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# A random walk in the literature on criminality: a partial and critical view on some statistical analysis and modeling approaches

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## 1 Introduction

Criminology is intrinsically a pluri-disciplinary subject. Along history different schools of thought have proposed different and sometimes conflictive ways of considering crime. We are interested in the possible contributions of mathematical modeling. In an attempt to discover stylized trends worth of being understood through simple models, we referred to numerous and quite recent papers that analyze and discuss empirical data. In the following we summarize part of this literature trying to extract general conclusions. Then we present some recent modeling attempts that may help to understand the large variance in the statistical based conclusions. Certain models are developed more in depth in other papers of this issue.

The reasons why crimes are committed and how and when they should be punished have always been questions of main concern in organized societies. Different attitudes have been adopted to prevent or deter crime, from the *lex talionis* principle (“an eye for an eye...”) and death penalty to different kinds of sanctions more or less correlated with the gravity of the offense, like incapacitation by imprisonment, fines, etc.

One would expect modern law enforcement policies to be based on theories, supported by empirical evidence grasped from statistics but this is far from being the case. For example, there is a widespread belief that a hard sentence not only incapacitates the convicted criminal, but would have a deterrent effect on potential offenders and help to prevent recidivism. Yet, as discussed in section 2, the deterrent effect of punishment is a highly polemic subject. Also, law experts diverge with respect to whether offenders may be rehabilitated or should simply be punished (see for example the recent public discussion in France [1, 2]).

Thanks to data availability in the USA after World War II, showing a dramatic raise of criminality in the mid of the 60’s (see figure 1), crime modeling emerged as

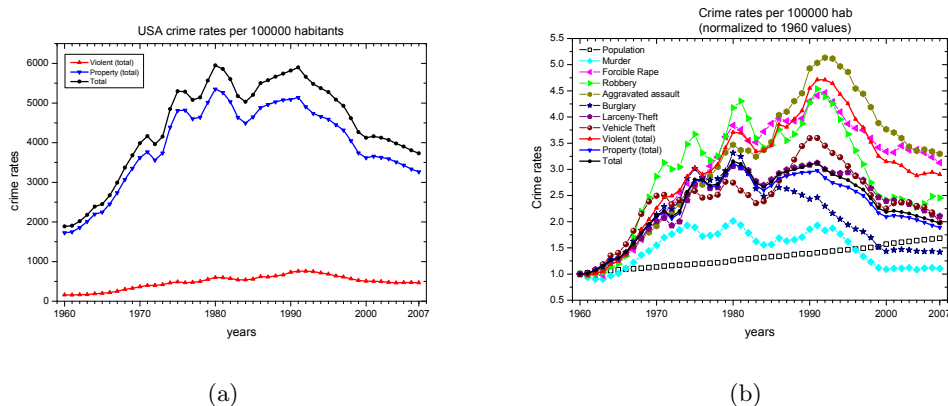


Figure 1: Crime rates in USA (data from U.S. Department of Justice, Bureau of Justice Statistics): (a) Total values per 100 000 inhabitants, (b) Normalized to values in 1960

a field worth of being investigated. Similar trends have been reported in countries of western Europe.

This increase of criminal activity has led some sectors of the population and politicians to ask for harder penalties. In the USA some states implemented harsh law enforcement policies invoking a famous paper called “*Broken windows: The police and neighborhood safety*” published in 1982 by James Q. Wilson and George L. Kelling [3]. The authors claimed, without clear empirical evidence (see Harcourt and Ludwig[4] for a discussion) that “... if a window in a building is broken and is left unrepaired, all the rest of the windows will soon be broken”. The conclusion was that “... the police — and the rest of us — ought to recognize the importance of maintaining, intact, communities without broken windows”. Quite naturally it was deduced that a policy of zero tolerance was the way of preventing crime. This policy, first implemented in New York in 1990, has been followed by Chicago and Los Angeles. In fact, crime rates have dramatically decreased in the following years ... although not only in these but in all (or almost all) the US largest cities [4]. The soaring crime rate between the 60’s and the 80’s and the unexpected quasi monotonic decrease beginning at the 90’s are subject of intense investigation and debate. Why this drop took place and whether there is a marginally larger drop in cities that applied the zero tolerance policies is a question that has not received a clear answer yet.

## 2 Empirical results

Early attempts to discover relationships between crime and social factors using statistics may be traced back to Quetelet, who coined the term “Social Physics” in 19th century [5]. However, it is only recently that crime is being investigated systematically, with attempts to explain observed trends based on statistics and on different life-course surveys. We discuss these approaches in section 2.1.

Other valuable sources of data are longitudinal studies of cohorts, mainly of young

men, containing detailed individual trajectories or “criminal careers”. These approaches are considered in section 2.2.

## 2.1 Statistics

It has become commonplace to argue that statistical data are voluminous and inconclusive. However, statistics are essential to discover stylized facts and validate theories. In this section we summarize some published results and we try to understand the faced difficulties.

A very large production of statistical analyses since the early 70’s try to relate crime rates to possible explicative variables, generally through least squares linear regressions. The models assume that

$$crime\ rate = f(\text{explicative variables}) \quad (1)$$

where  $f(\cdot)$  is a linear function and the explicative variables are selected by the authors depending on the question they are interested in and on data availability. Among the explicative variables considered in the literature we find average income, inequality, gender, age, education level, race and/or any other variable thought to be related to delinquency.

Geographically aggregated statistical data, based on justice or police contacts — sometimes corrected for systematic biases and underreporting—, are obtained from governmental agencies. They may present some inaccuracies due to the fact that they are not specifically collected for research. Many of the conclusions in the literature are drawn from American data issued of the Uniform Crime Reports (UCR), collected annually since 1930 by the FBI (Federal Bureau of Investigation). Maltz and Targonski [6] cautioned in 2002<sup>1</sup> that, although available, data from the UCR at the City and County levels provided by the FBI’s UCR Program are unreliable. They may have non-systematic errors, which cannot be corrected, due to a lack of reports from some local agencies. Apparently, few researchers were aware of these problems, which may have spoiled investigations at the level of individual cities.

Data sources of other countries are less easily available, and have been much less investigated, but the conclusions drawn by different authors are contradictory. For example, in a survey of crime factors in Germany, with data of 11 German Laender over 1975-1996 (all types of crime), Entorf and Spengler (2000) [7] conclude that higher income and also higher urbanization *are* associated with higher crime rates, while Fajnzylber et al (2002) [8], based on data of homicide and robbery from  $\approx$  45 countries over 1970-1994, conclude that average income is *not* correlated with (violent) crime, and that higher urbanization is associated with higher robbery rates but not with homicide rates.

As Eide [9] points out in an interesting survey of statistical approaches of criminal behavior, equation (1) is meaningful only if the explicative variables are not simultaneous with the crime rates. Otherwise, one cannot consider the arguments of the function as actually “explicative” ; they may at best (if the coefficients are significative) be correlated with crime rates, but such an equation does not allow to deduce that

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<sup>1</sup>A warning exists now on the FBI’s web site

these variables are “causes” of crime. For example, if crime rates and probabilities of punishment are negatively correlated, one cannot distinguish between the hypothesis that higher probabilities of punishment cause lower crime rates or the hypothesis that higher crime rates cause lower probabilities of punishment (because of police overloading). However, many studies consider simultaneous (cross-sectional) data.

In some cases the data corresponding to the explicative variables of the theory are unavailable and authors use more or less ad hoc “proxies” whose correlation with the variables they are supposed to represent is not always convincingly demonstrated. These procedures introduce biases that are seldom if ever discussed. A non exhaustive list of proxies used in the literature includes female participation rate in the labor market as a proxy for the proportion of unguarded homes [10] ; number of police personnel per 100,000 inhabitants and the existence of death penalty as a proxy for severity of punishment [8] ; per capita drug arrests as a proxy for certainty of punishment [11] ; death rate among prisoners as a proxy for prison conditions [12] ; demographic characteristics of those who get arrested as a proxy of the demographic characteristics of those who offend [13] ; false arrests serve as a proxy for the degree of harassment of the community by the police [14] ; arrest information as a proxy for criminal offending – which is common in the crime literature (error-free criminal offending data does not exist) ; the arrest date is a proxy for the date of crime commission (due primarily to a reporting problem – officers do not always submit information on the offense date [15]) ; the mean family income as an indicator of illegal income opportunities for criminals [16, 7].

A same proxy may be given almost opposite interpretations in different investigations: *“Because illegal income opportunities cannot be directly measured, a proxy is needed. Ehrlich (1973) [16] proposes the mean family income as such a measure. He argues that higher income means a higher level of transferable assets in the community and, thus, more lucrative targets for potential criminals. Other authors use the same variable to measure legal income opportunities. They argue that higher absolute wealth is an indicator for more rewarding legal jobs. Which interpretation is more appropriate?”* [7].

Nevertheless, some results of the statistical analysis are considered as being robust. For example, there is strong agreement on the fact that crime is correlated with age [17] (most criminals are young) and gender [18] (7% of the US workforce were men under supervision of the criminal justice system in 1993).

Methodological problems may also be the source of disagreement between studies. Recently, Spelman [19] discussed in details the relationship between crime and prison rates, comparing conclusions of 13 published studies on similar data using different technical specifications for the analysis. He shows that the discrepancies in the results depend on the methods rather than on real differences in the data sets. The author shows that, not surprisingly, there is a strong time auto-correlation of both variables (crime rates and prison population): most criminals active at some year were active the preceding year and will remain active in the next future, and similarly for the number of inmates. Thus, changes can only be expected on the long run (7 to 10 years), depending on the characteristics of the younger cohorts and the implemented policies. The divergence in the findings are mainly due to the treatment of time series

correlations. Some studies relied on differences (variables are expressed in the form  $\Delta X_t = X_t - X_{t-1}$ ), others used the values  $X_t$  and adjusted for time correlations, and others did not account for serial correlation at all. Although in the short run current crime rates and prison populations may appear correlated, it is not clear whether there is any causality link between them.

The conclusions of regressions on available data are thus considered with circumspection. There is a long list of discrepancies between studies. Among the contentious variables thought to be relevant to explain crime rates we find:

*Punishment and deterrence.* There are two different aspects of punishment: the frequency at which illegal actions are punished (which corresponds to the punishment probability in the models), and the severity of the punishment itself. The main question in debate is whether frequency and severity have any deterrent effect (besides incapacitation during the imprisonment period) on the punished offender and on would-be offenders. Early results by Ehrlich [16, 20], who found a deterrent effect of capital punishment, have been shown to be very sensitive to statistical treatments [13]. A recent meta-analysis of 104 published studies corroborates this unexplained sensibility: conclusions from time series and panel data give evidence for a deterrent effect of executions while cross sectional data do not [21].

There is some agreement that crime rates are (negatively) correlated with the probability of punishment but not with punishment severity [9]), but it is not clear whether the deterrent and incapacitation effects of incarceration are greater than any criminalization effects of the incarceration.

More recent investigations cast doubts on the effectiveness of incarceration itself as a crime control policy. For example, based on a meta-analysis of 117 published studies involving slightly less than half a million offenders, Smith, Goggin and Gendreau [22] conclude that different types of sanctions (incarceration or community-based sanctions) did not produce decreases in recidivism neither on juveniles, females, or minority groups. Moreover, there were some indications that increasing lengths of incarceration were associated with slightly greater increases in recidivism. They conclude that prisons and intermediate sanctions should not be used with the expectation of reducing criminal behaviour.

Villetaz, Killias and Zoder [23] compare recidivism of offenders according to whether they have been incarcerated or punished through an alternative non-custodial sanction (typically probation). After examination of more than 3 000 published and unpublished abstracts they keep only 27 studies considered sufficiently rigorous. They find that the rate of re-offending after a non-custodial sanction is significantly lower than after a custodial sanction in 11 studies, in 14 there is no significant difference on re-offending between both sanctions, and two out of 27 comparisons are in favour of custodial sanctions. Despite these results, that favor non-custodial sanctions, no significant difference in recidivism is found in a meta-analysis restricted to four available controlled (randomized) trials and one natural experiment judged by the authors to have reached the required standard. In contrast with the study by Gendreau and collaborators cited above, prevalence of offending is found to decrease after any type of sanction or intervention, independently of its severity.

A very recent study based on a natural experiment in Italy [24] corroborates the general deterrence hypothesis: increasing the expected punishment lowers the propensity to recommit a crime (at least within the seven months following release). However, this effect decreases with the length of the prison spell: more severe punishments are found to decrease the sensitivity to the threat of future punishment.

One has to keep in mind, when comparing results on deterrence, that they crucially depend on the time periods considered to observe recidivism.

*Police.* The belief that strong police forces reduces crime has been challenged. Realizing that the number of police officers in the U.S.A. increases mostly in election years, Levitt [25] considered the consequences of this variation (uncorrelated to crime) on crime rates. He finds that such increases substantially reduce violent crime, but have a small impact on property crime. These results have been criticized [26] and the debate that followed [27] shows how difficult it is to assess the impact of police on crime. On the other hand, the social benefit of reducing crime is not necessarily larger than the cost of hiring additional police. For example, Freeman [18] estimated that the overall cost of crime in the US in 1993 was of the order of 4 percent of the GDP, 2 per cent lost to crime and 2 percent spent on controlling crime. This amounts to an average of about 54,000 dollars/year for each of the 5 million or so men incarcerated, put on probation or paroled in that year. Interestingly, this number grew up to about 7,4 million in 2006. If the unit costs remained the same, the overall cost of crime would reach 400 billion dollars per year.

There is also a long standing debate about the zero-tolerance policy in New York, which, according to Kelling and Sousa [28], had a positive impact. In a recent publication, Harcourt and Ludwig [4] criticize their data treatments and, using evidence from a social experiment involving 4 800 low income families, conclude that there is no support for a simple first-order disorder-crime relationship, and that broken windows policing is not the optimal use of scarce law enforcement resources. Although recent experiments show that disorder helps spreading of disorder [29], it is not evident that misdemeanors and more serious crimes belong to the same category of delinquency, requiring the same policy.

*Unemployment.* There is no agreement on the effect of unemployment rates on crime rates [13]. Levitt [30] argues that it is difficult to obtain significative insight on this issue from aggregated time series data at the national level, because unemployment has a very large variability at the local level. Unfortunately, as already mentioned, local crime rates data in the US may be flawed.

In Europe, the mentioned study of crime in Germany [7] takes into account the diversity of unemployment figures in the different Laender, and finds that unemployment has no significant effects on crime rates. Unfortunately, the worldwide panel study by Fajnzylber et al. [8] does not consider unemployment among the independent variables.

*Income inequality.* The relationship between income inequality and victimization is also controversial. Becker's economic model of crime would suggest that as income distribution becomes wider, the richer become increasingly attractive targets for the

poorer, and early results [31] seemed to confirm that. In the mentioned cross-country analysis [8] both income inequality and economic growth turn out to be robust determinants of violent crime rates. As discussed below, a non linear model [32] suggests that what matters is the fraction of the population below some wealth threshold, and not the global measure of inequality used in standard linear regressions.

On the other hand, Levitt [33] finds that, probably because rich people engage in behaviors that reduces their victimization, the trends between 1970 and 1990 is that property crime victimization has become increasingly concentrated on the poor.

*Others.* There is a famous controversy concerning the evidence put forward by Donohue and Levitt [34] that legalization of abortion in the 70's explains 50% of the drop in criminality starting at the 90s, roughly eighteen years after. The authors argue that the drop is due to the lack of unwanted children that would have otherwise been raised in conditions that favor criminal behavior. To come up against criticism [35, 36], the authors provided new evidence in favor of this claim [37, 38].

Another subject thoroughly investigated is the influence of the education level. Ehrlich (1975b) [39] argues, using statistics of property crimes committed across the U.S. in 1960, that committing crimes against property is attractive for young age individuals of low educational level, but crimes involving fraud, embezzlement and illegal commercial practices bear a positive relationship with the average number of school years completed by the adult population. On the other hand, Tauchen and Witte [40] find that having a high school degree has no significant effect. More recently, Lochner and Moretti [41] found that schooling significantly reduces the probability of incarceration and arrest. However, these results are based on types of crimes that do not allow to discriminate between the two categories pointed out by Ehrlich.

In conclusion, after an overview of the literature we identify two different kinds of problems faced by statistical studies. On one hand, data sources are not always reliable because crime data are not "given", they contain gaps that are generally filled by the Agency collecting the data. One should care about how this is done, and whether the same methodology has been used for all the time covered by the analysis [6]. On the other hand, there are methodological issues concerning the statistical treatments of data, so that the results should be interpreted with care. Some authors claim that only well randomized controlled trials can give useful and unbiased information about the crime variables [42, 43].

Graphic representations may prove to be useful when analyzing data, although they are not currently used. Consider for example the discussion about the possible correlations of crime with inequality and unemployment in Buenos Aires [44, 45]. Publication [45] performs different regressions, including dummies to test in particular whether the Mexican tequila crisis affected the Argentine economy, and others, concluding that crime is better correlated with inequality than with unemployment. Clearly the time series is too short to extract general conclusions but, since the data have been published together with the regression analysis, we use them here to exhibit the interest of graphic representations. Figures 2.1 represent the bare data (inequality – measured by the Gini index –, unemployment rates and inflation) together with crime rates, as a function of



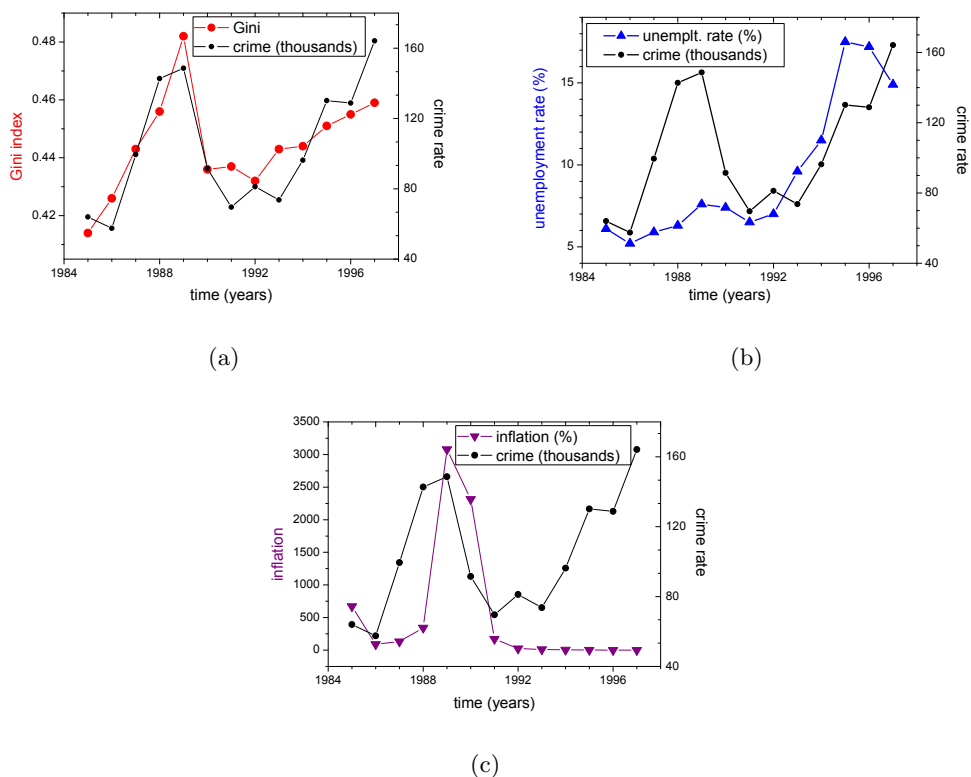


Figure 2: Graphic representation of Buenos Aires data

time. It is clear that, at least qualitatively, crime rates over the *overall* period are correlated with inequality more closely than with unemployment or inflation. However, what is most striking on these figures is that the phenomena follow very different patterns before and after 1991, the year at which the Argentine Government established the parity peso-dollar, a huge shock in the economy, in order to control hyperinflation. Before 1991 crime rates are extremely well correlated with inequality and there is also a positive correlation with one-year-lagged inflation (surprisingly, and probably meaningless, increasing crime rate seems to prefigure inflation growth), but *not* with unemployment. After 1991 crime rates are correlated with both Gini index *and* unemployment, but not at all with inflation, because due to the parity the latter remains almost constant. This change was probably captured by the dummy D90 introduced without any comment in some regressions. We see that the graphic representation allows to discover just by visual inspection all the results obtained using state-of-the-art statistical analysis. The latter, guided by these visual tools, helps quantifying the qualitative findings.

LaFree [46] pointed out that one of the more striking facts in criminological studies is ahistoricism, rooted in the assumption that theory and history should be separated. This example also shows dramatically that economic shocks (or others, like beginning or ending of wars, etc.) may give raise to phase transitions, i.e. sudden changes in the behavior of the system, and should not be ignored. Moreover, phase transitions between

different regimes may arise endogenously, due to social interactions. Regressions in that case should be done separately for each regime. Linear regressions on all the data may completely miss such nonlinearities.

## 2.2 Criminal careers

Micro-level longitudinal research designs give information about how criminal behavior evolves over time within individuals (see the special issue of the *Journal of Quantitative Criminology* for recent contributions [47]). In these studies, a set of individuals are followed-up along a portion of their lives. They raise ethical problems due to fear of their use for forecasting individual trajectories based on very early “symptoms”, without taking into account that statistical data cannot predict individual behavior. Interestingly, Quetelet warned against such use of statistics in the introduction to his book [5]: *Ces lois, par la manière même dont on les a déterminées, ne présentent plus rien d'individuel et par conséquent, on ne saurait les appliquer aux individus que dans de certaines limites. Toutes les applications qu'on voudrait en faire à un homme en particulier seraient essentiellement fausses ; de même que si l'on prétendait déterminer l'époque à laquelle une personne doit mourir, en faisant usage des tables de mortalité*<sup>2</sup>. Recently, T. Mathiesen [48] has strongly criticized the use of statistical results as justifications of Court decisions.

Longitudinal data of criminal careers, including self-reported delinquency from systematic cohort studies, are available mainly in the USA and in Great Britain. For example, the Pittsburgh Youth Study [49] (started in 1987 and ending in  $\approx$  2000-01, with data of about 1 500 boys of three different school grades that were periodically interviewed at annual intervals) contains the number of self-reported acts of delinquency committed by each boy. There are two Philadelphia Cohort Studies [50, 51] that contain social and demographic data of more than 10 000 boys born in 1945 (followed from ages 10 to 18) and in 1958 (followed from ages 4 to 26) respectively. The Rochester Youth Development Study (RYDS) is a multi-wave panel study (starting in 1988) of the development of delinquent behavior among adolescents and young adults ; self-reported acts of a panel of 1 000 adolescents, among which 12% are non-offenders, span a period from when the average age of sample members was 13.5 to when the average age was 22. The Cambridge Study [52] contains data of  $\approx$  400 young males in Great Britain over the period 1961-1981, selected because of the prior expectation of a high prevalence of convictions (about a quarter) among them. Interestingly, about 60% were never convicted during the 20 years spanned by the study ([53], see also [54]) meaning that the sample contains a relatively low number of offenders (about 160 individuals), a problem that also exists in the other data bases. Besides the small number, there are other difficulties with longitudinal data. First, some rely on offenders self-reported criminal behavior, which is not above suspicion [48]. Another is attrition, which may introduce a possible bias on studies because one cannot exclude that individuals who

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<sup>2</sup>These laws, due to the way they have been obtained, do not represent anything individual and consequently should be applied to individuals only within certain limits. All the applications to an individual in particular would be essentially false ; in the same way as if one would pretend to determine the time when a person has to die using mortality tables.

drop out of a longitudinal study differ in important ways from individuals who do not [55].

In the following we present some published results.

An investigation carried out on the Cambridge data base considered the number of convictions  $n$  of each individual. The frequency  $f(n)$  with which each number of convictions was observed follows a power law as a function of the number of convictions,  $f = \alpha n^\beta$  [54]. This corroborates the general observation that a small number of offenders are responsible for a disproportionate share of total crime [18]. For example, in the Philadelphia 1946-cohort study, there are 3 595 juvenile offenders who are responsible of 10 214 offenses. A power law distribution may reflect the existence of social interactions [56]. However, one should keep in mind that these conclusions are based on a small data set and for slightly more than one decade of the independent variable  $n$  ( $0 \leq n \leq 14$ ). The most conspicuous characteristic of a power law distribution is that it has a fat tail at large values of  $n$ , but here the range of the latter is intrinsically restricted, since it cannot be larger than the total number of offenses committed by a single individual. Interestingly, the self-reported offenses of the Pittsburgh Youth Study follows a similar distribution [54].

Policy makers have been seeking to reduce crime more efficiently by targeting corrections at the frequent offenders. The utility of this selective incapacitation has not been demonstrated yet. Individual longitudinal data have been used for several studies. In particular, a study based on the Philadelphia Birth Cohort data fails to find any evidence to support the argument that selective incapacitation is a practical strategy for crime reduction [57]. In a recent simulation study, Blokland and Nieuwbeerta [58] considered data from the “Criminal Career and Life Course Study” carried out at the Netherlands Institute for the Study of Crime and Law Enforcement, charting the complete criminal careers of a large number of individuals. The authors estimate, through simulations, the incapacitative effects of alternative selective prison policies. According to their results, costs of imprisonment typically exceed benefits gained from crimes prevented. In other words, selective incapacitation may not yield a positive societal result, a conclusion reached also by other studies [59]. A recent article by French and Heagerty [60] reviews available statistical techniques for analyzing longitudinal data in the context of evaluating a policy change. Piquero and Blumstein [61] argue that improved estimates of incapacitation will come about only through greater knowledge about individual crime-committing behavior.

Another kind of studies try to identify typical patterns in criminal trajectories to implement selective incapacitation. The purpose of selective incapacitation is to “select” those particularly prone to violence and to incapacitate them. Longitudinal studies might help to predict individual behavior by looking at the corresponding offending career [62, 47]. Clearly this poses ethical and methodological problems, discussed in particular by Mathiesen [48] in a controversial paper. He argues that predictions are highly questionable in criminology because they are based on very inaccurate data. Experiments involve a large number of false predictions which in turn raise serious ethical problems because such predictions “... involve the intended infliction of pain, in the form of punishment or penal sanctions, on the individual level.”

Some methodological problems are related to the non-uniqueness of the results. In order to extract general conclusions from longitudinal studies, one needs to cluster together similar individual trajectories. The properties of clustering methods and algorithms are mostly studied by the community of Machine Learning research [63, 64, 65, 66]. Clustering algorithms need much larger amounts of data than those available in the above mentioned databases. They use a similarity measure to compare couples of data. Results highly depend on this measure. Even mild differences in data encoding, like using either offending frequencies or numbers of offenses, may result in quite different clustering results. Also, depending on the clustering algorithm and in some cases on the order of the data, very different solutions may be reached. These techniques, mostly implemented for analysis of large data sets, pattern classification, dimension reduction, etc. have been used in recent papers to characterize prototypical criminal behaviors [67, 68, 53]. However, due to the small size of the data sets, some clusters in these studies have too few individuals, with the risk of being artifacts.

Finally, let us mention recent attempts to use graphical methods for studying longitudinal statistical and idiosyncratic trajectories [69, 70].

### 3 Models in criminology

Mathematical models using tools or concepts from economy, biology, statistical physics, as well as multi-agent simulations, contribute to the field of criminology with their own methods. These models do not try to predict actual data but to explain stylized facts and understand the consequences of different hypothesis. In his paper “Connecting the dots”, R. Rosenfeld [71] points out that *“The role of theory in criminal justice is no different than in any other policy endeavor. Theory sets priorities for research, organizes otherwise disparate research findings, and links research to policy and practice. The relationship between theory and research is not a chicken-and-egg proposition. Theory comes first”*.

In a review paper, Alfred Blumstein [13] traces back the recent interest in crime modeling to 1966, when the USA President’s Commission on Law Enforcement and Administration of Justice created a Task Force on Science and Technology. Composed mainly by engineers and scientists, its aim was to introduce simulation modeling to evaluate the resource requirements and costs associated to a criminal case, from arrest to release, by considering the flow through the justice system. For example, it estimated the opportunity of incarceration of convicted criminals and the length of the incarceration time. This kind of approaches belong to “engineering modeling” [72]. Here we are mainly interested in mathematical “scientific models”, concerned with understanding the mechanisms at work in reality. Although the boundaries are not strict, in the following we distinguish economic approaches from what we call hereafter “social models”, that include non-linear effects and social interactions explicitly, like statistical mechanics, differential equations and multi-agents modeling.

### 3.1 Economic models

Although the idea that the decision of committing a crime results from a trade off between the expected profit and the risk of punishment dates back to the eighteenth and nineteenth centuries (Beccaria, Bentham), it is only in 1968 that the modern economic approach to crime modeling was initiated by G. Becker [73].

Economic theory of criminality describes illegal behavior as the result of a rational choice. Becker postulates a social loss function  $L$  which includes costs and benefits of crime. Its minimization determines how many resources and how much punishment should be used to enforce the law.  $L$  depends on the number of offenses  $O$ , the probability of conviction  $p$  and the costs to offenders due to punishment  $f$

$$L = D(O) + C(p, O) + fpO \quad (2)$$

where  $D(O)$  is the social loss (damages) from offenses (produced harm minus gain to offenders),  $C(p, O)$  is the social cost of incapacitation factors (arrest and conviction), and the last term represents the loss to the  $pO$  convicted criminals.

At the minimum of the social loss neither criminals, private individuals nor government can expect to improve their benefits by changing their behaviors. This corresponds to the “equilibrium” between the “demand” of (or tolerance to) crime – reflected by the expenditures for protection and law enforcement – and the “supply” of crimes – reflected by the cost to offenders.

Under simplifying assumptions about the convexity of functions  $C$  and  $D$ , the model determines how many offenses should be permitted and how many offenders should go unpunished at the minimum of  $L$ . The model enables to think about the consequences of modifying policies. It allows to include relevant quantities (it may even take into account the private expenditures against crime, the social cost of punishing innocent persons and other policy drawbacks) in the definitions of  $D$ ,  $C$ , etc.

Validation of the model would require enough data to determine the coefficients of the different terms in a development to (at least) second order of (2), and some attempts have been made. As we discussed in section 2, because the interpretation of crime statistics is still unclear, the mentioned coefficients are difficult to estimate. On the other hand, the conclusions of the model are very dependent on the assumed functional forms, which sometimes have no other justification than mathematical simplicity or intuition. In particular, the model does not consider possible non-linear effects due to social influences.

Economic theories consider crime like any other good. There is a “market” of crimes and individuals make rational decisions: a person commits an offence if his expected utility exceeds the utility he could get with legal activities. The basic equation underlying individual decisions, introduced by Ehrlich [16], relates the expected utility of committing a crime  $E[U]$ , to the probability of capture and punishment  $p$ :

$$E[U] = pU(b - f) + (1 - p)U(b). \quad (3)$$

The utility  $U(x)$  depends on the income  $x$ ; if the crime is not detected the income is  $b$ , if it is punished, the income is  $b - f$ . All incomes, including psychic components such as fear, excitement, pain, are assumed to be convertible to monetary equivalents [74].

Depending on the assumptions about the quantities entering equation (3), i.e. whether they depend on income, or on inequality, or any other variable, different authors reach different and sometimes contradictory results. For example, Ehrlich [16] concludes that there is a strong positive correlation between income inequality and crimes against property. A subsequent study by Deutsch et al. [75] consider different sources of inequality. They show that the results depend on whether the wealth inequality increases because of a personal decline or because rich become richer. Considering various scenarios, they conclude that an increase in wealth inequality has an indeterminate outcome both with respect to the decision of the poor on whether or not to enter the crime “industry” and with respect to the decision of those already participating in illegal pursuits to increase or decrease their level of activity. Notice that, since the basic assumption of the economic models is that individuals are payoff maximizers, the equilibria generally correspond to stereotyped populations partitioned into poor criminals and rich law abiders.

There is a huge number of theoretical publications in the economics literature of crime. Among the subjects considered are the relation between education and crime [39], optimal law enforcement [74, 76, 77], the consequences of criminal’s uncertainty in the estimation of punishment probabilities [78], the difference in crime rates and law enforcement expenditures between the US and Europe [79].

### 3.2 Social models

Economic theory has been very fruitful in producing predictions that encouraged many of the statistical studies described in the preceding sections. However, besides the validation difficulties of economic models, there is a deeper reason why they may fail in explaining crime trends in societies. Like in standard equilibrium theories, the economics approaches to crime modeling assume constant signs for the first and second order derivatives of the loss function. That is, they consider “well behaved” monotonic supply and demand curves, which ensure the existence of a single equilibrium. Such theories cannot account for phenomena where social interactions are important, as is certainly the case in the domain of crime. Social interactions may be responsible of bandwagon effects [80, 81], non-monotonic demand curves in economy [82, 83, 84], hysteresis in the dynamical behaviors [85], and more generally, positive feedback loops in the dynamics of the social system.

The models presented in this section take into account the existence of social influences, either explicit or indirect, among individuals. These introduce nonlinearities in the equilibrium and the dynamical equations, which give raise to multiple equilibria. The state of the system depends then on its history. Glaeser et al [86] suggested that social interactions may explain the large variance in crime on different cities of the US. A non-linear economics model that takes into account the interaction between workers and firms displays multiple equilibria due to endogenous wage dispersion [87]. In the model, rational individuals that are either unemployed or workers, choose whether to commit crimes or not, and whether to accept wage offers from firms. The wage offers posted by the firms depend on the fraction of offenders in the population, which in turn depend on the wage offers. This entails the existence of multiple stereotyped equilibria

where all the unemployed are criminals, and differ in the proportion of workers that choose to commit crimes.

More recent approaches use concepts from ecology, epidemics, statistical physics, and implement agent based simulations. Here we give a short overview, since there are many examples in the present volume.

One of the first attempts to understand the spread of social problems using an epidemic contagion model is due to Crane [88]. Using ideas from Schelling's tipping model [89], he assumes that social problems are contagious and spread through peer influence. The model is tested on data of school dropout and teenage childbearing. The probability that a child will drop out depends on individual characteristics and on the quality of his neighborhood. There is a jump of the probability of dropout (and of childbearing) with decreasing neighborhood quality that corresponds to the incidence of social problems reaching a critical point. Beyond that point the spread process through contagion explodes. Another model that exhibits multiple equilibria is the model of gang formation proposed more by Crane et al. [90].

Campbell et al. (2000) [91] also treat criminality as an epidemics problem using differential equations. Criminal activity is viewed as a social activity where "susceptible" individuals  $S$  in contact with criminals  $C$  are prone to commit crimes, while non-susceptible individuals  $N$  are not. The dynamics of the crime rate growth is described by differential equations, inspired from epidemiology and ecology, that include a social pressure against  $C$ , and contamination of  $S$  by  $C$ . They consider the following set of differential equations:

$$dN/dt = -\theta N + \mu S + \beta(N)C, \quad (4)$$

$$dS/dt = \theta N - (\mu + \alpha)S - \lambda SC, \quad (5)$$

$$dC/dt = \alpha S - \beta(N)C + \lambda SC, \quad (6)$$

with the condition that the overall population remains constant ( $N + S + C = 1$ ). The different coefficient in (4) correspond to different factors that affect crime: demographics and economic conditions ( $\theta$ ,  $\alpha$ ), deterrence and social disapproval ( $\mu$ ,  $\beta(N)$ ), social interactions ( $\lambda$ ). The nonlinearities in the level of crime associated with different combinations of the parameters give raise to wide differences in the composition of the population.

Another point of view comes from the ecological approach (see Nuño et al. (2008) [92]) which studies the the society as a predator-prey system with three kinds of individuals: owners  $O$  are preys, criminals  $C$  are predators of  $O$ , the guards  $G$  are in turn predators of both  $O$  and  $C$ . The evolution of the populations depends on the efficiency of the guards  $G$  and on the competition between  $C$  and  $G$ . This kind of models allow to study mathematically different aspects of the routine activity theory [93], which emphasizes that in order for a crime to occur there must be convergence of an offender, a target and the lack of a guardian.

Models of social interactions within the tradition of statistical physics allow to investigate stylized facts parsimoniously, i.e. using as few parameters as possible. Although published in the economics literature, the model proposed by Bourguignon et al. [32] to

explain the weak correlations of crime and inequality found in economic cross-sectional analyses belongs to this category. The model assumes that the individuals in the population have idiosyncratic honesty levels and that the expected loot is a function of this honesty level. The crime rates depend non-linearly on the honesty distribution, and only those individuals whose income is below some fraction of the average income are determinant to explain crime rates. One of the main conclusions of this work is that inequality changes affecting people that are above this limit are likely to have no significant influence on the crime rate. This may explain the contradictory results of the statistical analyses mentioned in section 2, which consider overall measures of inequality.

Considering a population where individuals have inhomogeneous wages and idiosyncratic propensities to commit crime, Gordon et al. [94] study the influence of different punishment probabilities on crime rates. The model assumes that the honesty level of the population changes according to whether criminals are or not arrested. There is a critical value of the probability of punishment at which the system exhibits a phase transition between a high-criminality situation and a relatively low criminality one. The dynamics of the honesty levels are studied in another paper of this issue.

Multi-agents simulation approaches are useful to treat spatio-temporal patterns, which are difficult to study analytically. They allow to explore large ranges of parameters and make predictions of collective behaviors by taking into account idiosyncratic characteristics of the individuals, the probability of encounters, the type of network organization, etc. Several papers propose and explain general implementations (see for example [95]) but there are few results published in the literature [96, 97, 98, 99]. The strengths of these models are also their weaknesses: they have many adjustable parameters, and it is not easy to understand what are the causes of the obtained behavior.

Another type of models focuses on criminal networks. Ballester et al. [100] argue that optimal policy against crime should remove the so called key-player”, which is the criminal who, once removed, leads to a maximum reduction in the aggregate crime. The authors provide a characterization of such key player knowing the links in the criminal network. This policy requires thus a very good knowledge of the criminals’ organization.

## 4 Conclusion

Crime modeling should help to understand typical facts and predict how crime rates are expected to vary when some parameters are modified. This is why in the first part of the paper we looked for statistical evidences. However, we found large divergences in the empirical conclusions, mostly based in linear regressions. Part of the difficulties come from inherent difficulties in data collection, part from technical issues in the statistical treatments, and also from a lack of models that include the non linearities that exist in social systems.



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