$

Now we can compute the lag in the direction of the principal embedding of the time of a particle of type G_2 compared to the one of the electron. This "lag" will allow us to get the $\cosh t$ that has to be used, and then we will get the mass of the particle considered.

In the mobile frame of the electron, the change in the coefficient is therefore given by

$$t = (1 - \cos a) \times 2\frac{\sqrt{1-\alpha^2}}{\alpha} \times \frac{2}{\pi} \times \frac{1}{i} \cong 0.386181207486$$

The appearance of the coefficient $\frac{2}{\pi}$ was explained earlier. Taken into account the precession, we find for this particle, that we call the neutron, the following mass m_n, if $m_e = 9.10938215(45).10^{-31}\ kg$ is the mass of the electron

$$m_n = 2\cosh t \times \frac{m_e}{\alpha'(1+\alpha)} \cong 1.67488836.10^{-27}\ kg$$

to be compared with the measured mass $m'_n = 1.67492729(28).10^{-27} kg$, giving a ratio of $\frac{m'_n}{m_n} \cong 1.0002$, next to 0.002 % of error.

If we calculate now on the other face of $SO_4(\mathbb{C})$, using a rotation of $\frac{\pi}{2}$ compare to the initial direction of the electron, we should see a particle with the appearance of an electric charge opposite to the one of the electron, and for the calculation we have to use the coefficient $\psi \cong 1.0015156511$ defined and calculated earlier, replacing α by $\frac{\alpha}{\psi}$ everywhere it appears.

So the new "lag" t' becomes

$$t' = (1 - \cos a) \times 2\frac{\psi\sqrt{1-(\frac{\alpha}{\psi})^2}}{\alpha} \times \frac{2}{\pi} \times \frac{1}{i} \cong 0.38676658329$$

and the mass of the particle that we can call the proton

$$m_p = 2\cosh t' \times \frac{m_e\psi}{\alpha'(1+\frac{\alpha}{\psi})} \cong 1.672744.10^{-27} kg$$

which has to be compared with the measured value of $m'_p = 1.672621637(83).10^{-27} kg$, giving a ratio of $\frac{m_p}{m'_p} \cong 1.00007$, next to 0.0007 % of error.

3.4 Mass calculations for the electrons τ and μ

Here, the methodology is the same than before, but simpler because we do not have to compare the metric on the embedding with the one of g_2. We just move the copy of sl_2 in g_2 by the Weyl rotation of the g_2 root diagram of angle $\frac{\pi}{3}$. The first application of this rotation makes us cross the first zone (angle $\frac{\pi}{4}$) of the root diagram of so_4, which implies to use the dual, and this has the following consequences :

1. Use the $\cosh^{-1}$ instead of the cosh to calculate the "lag" t.
2. Use the factor $\frac{1}{2}$ instead of 2 in front of the $\cosh t$.

Indeed, the $\frac{\pi}{2}$ rotation of the roots of the realified of sl_2 needed to pull back the Weyl chamber of sl_2 in a position that contains the μ direction is equivalent to the application of the Weyl symmetry that corresponds to the change to the dual.

To determine the position on the embedding of the electron, we just have to project on it with the $\frac{\pi}{3}$ angle. Therefore,

$$t = \cos\frac{\pi}{3} \times \cosh^{-1}\frac{1}{\alpha} \cong 2.8066887241$$

Then we use the electromagnetic mass of the electron, in GeV/c^2 units, that is $m_e \cong 0.440023544386\ GeV/c^2$ to get the mass m_μ of the electron μ,

$$m_\mu = m_e \times \frac{2}{\cosh t} \cong 0.105931407278\ GeV/c^2$$

to compare with the measured mass of $0.1056\ GeV/c^2$.

For the electron τ, after another application of the Weil rotation of angle $\frac{\pi}{3}$ of the root diagram of so_4, we again change the second zone of it by going through the angle $\frac{\pi}{2}$, which implies we go again to the dual and have to multiply by $\frac{1}{2}\cosh t$ instead of dividing by it, and use also the ψ coefficient.

The "lag" becomes

$$t' = \cos\frac{\pi}{3} \times \cosh^{-1}\frac{\psi}{\alpha} \cong 2.80744603698$$

and therefore the mass m_τ of the electron τ is

$$m_\tau = m_e \times \frac{1}{2}\cosh\frac{t'}{\psi} \cong 1.82146\ GeV/c^2$$

to compare with the measured value of $1.784\ GeV/c^2$.

4 Appendix I : On the foundations of the Absolute Relativity Theory

The transcendental conditions referred to in the first section can be expressed in a formal mathematical way by using the language of categories.

Since it is not the main purpose of the theory, we only outline here shortly the framework of such a mathematical formulation that has been useful to made the main choices that lead to the ART.

4.1 From graphs to categories

Representing the objects and morphisms used by a theory as points and arrows of a graph seems to be the most general algebraic way to begin with a theoretical construction.

It is then natural, as mentioned in section one, to define **the point of view of an object** a **on an object** b as the arrows that have b as domain (or source) and a as codomain (or target). *We will assume that any so defined point of view is a set.*

If the objects of the graph are distinct, applying this operation on all the objects of the graph generates sets of arrows that are not intersecting, which forbids to make any comparisons between the different points of view of those objects. It is thus impossible to refute the theory by experimenting contradictions between the different points of view represented, which could lead to solipsistic approaches.

Furthermore, there could be isolated objects that have no connection with everyone including themselves, and such objects do not have any point of view and are not part of anyone. Thus, their withdrawal would not change the theory that therefore is not well defined.

By contrast, if we assume that the graph is equipped with a law of composition of arrows, there begins to be a possibility to make tests of consistency since, if three objects a, b and c are linked by arrows $\rho_{c,b}$ from c to b and $\rho_{b,a}$ from b to a, there is a necessary relation between the points of view of a and b, namely : the composed arrow $\rho_{b,a} \circ \rho_{c,b}$ has to belong to the set $\hom(c, a)$ that represents the point of view of a on c. This gives a possibility to test the consistency of the points of view of the object a and b on the object c, but does not allow any consistency test of those points of view on arrows.

We have thus to consider a fourth object d and an arrow $\rho_{d,c}$. From the point of view of b, the arrow $\rho_{c,d}$ associates an arrow belonging to $\hom(c, b)$ with any arrow $\rho_{d,b}$. The same applies to the point of view of a on the same arrow $\rho_{d,c}$. If we restrict ourselves to the arrows from the point of view of a are defined through b, it is now easy to express the consistency conditions that is :

$$\rho_{d,a} = \rho_{d,b} \circ \rho_{b,a} = (\rho_{d,c} \circ \rho_{c,b}) \circ \rho_{b,a} = \rho_{d,c} \circ (\rho_{c,b} \circ \rho_{b,a}) = \rho_{d,c} \circ \rho_{c,a},$$

which corresponds to the associativity of the composition law.

Therefore, requiring the composition law to be associative is a necessary and sufficient condition to make consistency tests on both objects and morphisms.

We will also assume that there is an identity arrow attached to each object which means that each object has a non empty point of view on itself and that one of the corresponding arrows acts as an identity for the operation of composition of arrows. Those conditions are those that define a metacategory. For technical reasons, we will assume that this metacategory is in fact a category and that this category is small enough to see each object as element of a "big enough" set.[36]

We will call **preuniverse** this category that contains those objects and arrows used to describe the theory, and we will call it $\aleph$.

The above operation of taking the point of view becomes a contravariant functor that we will call **the point of view functor**. According to Yoneda Lemma the preuniverse may be embedded in the category of point of view functors by the operation that associates the functor $\hom(., a)$ with any object a, and the corresponding natural transformation with the arrows between any two objects a and b. Yoneda Lemma implies that one can hope to access to the preuniverse through the category of all the point of view functors with the corresponding natural transformations.

A functor F in a category C will be said to be representable if there exists some object a belonging to C such that $\hom_C(., a)$ is naturally equivalent to F.

The point of view functor is contravariant which means that it is a covariant functor from the opposite category that is obtained by reversing all the arrows.

The next conditions we will require for the theory we are beginning to build is that it has "enough" objects to make usual composed functors represented : this will mean that the theory is required to be able to represents by an object the usual composed functors in a way we will precise now.

4.2 From categories to additive categories and R - modules

We follow here [S.M.L.] pp.198-201 and 209.

Since we want to work on set of arrows coming to a point, it is natural to require that the product of two hom-sets $\hom(b, a)$ and $\hom(c, a)$ defines a new object *d that does not depends on* a, which is, as above, a condition for authorizing comparisons between points of view. This correspond to the existence of a *coproduct* associated with any pair of objects. In the same way, by considering the opposite category, we see that the category need also to contain a *product* of any two objects.

[36]This refers to the notion of universe as described for instance in [S.M.L.], page 12.

One may ask also that there is an object the point of view of which contains all the objects of the category : otherwise one can formally add it in the category. Imposing the same condition to the opposite category leads to postulate the existence in the category of an initial object. In order to have some invariance by the "taking the opposite" operation, one postulates that those two objects coincide, which means that the category has a zero element.

Let us add two other technical conditions, namely :

- every arrow in the category has a kernel and a cokernel ;
- every monic arrow is a kernel, and every epi is a cokernel.

In an informal language, the first of those conditions is again a condition requiring the category to have enough objects to make the point of view functor precise enough in order to allow to represent those fundamental (abstract) characteristics of arrows. The second one asserts that the category also has enough arrows in order to make possible the reciprocal way.

The key point is that those conditions are sufficient to ensure that the considered category is an Abelian category.

Finally, an embedding theorem referred to as Lubkin-Haron-Feyd-Mitchell theorem in the above reference, allows us to consider the category as a category of R-modules for some suitable ring R.

We are therefore now working on such a category.

For an aim of simplicity here, we restrict ourselves to categories of R-modules that are also complex vector spaces.

4.3 From additive categories to monoidal categories of modules over quasi-Hopf algebras.

Since we access everything through the above point of view functor, one can ask that there should be also objects in the category that represents (as an appropriate adjoint) the successive applications of this functor.

This leads to consider that we have also to be able to define tensor products of the R-modules we are now working on. This means that R has to be a quasi-bialgebra. Since we want furthermore to have the possibility to exchange left and right, we need an antipode operator.

This leads to the idea *to restrict ourselves to the cases where R is quasi-Hopf algebra.*

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