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## A Brownian motion technique to simulate gasification and its application 2 to C/C composite ablation

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#### 1. Introduction 43

Carbon/carbon (C/C) composites, which keep excellent mechan-44 ical properties at high temperatures [1], are used as thermostruc-45 tural protections in various applications such as atmospheric re-46 entry (vehicle thermal protection systems) [2], propulsion (rocket 47 nozzle) [3], and experimental Tokamak reactors for nuclear fusion 48 49 (divertor and first wall armor) [4]. In these applications, the C/C composites are mainly ablated by some gasification processes (oxi-50 dation, sublimation) [2]. An efficient design of a thermostructural 51 protection relies on an accurate evaluation of the wall recession 52 caused by ablation. In particular, C/C composites develop surface 53 54 roughness features, which may display a strong coupling with the surrounding environment [5,6]. The surface roughness chiefly 55 arises from the heterogeneity of the composites, whose fibers 56 and matrices are made of distinct forms of carbon [7]. In a previous 57 work, the composite behavior has been analytically modeled in 58 59 steady state [6]. The results were in agreement with the experi-

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#### ABSTRACT

Ablation of carbon-carbon composites (C/C) results in a heterogeneous surface recession mainly due to some gasification processes (oxidation, sublimation) possibly coupled to bulk mass transfer. In order to simulate and analyse the material/environment interactions during ablation, a Brownian motion simulation method featuring special Random Walk rules close to the wall has been implemented to efficiently simulate mass transfer in the low Péclet number regime. A sticking probability law adapted to this kind of Random Walk has been obtained for first-order heterogeneous reactions. In order to simulate the onset of surface roughness, the interface recession is simultaneously handled in 3D using a Simplified Marching Cube discretization. This tool is validated by comparison to analytical models. Then, its ability to provide reliable and accurate solutions of ablation phenomena in 3D is illustrated.

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mental observations [7] and provided a good understanding of C/ C composite behavior during ablation. Nevertheless, in order to enable an analytical modeling, the composite architecture had been simplified as much as possible. Mainly, as a first approximation, the fiber section had been considered circular, while the actual section outline slightly undulates around a mean circular line. Moreover, a 3D modeling of the transient behavior cannot be attained through purely analytical methods. This paper aims to present a 3D numerical simulation tool implemented to model with more accuracy C/C composites and their surface roughening during ablation.

Physico-chemical ablation, studied at microscopic scale, typically belongs to heterogeneous reaction/mass transfer problems with a receding surface [2,6,8,9]. The dynamics of nontrivial pattern formation at small scales poses a serious challenge for numerical simulations. A precise integration of the equation of surface motion at the pattern scale is mandatory. For instance, Finite volumes/volume-of-fluid methods represent one possible solution for this type of problems, and are quite efficient for most of the cases [10]; but they may be extremely memory-consuming as soon as large images with possibly complicated patterns are to be dealt

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81 with. A multigrid approach may be a solution to this inconvenient, 82 since the gas concentration field, some distance away from the sur-83 face undergoes small 3D gradients compared to the surface. An-84 other way to tackle this problem is to use a Random Walk 85 description of mass transfer: this method is known to provide 86 excellent results when computing effective transport coefficients, 87 i.e. only an average gas behavior, inside large and complex 3D 88 images of porous media [11]. Random Walk methods are available 89 for the simulation of either continuum [12], transition or rarefied 90 gas transport [13]. However, the algorithms have to be extended 91 in order to account for the heterogeneous reaction. One of the solu-92 tions proposed in the past rests on the coupling of the Random Walk code far from the surface with a Finite-Difference solver close 93 to it [14]. Here it is chosen to address the gas-solid reaction di-94 95 rectly through the use of a suited sticking probability law. This rep-96 resents a lighter algorithm, and allows to handle easily the surface 97 evolution, thanks to a simplification of the Marching Cube method 98 [15] for the surface discretization.

In the first section, the mathematical form of the studied problem is presented. In the second section, the numerical tool itself is presented and validated by comparison to the analytical solution obtained in [6], in the case of circular fibers. In the last section, an ablation simulation is carried out on a realistic fiber section and compared to experimental observations of surface roughness features.

#### 106 2. Model and analytical solution

#### 107 2.1. Ablation model

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The ablation process leads to the recession of the material surface *S* under heterogeneous gasification processes, like oxidation or sublimation [2]. The macroscopic motion of this surface can be interpreted as an advancing wavefront. The surface position may be described in <u>Cartesian</u> coordinates by the following scalar equation:

116 S(x, y, z, t) = 0

such that the function *S*, which possesses first-order partial derivatives almost everywhere, acquires nonzero values at all points not
lying on the interface [16]. This equation may be derived with respect to time

$$\frac{121}{123} \qquad \frac{\partial S}{\partial t} + \mathbf{v} \cdot \nabla S = \mathbf{0}$$
(2)

where  $\mathbf{v} = v J \mathbf{n}$  is the surface local normal velocity, with v the local 124 solid molar volume of the ablated material [16]. In the case of het-125 erogeneous materials, v varies with space. J (in mol m<sup>-2</sup> s<sup>-1</sup>) is the 126 127 molar rate of ablation. In this work, it is assumed to follow a first 128 order kinetic law for oxidation and a Knudsen-Langmuir model for sublimation [2,17]. The Péclet mass number is low close to the 129 wall [7,17]. Under these hypotheses, oxidation and sublimation 130 are shown to be mathematically equivalent as far as gasification 131 132 and mass transfer are concerned [2,17]. Using oxidation notations, 133 the model writes

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$$J = -D\nabla C \cdot \mathbf{n} = -k \cdot C(S = 0)$$
(3)

137 where k (in m s<sup>-1</sup>) is the gasification rate of the material (like v, k138 is a function of space for heterogeneous materials), D (in m<sup>2</sup> s) the 139 diffusion coefficient of the reactant in the bulk fluid phase, and 140 C(S = 0) (in mol m<sup>-3</sup>) is the reactant gas concentration at the 141 material surface. Mass conservation of the reactant in the fluid 142 writes

$$\frac{\partial C}{\partial t} + \nabla \cdot (-D\nabla C) = 0 \tag{4}$$

Boundary conditions for concentration relative to the model domain have to be specified (see for example Fig. 1a). A Dirichlet condition  $(C = C_0)$  on the boundary layer top is shown to be suitable [6]. On lateral boundaries, a periodicity condition is retained. The interface position *S* as a function of space and time is then given by the simultaneous resolution of Eqs. (2) and (4). For heterogeneous materials, this model cannot be solved analytically in the general case.

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Let the analytical solution previously obtained for circular fibers 154 [6] be briefly recalled here since it is a perfect reference case for the 155 following 3D validation of a the simulation tool (see Section 3.2). 156 The general model of Section 2.1 has been applied to the cell rep-157 resented on Fig. 1aa. It represents a carbon fiber embedded in a 158 carbon matrix shown before ablation: the surface is initially flat 159 (no surface roughness). Translation conditions on lateral bound-160 aries and the Dirichlet condition  $(C = C_0)$  on boundary layer top 161 are imposed. Under these boundary conditions, the cell represents 162 a composite yarn, whose lateral extension is infinite, and ablated 163 perpendicularly to its principal direction. Both components are 164 gasified by a first-order reaction (oxidation of carbon by oxygen 165 for example), the matrix being more reactive than the fibers [7]. 166 The concentration gradient of reactant in the fluid phase has been 167 shown to be vertical in steady state [6], that is, along z (see Fig. 1a). 168 An analytical solution to this problem gives the following results 169 [17,6]: (i) Due to the higher recession rate of the matrix, the fibers 170 are partially stripped, become thinner, and acquire a needle shape. 171 (ii) Only two parameters are necessary to describe the steady state 172 surface roughness features of the yarn: a reactivity contrast A and a 173 Sherwood number Sh. The reactivity contrast between the matrix 174 and the fibers A is given by 175

$$A = \frac{k_{\rm m} v_{\rm m}}{k_{\rm f} v_{\rm f}} \tag{5}$$

where  $k_{\rm m}$ ,  $v_{\rm m}$ ,  $k_{\rm f}$ , and  $v_{\rm f}$  are, respectively, the matrix and fiber reac-178 tivities and solid molar volumes, which are assumed constant for 179 each phase. For the standard composites and according to literature 180 data, fiber and matrix densities are of the same order of magnitude. 181 while matrices are neatly more reactive than fibers; hence, one has 182 A > 1 [7]. The geometry of the steady state is strongly linked to this 183 contrast; the higher it is, the higher the denuded fiber size  $h_f^s$  (from 184 fiber tip to matrix level) is when steady state is reached. The Sher-185 wood number Sh writes 186

$$Sh = \frac{k_{\rm m}R_{\rm f}}{D} \tag{6}$$

where  $R_f$  is the fiber radius. It is an indicator of the three reaction regimes that can appear on this heterogeneous reactive surface : a diffusion-limited regime  $(Sh \gg 1)$ , a transition regime  $(Sh \sim 1)$ , and a reaction-limited regime  $(Sh \ll 1)$ . For small values of Sh, the fiber tip geometry is conical and only depends on A, with  $h_f^s = R_f \sqrt{A^2 - 1}$  (see Fig. 1b). When Sh increases but is lower than  $\sqrt{A^2 - 1}$ , diffusional effects tends to lower  $h_f^s$  and the fiber tips become ogival. For higher values of Sh, fibers tips display a plateau and  $h_f^s = R_f (A - 1)/Sh$ . Hence, in diffusion-limited regime,  $h_f^s$  tends towards zero.

This analytical model seems then of great interest for a validation of the numerical simulation tool, as well as for the diffusion and reaction part (all reaction/diffusion regimes are scanned) than for the front tracking because 3D surfaces including singularities (fiber tips) are featured.

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(a) 3D elementary cell : initial time (b) Analytical result : steady state (c) AMA result: steady state

Fig. 1. Validation of AMA by comparison to a 3D analytical model in reaction-limited regime (A = 5).

## 204 3. Numerical solution

AMA has been developed on a Monte-Carlo Random-Walk prin ciple [18–23]. AMA is the acronym for: *Ablation Marches Aléatoires* (Random Walk Ablation in French).

208 3.1. AMA algorithm presentation

A scheme of the general algorithm and a sketch of AMA main features are presented on Fig. 2 a and b. First, a 3D image containing several phases (fluid/solids) is generated. It is discretized as grey-211 scale values on a 3D cubic grid. Then, the fluid/solid interface 212 (Eq. 1) is determined by a Simplified Marching Cube approach 213 [20], which is a linear approximation of the actual surface in each 214 voxel (3D pixel) of the interface. Mass transfer from the bulk gas to 215 the wall (Eq. 4) is simulated by a Random Walk based on the 216 Brownian motion simulation technique, which is a continuum 217 (grid-free), rapidly converging and memory-sparing method for 218 diffusion in a continuous fluid [18]. In order not to degrade the 219 numerical accuracy of the method, it is most convenient to give a 220



Fig. 2. Schematic description of AMA.

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221 constant representativeness to each Random Walker (i.e. a con-222 stant number of actual gas molecules associated to each walker). 223 In addition, one has to satisfy the Dirichlet boundary condition 224 on the upper part of the fluid phase. This is achieved through the 225 use of a buffer zone in which a constant number of walkers is en-226 sured. The total number of walkers in the rest of the fluid phase re-227 sults from the diffusion/reaction process, that is, the coupled resolution of Eqs. (3) and (4). As represented on Fig. 2b, the path 228 229 followed by a walker is not the physical one dictated by gas kinet-230 ics, but another one based on Brownian motion simulation technique that requires less computational time [18]. One of the 231 232 properties of Brownian motion is that, in average, the space increment  $\Delta r$  covered by a molecule is related to the time increment  $\Delta t$ 233 by the following equation: 234 235

$$\Delta t = \frac{\Delta r^2}{6D} \tag{7}$$

238 where D is the diffusion coefficient. Using this property for simula-239 tion purpose, the size of the space increment has obviously to be 240 chosen as large as possible to decrease the computational time, 241 but small enough when compared to the characteristic size of the 242 problem. A parametrical study must then be carried out to deter-243 mine the relevant space increment; it is the equivalent of a mesh 244 study for finite element methods. The trajectory of each walker is 245 followed individually during a given time interval  $\Delta t_e = N_s \Delta_t$ , 246 where  $N_s$  is a chosen number of steps. The value of  $\Delta t_e$  is evaluated 247 to ensure a walk duration leading to a probability of about 50% to 248 hit a wall. In general,  $N_s$  is rather large. At each step, the walk direction is determined randomly and equiprobably in 3D space. The 249 250 heterogeneous first-order reaction on the wall (Eq. 3) is simulated 251 using a sticking event based method [19]. Let us assume that a 252 walker hits the surface after a walk duration  $\Delta t_w$  ( $0 < \Delta t_w < \Delta t_e$ ). 253 A random number between 0 and 1 is generated from a uniform dis-254 tribution. If that number is higher than a previously determined 255 sticking event probability, the walker bounces back and its walk goes on until either it hits the wall again or  $\Delta t_e$  is reached. In the 256 converse case, the walker is assumed to stick; another walker is re-257 258 leased in the buffer zone, for a walk duration of only  $\Delta t_e - \Delta t_w$ . This 259 ensures that in steady state conditions the flux received by the wall 260 is equal to the flux entering the simulation box by its upper side.

261 Every sticking event is associated to the solid node which lies 262 nearest to the exact location where it takes place. The amount of 263 sticking events is recorded for every solid node of the fluid/solid 264 interface. Whenever for a given node the cumulated number of 265 sticking events reaches a given threshold, that is, when the re-266 quired quantity of reactant to gasify one voxel is reached, it is con-267 verted into fluid. Then, the surface is modified and has to be 268 updated (Eq. 2). When a significant number of solid voxels is 269 turned into fluid, an image is recorded. When an appreciable num-270 ber of fluid voxels has been created (say, the volume of one hori-271 zontal slice of the image), the resolution domain is shifted 272 downwards by the corresponding amount (one voxel height), in or-273 der to preserve a constant fluid volume throughout the simulation. 274 The average ablation velocity can be inferred using the frequency 275 of geometry shifts. The average ablation velocity and reactant flux 276 are compared with the value obtained at the previous geometry 277 shift operation. When the flux is constant from one run to another, 278 this means the steady state is reached, and then the simulation 279 ends. The steady state may also be visualized by 3D rendering 280 and characterized by image analysis. Let some critical steps of 281 the algorithm be further detailed.

To handle accurately the Dirichlet boundary condition, a parameter variation study has shown that the size of the buffer zone should be at least twice as large as the space increment  $\Delta r$  of the Random Walk. As sketched on Fig. 2b, using a buffer zone, the boundary condition is applied in practice at a position  $\Delta r$  inside that zone, the size of the concentration boundary layer  $\delta_c$  being then obtained by the sum of the fluid phase height (buffer excluded) and  $\Delta r$ .

The choice of the space increment size is critical as far as accuracy and computational time are concerned. In the bulk fluid phase,  $\Delta r$  has to be taken as large as possible, in order to lower the computational time. Conversely, close to a wall, a parameter variation study has shown that  $\Delta r$  should be smaller than the characteristic size of the surface roughness (fiber radius in our case for example). Indeed, in the converse case, some zones may be numerically shaded. Moreover, some conditions have to be respected for the walker diffuse reflection to insure mass conservation. When a walker hits the wall, its last space increment  $\Delta r_w$ , which leads to collision, is smaller than  $\Delta r$  (or at most equal). In this algorithm, the walker bounces back according to a diffuse emission law (Knudsen's cosine law) with a space increment  $\Delta r_{w}$ . Using this bouncing rule, the average space increment size close to the wall automatically decreases, leading to a more accurate simulation of surface recession. Compared to finite element, this is the equivalent of an adaptative mesh refinement that follows the wall recession.

The heterogeneous first-order reaction on the wall is simulated by a sticking event with a probability  $P_s$  adapted to the Brownian motion simulation technique. By definition, the sticking probability is the ratio between the number of molecules that are consumed (or stuck) on the wall and the number of molecules hitting the wall. In term of molar fluxes, this writes [19].

$$\tilde{P}_{\rm s} = \frac{\Phi_{\rm R}}{\Phi_{\rm MT} + \Phi_{\rm MA}} \tag{8}$$

where  $\Phi_{\rm R} = Ck$  is the molar flux consumed on the wall.  $\Phi_{\rm MT}$  is the 317 molar flux transported to the wall by mass transfer, which is equal 318 to  $\Phi_{R}$  in steady state (mass conservation). In the case of ablation or 319 condensation, the solid surface velocity is slow compared to mass 320 transport because of the low solid to gas molar volume ratio; hence, 321 surface evolution and mass transport are decoupled and mass 322 transport can be considered as a steady state phenomenon.  $\Phi_{MA}$  is 323 the molecular agitation flux. In the case of Random Walks following 324 Maxwell distribution, this flux is called half Maxwellian flux and 325 writes  $1/4C\langle v \rangle$ , where  $\langle v \rangle$  is the molecular agitation mean velocity. 326 The numerical velocity of a walker hitting the wall is  $v = 6D/\Delta r_w$ 327 (from Eq. 7). The mean quadratic length  $\langle \Delta r_w \rangle$  is potentially a func-328 tion of the surface roughness or the presence of notches. Therefore, 329 to maintain the accuracy of the simulations in the case of complex 330 geometries, it is important to keep  $\Delta r_{\rm w}$  itself in the numerical meth-331 od, the mean quadratic length being computed during the simula-332 tion. To account for the fact that the Brownian motion Random 333 Walk implemented does not follow a Maxwellian distribution law, 334 the introduction of a correction coefficient  $\gamma$  in the relation of the 335 molecular agitation flux is proposed 336 337

$$\Phi_{\rm MA} = 3/2\gamma CD/\Delta r_{\rm w} \tag{9} 339$$

Combining Eqs. (8) and (9),  $P_s$  is found to write [24]

$$\tilde{P}_{\rm s} = \frac{1}{1 + \frac{3\gamma D}{2k\Lambda_{\rm Fw}}} \tag{10}$$

A numerical study is carried out in the validation section to validate 344 the existence of  $\gamma$  for the Random Walk implemented and to assess 345 its value. 346

For ablation study, the key outputs are the local surface recession, whose fluctuations lead to surface roughness, and the global ablation velocity. Since the molar volume of the material is low compared to the gas, the surface evolution is slow compared to gas diffusion. In actual experiments, a huge number of reactant 351

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**Fig. 3.** AMA validation by comparison to a 1-D model ( $\Phi_0 = DC_0/\delta_c$ ).

molecules may be necessary to gasify the volume of one elementary
 cube of solid. Using numerical simulation, it is impossible to have

such a fine description, which fortunately is in no way necessary, especially in view of the discrete mode of geometry representation [19]. Then, the number of walkers can be restricted, using for each walker a representativeness of many reactant molecules estimated as follows. An elementary cube is assumed to be ablated when the corresponding number of sticking events reaches a given number *N*. *N* has to be chosen large enough so that statistical fluctuations are small, but is otherwise arbitrary. Theoretically, the statistical precision is roughly given by  $N^{-1/2}$ ; a practical value for *N* is then higher than 100 [19]. The computational time being approximatively proportional to *N*, the interest of choosing *N* as low as possible is obvious. A parametrical study is also useful here to optimize the computational time, without decreasing the accuracy of the result. This is addressed in the next subsection.

## 3.2. AMA validation

The first step is to try and find a value for  $\gamma$ . Let us use for this purpose the classic 1-D steady state diffusion/reaction model [24], which can be reduced from the general model proposed in Section 2.1. The 1-D reactant flux ( $\Phi$ ) at the interface can be analytically obtained [6] and writes

$$\Phi = \frac{D}{\delta_c} C_0 \frac{1}{1 + \frac{1}{D_a}}$$
(11) 376

where  $Da = k\delta_c/D$  is a Damköhler number. The evolution of the flux 377 as a function of the Damköhler number is illustrated in Fig. 3b. For 378 small Damköhler numbers, the flux is not a function of the reactiv-379 ity (or of  $\gamma$ ); this regime is called **diffusion-limited**. At the other limit 380  $(Da \gg 1)$ , the consumed flux is only a function of the reactivity and 381 the boundary layer concentration. A parameter variation study on  $\gamma$ 382 in reaction-limited regime (Da = 0.001) is presented in Fig. 3a. It 383 shows that if  $\gamma$  exists its value must be 0.8. In the reaction-limited 384 regime, the normalized flux must display a variation proportional to 385  $1/\gamma$  (from Eqs. (11) and (10)). This is verified and shows the 386 existence of  $\gamma$ . The 2% uncertainty in reaction-limited regime is 387



Fig. 4. Fiber morphology at steady state as a function of Sh (with A = 5): qualitative comparison of analytical solution (top) with AMA simulation (bottom).

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Fig. 5. Ablated fiber sections for different reaction/diffusion regimes: comparison of analytical (black lines) and numerical (grey-filled parts) results.

388 attributed to the Simplified Marching Cube description of the sur-389 face that saves computational time but is only a linear approximation of the interface position, leading to a slight error on the 390 391 estimation of the surface area. Using  $\gamma = 0.8$ , a parameter variation 392 is carried out on Da. As shown in Fig. 3b, a correct agreement is ob-393 tained for all reaction-diffusion regimes. The uncertainty is found 394 to be lower that the symbol size for any value of Da. This comes 395 from the fact that the uncertainty decreases with Da because the 396 diffusion process, which progressively dominates with the increase of Da, is independent of the local surface description. 397

Random Walks have the particularity to converge on average at 398 399 a rate proportional to the square root of the number of random drawings. The convergence has been obtained for  $\Delta t_e \ge 100 \Delta t$ 400 for diffusion and  $N \ge 10$  for reaction. In further simulations, these 401 402 numerical conditions are systematically implemented using a pre-403 processor. Increasing the ratio between  $\Delta t_e$  and  $\Delta t$ , that is, decreas-404 ing  $\Delta r$  and increasing N, would lower the amplitude of the 405 oscillations, but would dramatically enhance the computing time 406 without changing the average value. For example, for a cubic cell 407 of 100 pixel side length, it is more efficient and valid to use the

average flux as a reference value, using the following numerical parameters:  $\Delta t_e = 100 \Delta t$ , and N = 10.

The quality of the moving interface description has also to be 410 checked in 3D. The model of Section 2.2 has been solved using 411 AMA, in all regimes for A = 5 (see Fig. 1c where  $Sh \ll 1$ , and Fig. 412 4-bottom). These results are in qualitative agreement with the ana-413 lytical results of Figs. 1b and 4-top, respectively. To enable a quan-414 titative validation, sections of the analytical (black line) and of the 415 numerical solutions (grey-filled part) are compared at Fig. 5. The 416 overall agreement between the two solutions is excellent: the error 417 is not larger than one pixel width. In the computed cases, the aver-418 age horizontal section of the numerical cells is around 100 square 419 pixels, which is a correct compromise between computational time 420 and the quality of the interface description. The computational 421 time is about 24 h with a 3.2 GHz CPU computer. 422

## 4. Application to ablation

At microscopic scale, some roughness features take place on the 424 stripped and pointed ablated fibers. In particular, ex-PAN fibers of 425

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(a) Fibers sections : observation (SEM)



(b) Fiber section : model (AMA)



Fig. 6. Faceted fibers: comparison of experimental observations with a model of irregular fiber section.

3D C/C composites have been shown to be roughly conical but also 426 faceted after ablation [25,6]. On Fig. 6c, some fibers from a 3D C/C 427 428 oxidized under dry air at atmospheric pressure show such facets. The oxidation process as well as the composite are fully described 429 in [7]. The ablation regime is shown to be reaction-limited ( $Sh \ll 1$ ) 430 [6]. In this case, when steady state is reached, fibers with circular 431 cross-section display a regular conical shape (see Fig. 1b and c). 432 Nevertheless, SEM (scanning electron microscopy) analyses of the 433 fibers cross-section clearly show that it is irregular (see Fig. 6a). 434 435 This may have an effect on the steady state geometry and behavior 436 of the fibers. To verify this hypothesis, the irregularities of the fi-437 bers cross-section have been introduced in the model presented in Section 2.2, as represented on Fig. 6b. The steady-state result 438 of the simulation is presented on Fig.  $6d^1$ . As a whole, the large fac-439 ets which are experimentally observed on the fibers (from fiber tip to 440 bottom) are qualitatively well reproduced. The numerical approach 441 thus proves that they are originated from the irregularities of fibers 442 sections. However, some facets, which are smaller, cannot be ex-443 plained by this model. They may be linked to anisotropic nature of 444 the graphitic carbon microstructure [26]. Indeed, a modified version 445

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<sup>&</sup>lt;sup>1</sup> A movie showing the transient regime of ablation, from a flat surface to the steady-state regime, is provided online (see Elsevier website).

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446 of AMA accounting for anisotropic, orientation-dependent reactivity has allowed to show that the microstructure of a polycrystalline 447 448 graphite acquires under ablation a characteristic roughness linked 449 to its grain distribution [26].

#### **5.** Conclusions 450

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A simulation tool useful for the modeling of surface roughness 451 452 onset during ablation has been introduced in its context, pre-453 sented, and validated. First, a general model for receding surfaces 454 under a first-order gasification process with a mass transport controlled by diffusion, which has been shown to explain well the 455 ablation phenomenon, has been presented. To solve this model in 456 457 the general case, numerical simulation tools are useful. AMA, 458 which an efficient solution to solve reaction/diffusion problems 459 with a receding surface based on a 3D numerical simulation code 460 developed on a Monte-Carlo Random Walk principle with a Simpli-461 fied Marching Cube surface discretization scheme, has been pre-462 sented. AMA includes an automatic step refinement close to the 463 wall that lowers the computational time. The reactivity is simu-464 lated using a sticking probability adapted to the Random Walk used (Brownian motion simulation technique). The resulting inter-465 466 face evolution in 3D has been validated by comparison to an analytical model obtained in a previous study. As an application, the 467 ablation of cylindrical fibers with a non-circular basis, embedded 468 in a more reactive matrix, has been studied using AMA. An inter-469 470 pretation of the origin of the large facets experimentally observed 471 on sharp ablated fibers has been proposed. The large facets basically arise from the irregularities of fibers section which are inten-472 sified by ablation. This numerical simulation tool is then useful and 473 474 efficient to analyse and understand surface roughness onset during 475 ablation of composite materials.

476 As an opening, this code could be used as well to simulate other phenomena related to mass loss or mass deposition, including for 477 example crystal growth, surface coating processes, sedimentation, 478 and clogging that involve a similar physics but with a growing wall 479 480 instead of a receding surface.

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#### Appendix A. Supplementary data 486

Supplementary data associated with this article can be found, in 487 the online version, at doi:10.1016/j.commatsci.2008.07.015. 488

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