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## MODELLING OF THE NEGATIVE LIGHTNING DISCHARGE WITH AN EQUIVALENT ELECTRICAL NETWORK

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**Abstract** - In the present work a simplified model of negative lightning discharge taking into account the different phases of the propagation (i.e., the initiation of the first corona, the negative and space leaders and the junction of leaders) is presented. This model uses *LCR* line electrical network parameters derived from electromagnetic field, physical laws and gas discharge theories. Assuming the discharge channel to a long conductor and taking into account criteria for instabilities and some atmospheric conditions, a mathematical model is derived that enables to determine the discharge main macroscopic parameters. The computed results are found to be in good agreement with data reported in the literature.

### 1 INTRODUCTION

While the positive lightning discharge has been the subject of numerous studies, there are only few quantitative results concerning the negative lightning discharge. The literature concerning this later remains very poor in comparison with the positive one. The aim of these numerous studies carried out on both positive and negative lightning discharges was to improve our basic knowledge onto the involved processes and consequently to prevent the dangerous effects on engineering systems (direct hits and radiated electric fields resulting of the leaders and return stroke which can also have devastating effects).

The study of the negative lightning is very important since most cloud to ground lightning in temperate regions of the world (about 90% of lightning discharges) are of negative polarity. Contrary to the positive lightning for which many theoretical models are available [1 - 4], researches to derive models for the negative lightning are still at their earlier stages, due to the complexity of the mechanisms involved in the negative lightning propagation process.

This contribution is the continuity of our previous work related to the positive long discharge in air gaps and positive lightning [1]. A dynamic mathematical model of negative lightning discharge based on the idea that a

substantial similarity exists between the lightning phenomena and the discharge in large air gaps is presented. This model uses a *RLC* dynamic equivalent electrical network the parameters of which vary with time according to the discharge characteristics and geometry (*R*, *L* and *C* being respectively the resistance, the inductance and the capacitance). The equivalent resistances of streamers and leaders are calculated from simplified physical relationships. The equivalent inductances leader channels are deduced from the electromagnetic energy stored in the channel and the electromagnetic field radiated by both positive and negative leader channels. The capacitance of the discharge corresponds to the interval between the head of discharge and the opposite electrode (ground). Assuming the discharge channels to a long conductors and considering criteria for instabilities and some atmospheric conditions, a predictive model of the negative lightning enabling to determine the discharge macroscopic parameters (current and the corresponding charge, propagation velocities, potential drop along its channel, power and energy injected into the gap and a trajectory of the discharge plotted in 3D in real time obtained from a probabilistic distribution) is derived. This model takes into account the different phases of the propagation (i.e., the initiation of the first corona, the negative and space leaders and the junction of leaders).

### 2 GENERAL DESCRIPTION OF THE MODELLING

In recent contribution [5, 6], we have developed a mathematical model enabling to describe satisfactorily the processes in negative long air gaps. The general idea consists in representing the discharge channel by an equivalent electrical network as shown in Figure 1; where  $R_0$  is the plane electrode resistance and  $n$  is the number of leader steps. It assumes the discharge channel to be a long conductor and uses a typical single *RLC* conductor line (*R*, *L* and *C* being respectively the line resistance, inductance and capacitance). *R*, *L* and *C* vary with time

according to the channel characteristics and discharge geometry. The value  $R_0$  is set to  $100 \Omega$  [7]. Note that the pilot system is also represented by a  $RLC$  equivalent network.

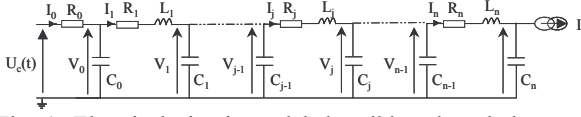


Fig. 1: Electrical circuit model describing the whole negative discharge propagation.

The electrical equations associated to a unit cell  $R_j L_j C_j$  (Fig. 2), corresponding to a partial displacement, are derived

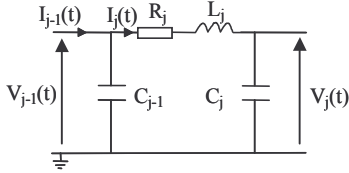


Fig. 2 : The equivalent network unit cell.

$$I_j(t) = C_j \frac{dV_j(t)}{dt} \quad (1)$$

$$V_{j-1}(t) = R_j I_j(t) + V_j(t) + L_j \frac{dI_j(t)}{dt} \quad (2)$$

$$C_{sj}(z) \cdot \frac{dV_{sj}}{dt} = I_j(t) - \frac{V_{sj}(t)}{R_{sj}} \quad (3)$$

$$C_{j-1} \cdot \frac{dV_{j-1}}{dt} = I_{j-1}(t) - I_j(t) \quad (4)$$

The above equations (system of first-order differential equations) are then written in a matricidal form using the state representation

$$\frac{dX}{dt} = AX(t) + BU(t) \quad (5)$$

$$Y(t) = CX(t) + DU(t) \quad (6)$$

$A$ ,  $B$ ,  $C$  and  $D$  are the state matrices corresponding respectively to the dynamic of the system, the control, the observation and the direct transmission.  $U(t)$  and  $Y(t)$  are the input and output matrices.  $X$  represents the current in resistances and inductances, and the voltage at the capacitances. The state matrices are calculated from the topology and values of the circuit parameters.

The resolution of these state equations gives the voltages  $V_o$  and  $V_j$ , and the currents  $I_o$  and  $I_j$ . Then, we compute the total charge  $Q(t)$  injected into the gap.  $Q(t)$  is equal to the sum of the elementary charges  $q_j$

$$Q(t) = \sum_1^n q_j = \sum_0^n \sum_0^t C_j V_j(t) \quad (7)$$

$V_j(t)$  being the voltage across the  $C_j$  capacitance and  $n$  the number of the leader steps.

And using the digital samples of the instantaneous voltage  $V_j$  and current  $I_j$ , one computes the power  $P_j$  and the corresponding energy  $W_j$  injected into the electrode gap

$$P_j(t) = V_j(t) I_j(t) \quad (8)$$

$$W_j(t) = \int_t^{t+dt} V_j(t) I_j(t) dt \quad (9)$$

## 2.1 Initiation and corona development

Because inception and development mechanisms of the first corona are similar to that of positive polarity [8], the electric field required for the first corona inception may be calculated using Peek's formula [ 9]

$$E_c = E_0 \left( 1 + \frac{0.0436}{(r_e \delta)^{1/2}} \right) \quad (10)$$

where  $E_0$  is the breakdown electric field for an air gap between two plane electrodes estimated at  $30 \text{ kV/cm}$ ,  $r_e$  is the equivalent curvature radius of the sharp-edged electrode, and  $\delta$  is the relative air density.

The electric field  $E_t$  at the head of the cathode (sharp electrode: point electrode here) is computed using the hyperboloidal approximation

$$E_t = \frac{2U_t}{r_e \ln 4 \left( \frac{D}{r_e} \right)} \quad (11)$$

where  $U_t$  is the voltage at the point electrode tip and  $D$  is the electrode gap.

The length  $l_s$  of the negative corona is calculated using the graphical construction by considering the electric field within the corona region as constant [10].  $l_s$  is determined by the intersection of the voltage of geometric distribution and that after the corona development (Fig. 3).

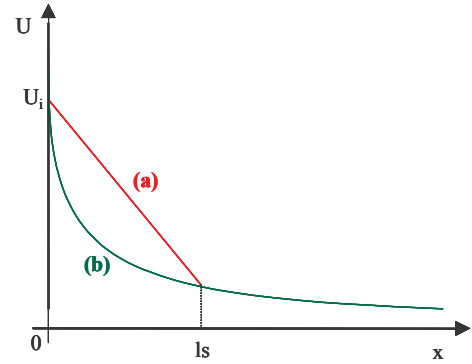


Fig. 3 : Voltage distribution before (a) and after (b) the development of the negative corona streamers.

## 2.2 Development of the pilot system

Both positive and negative coronas are initiated once the criterion related to the electric field at the head of the streamers is satisfied. The electric field at the head of streamers is calculated in the same manner as for the initiation of the first corona

$$E_{s,head} = \frac{2U_j}{r_s \ln \left( \frac{4z}{R_s} \right)} \quad (12)$$

$z$  is the uncrossed gap by the discharge ;  $U_j$  is the voltage at the streamers head ;  $r_s$  is the negative streamers radius

estimated analytically from the relationship giving the electric field of stability [11].

After the extinction of these newly formed coronas, another relaxation process takes place along the pilot system and so on.

### 2.3 Development of electrode and space leaders

The displacements of electrode and space leaders are governed by the current injected at the head of each leader by the corona and pilot systems. This current is calculated thanks to the equivalent electric circuit  $RLC$ . From the value of injected power into the gap (equation (8)), the instantaneous velocity can be computed [5, 12, 13]

$$v_j(t) = \left( \frac{2\beta}{\rho\pi r_l^2} P_j(t) \right)^{1/3} \quad (13)$$

where  $\rho$  is the air density ( $\rho$  is set to  $0.15 \text{ kg m}^{-3}$ ) and  $r_l$  is the leader tip radius at the time  $t_0$  at which it is formed.  $\beta$  is the fraction of the available energy used for the leader displacement ( $0 < \beta \leq 1$ ). Its value is estimated around 0.1.

Using the above relationship, the position of the leader at every time can be determined. Streamers are taken into account by suitable modifications of the equivalent circuit parameters essentially based on our previous modeling of positive discharge in long air gaps [7].

### 2.4 Attachment process

The propagation of the positive head of the leader towards the cathode leads to the junction with the negative leader propagating in the opposite direction. The distribution of the potential at the end of the re-illumination is calculated thanks to a geometrical method and by considering that the electric field in the channel of new leader is null, the injected current being sufficient to allow the thermalisation of the channel leading to the significant drop of the internal field [14]. Thus, the leader channel will be more conductive.

### 2.5 Discharge trajectory and dispersion

To take into account the hazardous propagation of the discharge, we use the same probabilistic law established for the long positive discharge by Hutzler and Hutzler [15]. The complete calculation leads to the determination of  $\varphi$  ( $\varphi$  is a random angle between the segment of the discharge trajectory and the electrode axis). This aspect is taken into account with a law of probability characterised by a dispersion parameter "a" such as

$$P(\varphi) = \frac{|\sin \varphi|}{2a \left( 1 - \exp\left(\frac{-2}{a}\right) \right)} \exp\left(\frac{\cos \varphi - 1}{a}\right) \quad (14)$$

with

$$a = 0.6(1+r)^{-1} \left( 0.8 + 10^{-10} \frac{dU_c(t)}{dt} \right)^{-1} \quad (\text{m, V, s}) \quad (15)$$

where  $r$  is the curvature radius of the equipotential surface at the considered point and  $dU_c(t)/dt$  (in V/s) is the variation rate of the applied voltage at the considered instant. This empirical formula expresses the fact that the tortuosity of the leader are more marked as the field is less uniform and as the voltage increases less quickly. As we work in 3D coordinate, we will take  $\varphi = 2\pi \text{rand}$ ; where  $\text{rand}$  generates a pseudo-random number ranging from 0 to 1 [16].

## 3 EQUIVALENT ELECTRICAL CIRCUIT PARAMETERS

### 3.1 The resistance

**The leader corona:** The leader corona consists of many filamentary channels of streamers. Let  $I_{si}$  be the current flowing in each branch, so that  $\sum I_{si} = I_s$ . Due to the high value of the streamer conductivity,  $I_{si}$  is very small with respect to  $I_s$ . In the following, we consider that the leader corona behaves as a leader portion of length  $S$ . Thus the resistance per unit length can be deduced from the following relationship

$$\lambda_s = \frac{E_s}{I_s} \quad (16)$$

where  $E_s$  is the electric field within the streamer zone.

**The resistance of both leaders:** It is also estimated in the same manner as for the streamer zone. By assuming the channels to cylindrical ones, one can compute the resistance per unit length of the negative leader

$$\lambda_L = \frac{E_L}{I_L} \quad (17)$$

where  $E_L$  and  $I_L$  are respectively the electric field within the leader channel and the current leader.

In the following, we will use a value of  $4.10^3 \Omega \text{cm}^{-1}$  for both negative and spatial leaders [16].

### 3.2 The inductance

To determine the inductance of the channel, we used the simplification proposed elsewhere [7, 16] by ignoring the end effects. Thus, the inductance  $L_u$  per unit length of the channel is the sum of two inductances [7]:

(i) the inductance (in  $\text{Hm}^{-1}$ ) due to the electromagnetic energy stored in the channel (the internal inductance)

$$L_1 = \frac{1}{4} \frac{\mu_0}{2\pi} \quad (18)$$

(ii) the inductance due to the electromagnetic field radiated by the current flowing through the channel

$$L_2 = \frac{\mu_0}{2\pi} \ln\left(\frac{D}{r}\right) \quad (19)$$

leading to

$$L_u = \frac{\mu_0}{2\pi} \left[ 0.25 + \ln\left(\frac{D_f}{r}\right) \right] \quad (20)$$

$D_f$  should be kept large ; a value of  $100 \text{ km}$  is plausible [17].

### 3.3 The capacitance

The capacitance  $C_j$  per unit length between the discharge tip and the opposite electrode (plane) is computed in the same manner as in our previous work by using a spherical approximation [17].

$$C_j = \Gamma_j \varepsilon_0 \frac{D}{L_T} \quad (21)$$

$L_T$  is the total length of the discharge;  $D$  is the electrodes gap; and  $\Gamma_j$  is the solid angle of the discharge (smaller than  $4\pi$ ).

## 4 RESULTS AND DISCUSSION

Let consider for example a cloud - earth distance  $D=2000 \text{ m}$ , this value being in the range of those used in the literature [14, 18]. It is well-known that before a cloud-to-ground discharge, the field at the ground level is of about  $15 \text{ to } 20 \text{ kV/m}$  [15]. Assuming that this field is roughly uniform through the space cloud base - ground, the voltage difference (before the discharge inception) between them can be estimated to a few tens megavolts. Therefore a gap length submitted roughly to a constant voltage of about  $U_c=25 \text{ MV}$  under a specific atmospheric conditions:  $P=100 \text{ kPa}$  and  $T=20^\circ\text{C}$  at the ground level have been considered. As concerns the cloud diameter, a kilometre constitutes a good dimension order [15]. Initially the system may be equivalent to an electrode with a radius  $r_i = 6 \text{ m}$  [18].

Figures 4 to 11 show some simulated results of the downward lightning leader characteristics. An example of trajectory is given in Figure 4. From those results, it can be observed that the simulated current (Figure 5), the charge (Figure 8) and energy injected into the gap (Figure 6) show a real stepping. This is a well known phenomenon concerning the propagation of negative lightning. The current values are in the same range as those observed experimentally [18, 19]. Indeed, the simulated pulse currents of steps range from  $I$  to some  $kA$  and according to the experimental results reported in literature, the steps have pulse currents of at least  $1 \text{ kA}$  [18]. Note that the simulated stepped leader duration seems to be higher of about one order than the average one estimated experimentally. The characteristics of the thermal radius are provided in Figure 9. Its growth is of a few centimeters as observed during natural lightning [18, 19]. The simulated leader instantaneous velocities are of some  $10^6 \text{ m/s}$  (Figure 10) and the simulated average value is of about  $2.10^6 \text{ m/s}$ . Such a value is in a good accordance with that estimated experimentally from streak camera measurements, for the bottom of the channel of stepped leaders (i.e., near the earth) [20 - 23].

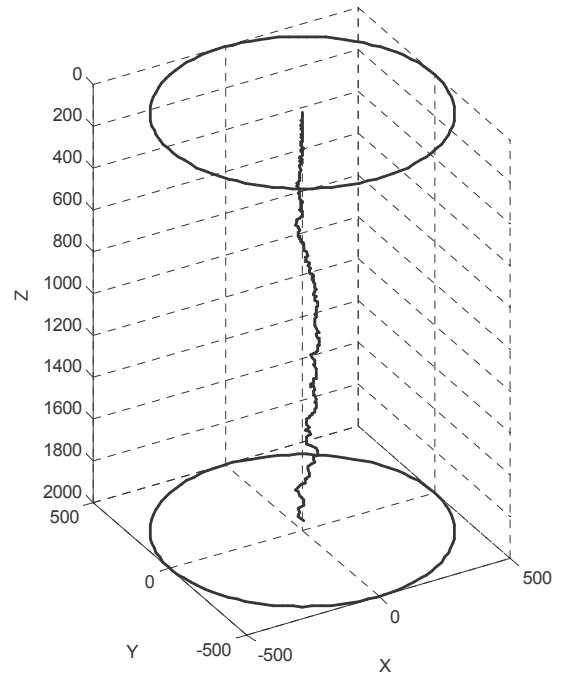


Fig. 4 : Simulated leader's trajectory in space.

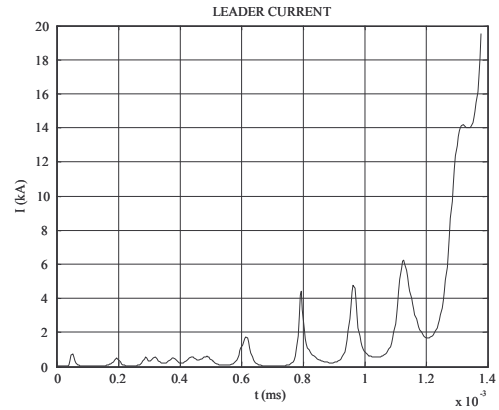


Fig. 5 : Simulated leader's current.

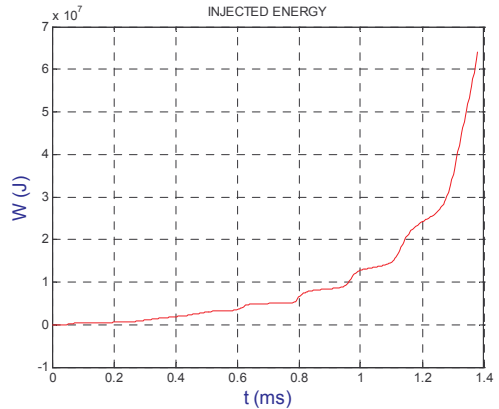


Fig. 6 : Simulated energy injected into the gap

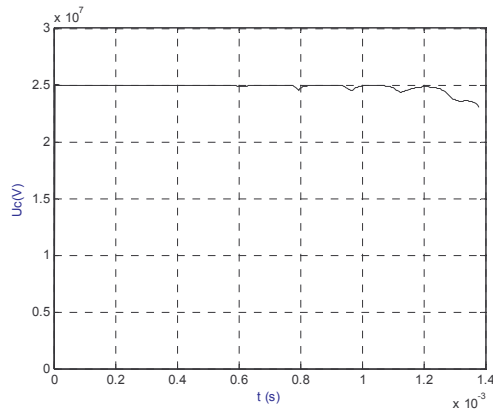


Fig. 7 : Applied voltage between cloud - ground

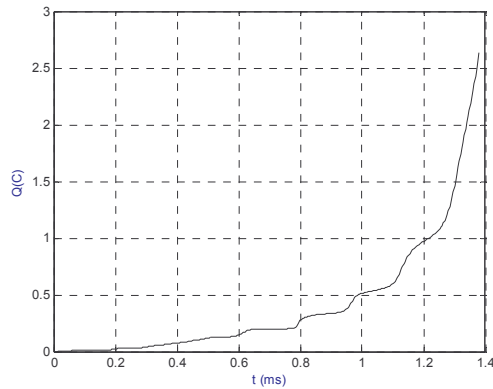


Fig. 8 : Simulated charge injected into air gap.

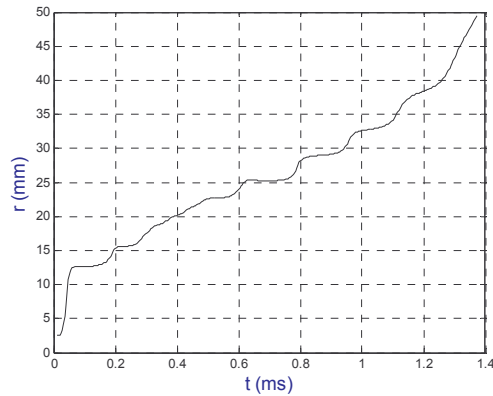


Fig. 9 : Main leader thermal radius evolution.

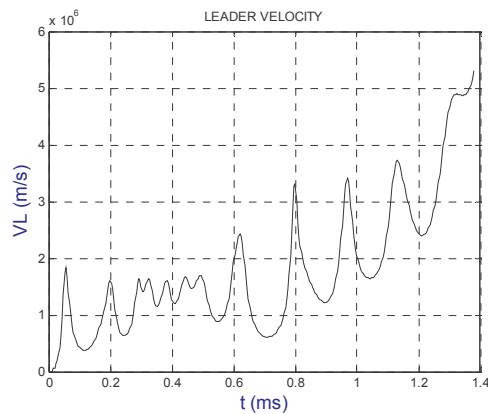


Fig. 10 : Main leader instantaneous velocities.

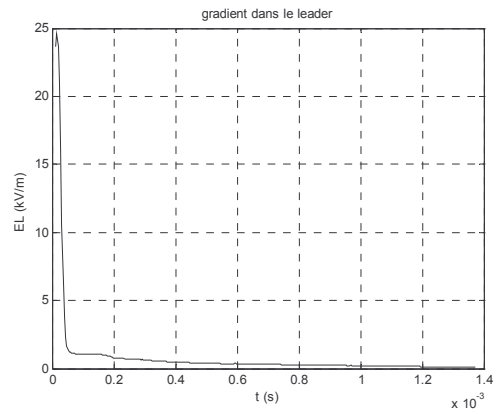


Fig. 11 : The gradient in the main leader.

It's higher of about one order than that estimated experimentally near the cloud base. However, the most often measured two-dimensional speed is between  $1$  and  $2 \cdot 10^5$  m/s [19]. The fact that some simulated results are higher than those experimentally estimated can be due to the values of the gap and the voltage we chose in our simulation.

## 5 CONCLUSIONS

This work demonstrated once more the utility of circuit models in simulating lightning discharge. The self-consistent mathematical model presented enables to predict the negative lightning stepped-leader characteristics (trajectory, current, charge, velocity, energy injected into the gap, leader thermal radius, local electric field, voltage evolution). The obtained results are found generally to be in a good accordance with those recorded during natural lightning. To be more accurate, this model needs to be improved by replacing, for instance, the resistance per unit lengths of leader channel by a value corresponding to real conditions.

The proposed simulator therefore constitutes an useful tool to determine the characteristics and the critical electromagnetic environment of negative lightning stepped leader.

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