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Thermal Chaotic Mixing of Power-Law Fluids in a Mixer with Alternately-Rotating Walls

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Abstract

We investigate the enhancement of both mixing and heat transfer in a two-rod mixer for highly-viscous non-Newtonian fluids. The mixer is composed of two vertical, circular rods in a cylindrical tank. Chaotic flows are obtained by imposing the temporal modulations of the rotational velocities of the walls. We study the effects of different stirring protocols, which lead to non-chaotic and chaotic flows, on the efficiency of both mixing and heat transfer for three different rheological fluid behaviors: shear-thinning, Newtonian and shear-thickening. For this purpose, we use statistical indicators that characterize the mean value of the fluid temperature and its homogenization. We find that chaotic mixing is suitable for shear-thickening fluids for which we observe a clear enhancement of the thermal mixing (heat extraction and homogenization). This is due to the increase in the apparent fluid viscosity in the vicinity of the rotating walls. This aspect confirms the relevance of chaotic mixing for highly-viscous fluids.

Key words: Chaotic mixing, Power-law fluids, Heat transfer, High Prandtl number, Thermal eigenmodes, Unstructured finite volume method

Nomenclature

c_p heat capacity ($J.kg^{-1}.K^{-1}$)
 $\underline{\underline{D}}$ Rate of deformation tensor

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k	consistency index ($Pa \cdot s^n$)
n	flow behavior index (dimensionless)
\vec{n}	normal-oriented unit vector
p	pressure (Pa)
R_3	tank radius (m)
R_1, R_2	rod radii (m)
t	time (s)
T	temperature (K)
U	tangential velocity
\vec{U}	velocity field ($m \cdot s^{-1}$)
X	rescaled dimensionless temperature

Dimensionless numbers

Pe	Péclet number
Re	Reynolds number
Re_{PL}	Generalized Reynolds number
T^*	dimensionless temperature

Greek symbols

ε	eccentricity (m)
ϕ	generic scalar variable
Γ	generic diffusion coefficient
ρ	fluid density ($kg \cdot m^{-3}$)
σ	standard deviation
τ	period of modulation (s)
$\bar{\tau}$	viscous stress tensor
Ω	angular velocity ($rd \cdot s^{-1}$)

Subscript

c	cell
m	mean
f	face of a cell

Superscript

*	dimensionless
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Introduction

Mixing processes are currently encountered in many practical engineering domains where enhancement of the heat, mass and momentum transfer are required. In the present study, mixing is achieved through the presence of

a laminar chaotic flow that ensures the efficient stretching and folding of material lines (in this case, for a 2D flow). The need for chaotic mixing is particularly interesting when high viscosity and/or shear-sensitive fluids are concerned [1]. In this case, classical laminar 2D time-independent flows are unable to give a good mixing performance and as a consequence, heat transport from the wall will be ineffective [2]. For example, such a situation exists for non-Newtonian molten polymers or polymer blends [3]. Thus, the processing of highly-viscous polymer-molecule networks can degrade in high shear regions of the flow. Also, undesired or ill-defined structures may be obtained for polymer blends. Furthermore, the problems of both heat dissipation and energy costs may become very important for highly-viscous fluids when classical impellers are used [4].

Many more works can be found in the literature that concern the relationship between chaotic mixing and heat transfer (e.g., [5, 6]) or chaotic mixing studies for non-Newtonian fluids (e.g., [7, 8]). Nevertheless, to date and to our knowledge, there is only one published work that considers the coupling of heat transfer and chaotic mixing for rheologically-complex fluids despite its industrial relevance [9]. However, it is of vital importance to study this interplay because the mixing of non-Newtonian fluids is strongly influenced by the local flow field [10].

The approach that this work takes is to analyze the combined effects of mixing and heat transfer in the case of shear-thinning and shear-thickening fluids and to compare the obtained results to those of Newtonian fluids.

The flow in this mixer was previously studied for Newtonian fluids [11, 12]. It is characterized by unsteady velocity fields and streamlines that are compared for each of the three rheological behaviors. Different mixing indicators and statistical tools are also used to compare the heat transfer efficiency for the different complex fluids.

1. Problem formulation

1.1. Geometry of the two-rod rotating mixer

A sketch of the mixer that was used in the study is presented in Fig. 1. It is composed of two circular rods with equal radii, which are maintained vertical inside of a cylindrical tank (a bounded domain). The tank and the rods are heated and can rotate around their respective revolution axis. This geometry is similar to the two-roll mills that were studied in the literature by Price *et al.* [13] ; however, in our case, the outer cylinder has the ability

to rotate. The rods and the tank rotate with an alternating or a continuous velocity modulation. They can also have different rotation directions. We have shown that for a Newtonian fluid [11] this two-rod mixer is suitable in order to obtain fully-chaotic flow without KAM regions, which is particularly interesting for industrial applications.

The geometry of the flow domain is characterized by the radii of the rod and the cylindrical tank, which are, respectively, $R_1 = R_2 = 10 \text{ mm}$ and $R_3 = 50 \text{ mm}$. The eccentricity of the rods is set to the value of $\varepsilon = 25 \text{ mm}$.

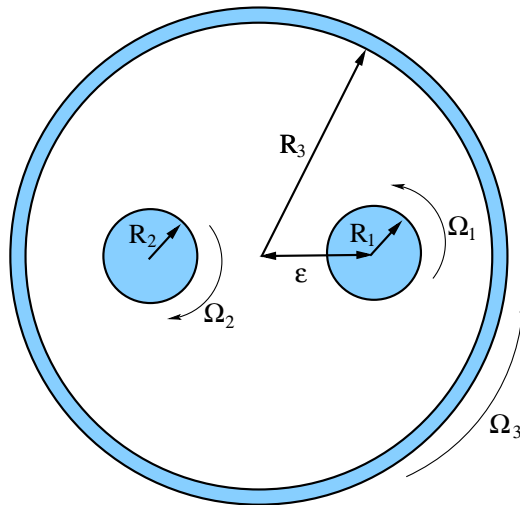


Figure 1: Sketch of the two-rod mixer.

1.2. Flow parameters: stirring protocols

Chaotic mixing flows are produced by varying, with time, the angular velocity of the rods and the tank by using a sine-squared waveform. The stirring protocols that were studied are defined by two parameters: the respective rotation direction between the rods and the cylindrical tank, and the duration of the time periodic modulation. Among the three possible stirring configurations that correspond to different flow topologies, in this study, we chose a stirring configuration with co-rotating tank and rods. We have shown in previous work [12] that this configuration gives globally-chaotic flows without the existence of elliptic KAM regions in the Newtonian fluid. In the same study, two other types of stirring configurations were studied but it was demonstrated that they were less efficient.

For chaotic flows, two types of temporal modulation are considered for the stirring protocols: continuous (sine-squared modulation of the wall velocities) and non-continuous or alternating. In the case of the non-continuous modulation, the rods are stopped together for half a period while the outer tank is rotating and then, for the next half-period, the contrary occurs. These stirring protocols are illustrated in Fig. 2. The maximum angular velocity is the same for the two types of modulation. The rods and the cylindrical tank follow the sine-squared modulations, which are defined by equations 1 (with a modulation period of $\tau = 30$ s):

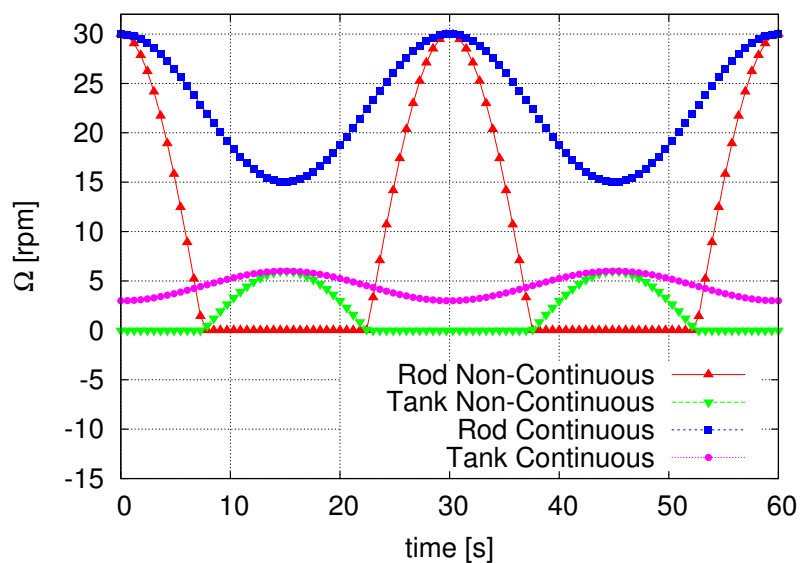


Figure 2: Temporal modulation of the angular velocity of both the rods and the tank for the continuous and non-continuous stirring cases. The modulation period, τ , is 30s.

$$\begin{aligned}
 & \text{Non-continuous case:} \\
 \Omega_{1,2} &= 30 - 60 \sin^2\left(\frac{\pi t}{\tau}\right) & \text{if } \Omega_{1,2} < 0 & \Rightarrow \Omega_{1,2} = 0 \\
 \Omega_3 &= 6 - 12 \sin^2\left(\frac{\pi t}{\tau} + \frac{\pi}{2}\right) & \text{if } \Omega_3 < 0 & \Rightarrow \Omega_3 = 0 \\
 & \text{Continuous case:} \\
 \Omega_{1,2} &= 30 - 15 \sin^2\left(\frac{\pi t}{\tau}\right) \\
 \Omega_3 &= 6 - 3 \sin^2\left(\frac{\pi t}{\tau} + \frac{\pi}{2}\right)
 \end{aligned} \tag{1}$$

The maximum angular wall velocities are fixed to $\Omega_1 = \Omega_2 = 30$ rpm for the rods and $\Omega_3 = 6$ rpm for the outer tank. Thus, the tangential wall velocity is the same and equal to $U = 31.41 \text{ mm.s}^{-1}$. In our modeling, we take into account inertial effects because the flow that is considered for this tangential wall velocity does not satisfy the quasi-steady hypothesis.

1.3. Rheological model

In order to model the flow of non-Newtonian fluids, we need to relate the shear stress tensor to the rate of deformation tensor. For generalized Newtonian fluids, the linear model, $\bar{\bar{\tau}} = 2 \eta \bar{\bar{D}}$, which has been established for purely-viscous fluids, is extended by replacing the constant viscosity by a function that depends on the shear rate. For Newtonian fluids, the relationship between the two tensors is simply linear, and the coefficient of proportionality is defined as the dynamic viscosity, η , of the fluid. The purely-viscous (i.e., inelastic) non-Newtonian character of the fluid that is studied here is represented by an Ostwald-de Waele power-law model for the case of both shear-thinning and shear-thickening fluids. The nonlinear relationship between the apparent viscosity, η , and the shear rate, $\dot{\gamma}$, is given by the constitutive equation:

$$\eta(\dot{\gamma}) = k (\dot{\gamma})^{n-1} \quad (2)$$

with empirical constants k and n , which are, respectively, the consistency index and the flow behavior index. The viscous stress tensor is then expressed as:

$$\bar{\bar{\tau}} = 2\eta(\dot{\gamma}) \bar{\bar{D}} \quad (3)$$

where

$$\dot{\gamma} = \sqrt{2 \text{tr} \left(\bar{\bar{D}}^2 \right)} \quad (4)$$

and

$$\bar{\bar{D}} = \frac{1}{2} \left(\bar{\bar{\nabla}}V + (\bar{\bar{\nabla}}V)^T \right) \quad (5)$$

We will consider three different flow indexes that are respectively associated with shear-thinning, Newtonian and shear-thickening fluids: $n = 0.5$, $n = 1$ and $n = 1.5$. The consistency index is adapted in each non-Newtonian case in order to give the same generalized Reynolds number as was con-

sidered for the Newtonian flow (i.e., $Re = \frac{\rho U \cdot 2 \cdot (R_3 - R_1)}{\mu} = 1.66$). This generalized Reynolds number, Re_{PL} , can be written for the power-law model as [14]:

$$Re_{PL} = \frac{\rho \cdot (R_3 - R_1)^n}{k \cdot U^{n-2}} \quad (6)$$

The thermodependence of the viscosity and the viscous heating were not taken into account in this study. The Newtonian fluid that was considered in this study has the thermophysical properties that are listed in Tab. 1.

Dynamic viscosity (μ)	1.5 Pa.s
Density (ρ)	990 kg.m ⁻³
Thermal conductivity (λ)	0.15 W.m ⁻¹ K ⁻¹
Specific heat (c_p)	1000 W.kg ⁻¹ K ⁻¹
Péclet number (Pe)	16,584

Table 1: Properties of the Newtonian fluid.

1.4. Thermal wall boundary condition

Dirichlet boundary conditions are imposed on the rods and the tank walls. They are kept at a constant *hot* temperature, T_{hot} . Before the start of the heating process, the initial uniform temperature of the fluid was set to a *cold* temperature, T_{cold} . We define the dimensionless fluid temperature as:

$$T^* = \frac{T - T_{cold}}{T_{hot} - T_{cold}} \quad (7)$$

Therefore, $T_{cold}^* = 0$ and $T_{hot}^* = 1$, and the maximum temperature difference between the walls and the fluid is always 1. The heated walls play the role of a variable heat source that is continuously dissipated in the 2D modulated flow field. This is a specific situation that is not normally encountered when dealing with the scalar dissipation of a concentration field. The Dirichlet boundary condition, which is imposed on the walls, implies that during the mixing of the fluid in the tank the parietal fluxes will change along the walls and with time, depending on the local flow conditions.

The other important dimensionless parameter for this non-isothermal mixing problem is the Péclet number ($= Re.Pr$). The Péclet number can

be seen as the ratio of the thermal diffusion time, τ_{td} , to the advection time, τ_{ad} , for scalar temperatures. The advantage of using the Péclet number, as opposed to the commonly used Prandtl number, is that the Péclet number is independent of the empirical power-law constants. Thus, its definition is valid for both Newtonian and non-Newtonian fluids. The limit $Pe = 0$ corresponds to the pure diffusion case. In our study, Pe is large (see Tab. 1) and hence, τ_{td} is 16,584 times greater than τ_{ad} . Thus, the need to speed-up the mixing of temperature scalars is clearly demonstrated.

2. Mixing and energy indicators

In order to quantify the efficiency of the heating process for the Newtonian and non-Newtonian fluids, we have used two instantaneous measures: the mean value of the dimensionless temperature, T_m^* , and its standard deviation, σ . These quantities are defined as (where the summation is made over all of the mesh cells, c , with area A_c):

$$T_m^* = \frac{1}{\sum_c A_c} \left(\sum_c A_c T_c^* \right) \quad (8)$$

$$\sigma = \left[\frac{1}{\sum_c A_c} \sum_c (A_c (T_c^* - T_m^*)^2) \right]^{\frac{1}{2}} \quad (9)$$

The evolution of the mean temperature can be seen as an indicator of the total energy that is supplied to the fluid during the mixing process [11, 15] while the standard deviation accounts for the homogenization level of the scalar temperature inside of the 2D tank. The mean temperature is asymptotically bounded by the fixed temperature that is imposed on the walls (i.e., T_{hot} or 1 for non-dimensional temperatures). Efficient thermal mixing requires good values for both of the aforementioned indicators (i.e., T^* near 1 and σ near 0).

3. Computational modeling

3.1. Governing equations

The unsteady Navier-Stokes equations that govern the flow of incompressible fluids as well as the continuity equation are considered in their integral

form:

$$\frac{\partial}{\partial t} \int_V \rho \vec{U} \, dV + \int_S \rho \vec{U} \vec{U} \cdot \vec{n} \, dS = \int_V -\vec{\nabla} p \, dV + \int_S \vec{\bar{\tau}} \cdot \vec{n} \, dS \quad (10)$$

$$\int_S \vec{U} \cdot \vec{n} \, dS = 0 \quad (11)$$

where $\vec{\bar{\tau}}$ is the viscous stress tensor, which is described in section 1.3 for both Newtonian and power-law fluids. The integration is over a volume, V , that is surrounded by a surface, S , whose orientation is described by its outward unit normal vector, \vec{n} . The energy conservation equation is considered in terms of the temperature:

$$\frac{\partial}{\partial t} \int_V \rho c_p T \, dV + \int_S \rho c_p T \vec{U} \cdot \vec{n} \, dS = \int_S \lambda \vec{\nabla} T \cdot \vec{n} \, dS \quad (12)$$

where λ is the thermal conductivity.

3.2. Numerical method

The conservation equations (10, 11 and 12) are solved by means of an in-house code called Tamaris. This code has an unstructured, finite-volume framework that is applied to hybrid meshes. Variable values (\vec{U} , p and T) are stored at cell centers in a collocated arrangement. Cell shapes can be of different forms (tetrahedral, hexahedral, prismatic or pyramidal).

Spatial schemes that approximate convective and diffusive fluxes are accurate to a second order. The convective fluxes are approximated by the non-linear high-resolution bounded scheme, CUBISTA, of Alves *et al.* [16] and by using the deferred-correction practice of Ng *et al.* [17]. Diffusive terms are treated with a centered-differencing scheme in conjunction with a treatment of possible non-orthogonality of the mesh [18].

Pressure-velocity coupling is ensured by the SIMPLE algorithm [19] while the mass fluxes at the cell faces are evaluated with the Rhie-Chow interpolation [20] in order to avoid pressure checkerboarding. The implicit three-time-step Gear's scheme with a second-order accuracy is used to discretize the unsteady terms at each iteration; the discretization technique that was presented above leads to a linear system with a non-symmetric sparse matrix for each variable. These linear systems are solved by means of an ILU preconditioned GMRES solver using the implementation of the IML++ library

[21].

This 3D code can deal with 2D computations (e.g., in (\vec{x}, \vec{y}) plane), without any change, by considering a single layer of computational cells (in \vec{z} direction) and by neutralizing the top and bottom faces (with respect to \vec{z}). In the scope of this work, all of the computational meshes were generated with the open-source software Gmsh [22].

3.3. Validation test cases

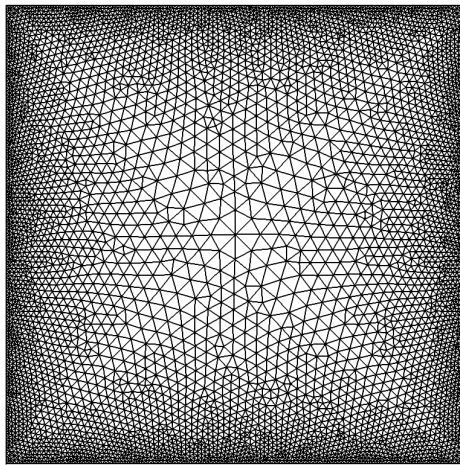


Figure 3: Computational mesh for the lid-driven square cavity mesh (10468 cells).

In this section, we perform calculations for some well-documented test cases in order to assess the accuracy of the code. We consider the flow in the lid-driven, square cavity for three different fluids: Newtonian, shear-thinning (with $n = 0.5$) and shear-thickening (with $n = 1.5$). The mesh that is used in each of the three cases (Fig. 3) is composed of 10,468 triangular cells. For the Newtonian fluid, we present the results for the case with a Reynolds number of $Re = 1,000$. The results are then compared to the results of Botella and Peyret [23], which were obtained by using a spectral Chebyshev method, and Ghia *et al.* [24], which were obtained by using a vorticity-stream function formulation. In Fig. 4, the velocity components u and v are both presented along the vertical (y) and horizontal (x) cavity axes, respectively.

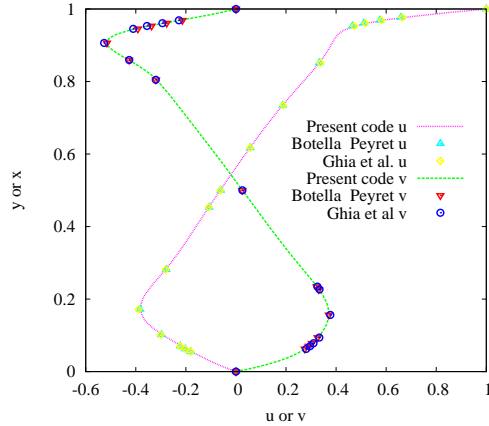


Figure 4: For the Newtonian fluid, the u and v velocity components are presented for the lid-driven cavity test case ($Re = 1000$).

The code is validated by comparing the results of the non-Newtonian power-law fluids to the results of Bell and Surana [25], which were obtained by using a p-version least-squares finite-element method. The flow that is considered has a power-law Reynolds number of $Re_{PL} = 100$ (Eq. 6) for both values of n . The results are shown in Fig. 5. For the two fluids that were studied, the results that are given by our code compare quite well with those of other authors.

4. Results and discussion

After a grid size-dependence study, a mesh of 10,680 computational cells is adopted. This mesh is shown in Fig. 6, where regular quadrilateral cells are used near the walls to enhance the resolution of the boundary layers.

4.1. Non-Newtonian flow patterns

In order to compare the flow topologies that have been obtained for the three different rheological behaviors, we present in Figs. 7 and 8 the patterns of the scalar temperature fields that are associated with the corresponding streamlines for both continuously- and non-continuously-modulated stirring protocols.

In the first case (continuous modulation), the upper part of Fig. 7 represents the flow situation at time $t = 120$ s, which is equivalent to $t = 4\tau$, while

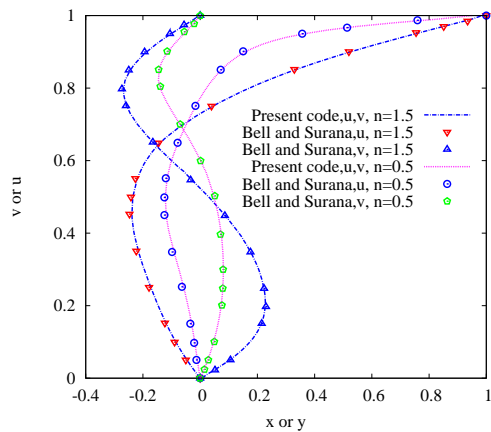


Figure 5: For the shear-thinning and shear-thickening fluids, the u and v velocity components are presented for the lid-driven cavity test case ($Re_{PL} = 100$).

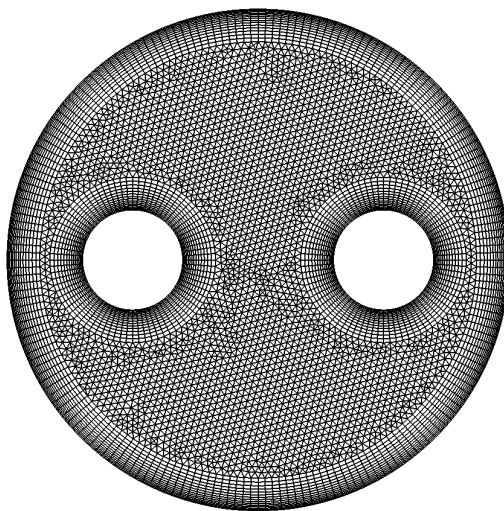


Figure 6: Computational mesh of the two-rod mixer.

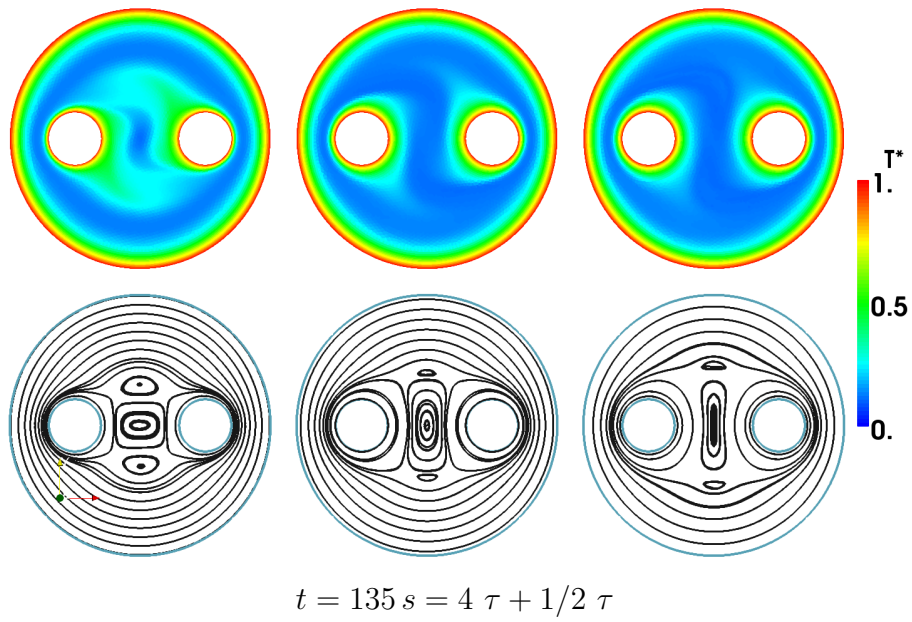
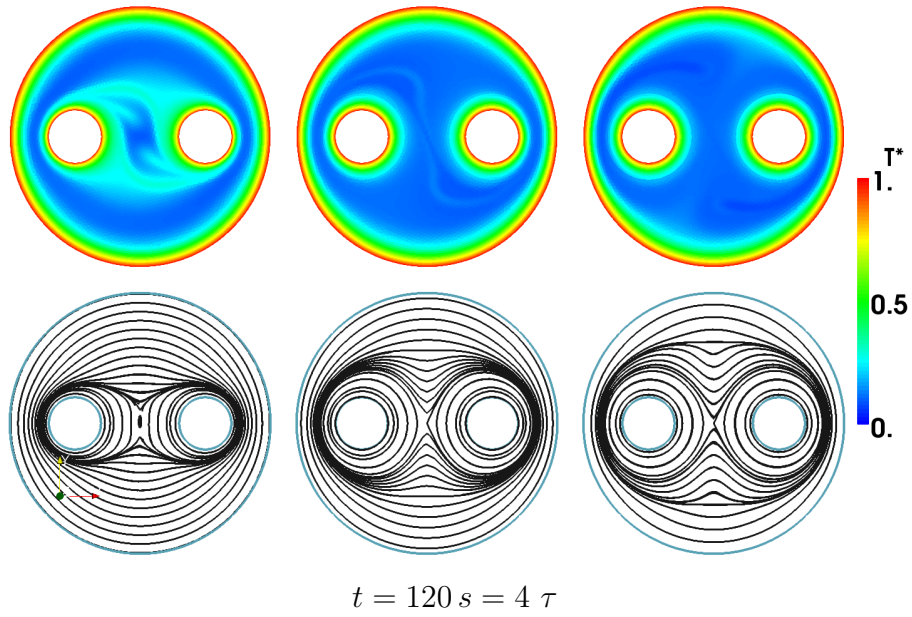


Figure 7: For the case of continuously-modulated stirring, the flow patterns (temperature fields and streamlines) at two different instants ($t = 120 s$, top half, and $t = 135 s$, bottom half) are presented for the three rheological fluid behaviors: shear-thinning fluid (left), Newtonian fluid (middle) and shear-thickening fluid (right). The period of modulation is $\tau = 30s$.

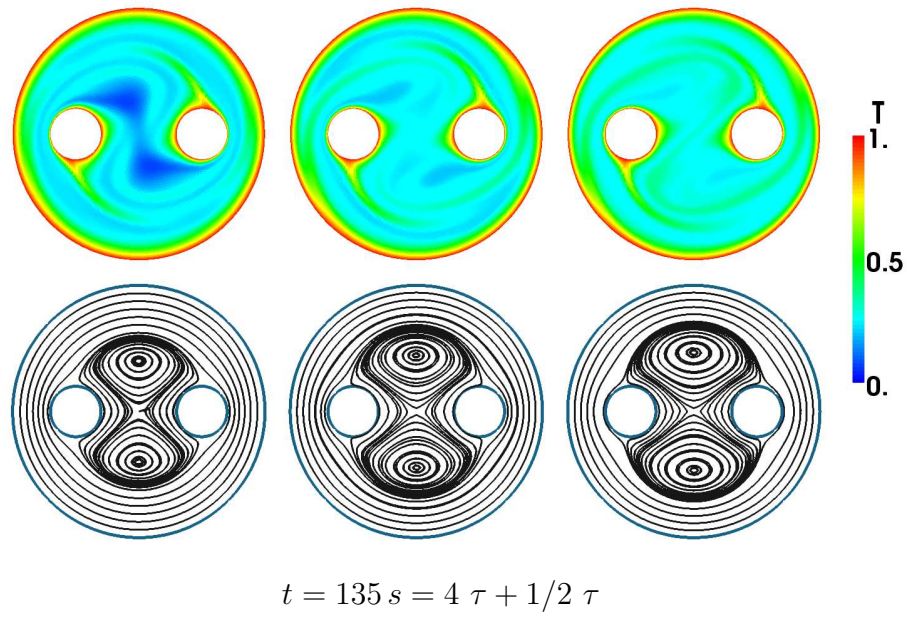
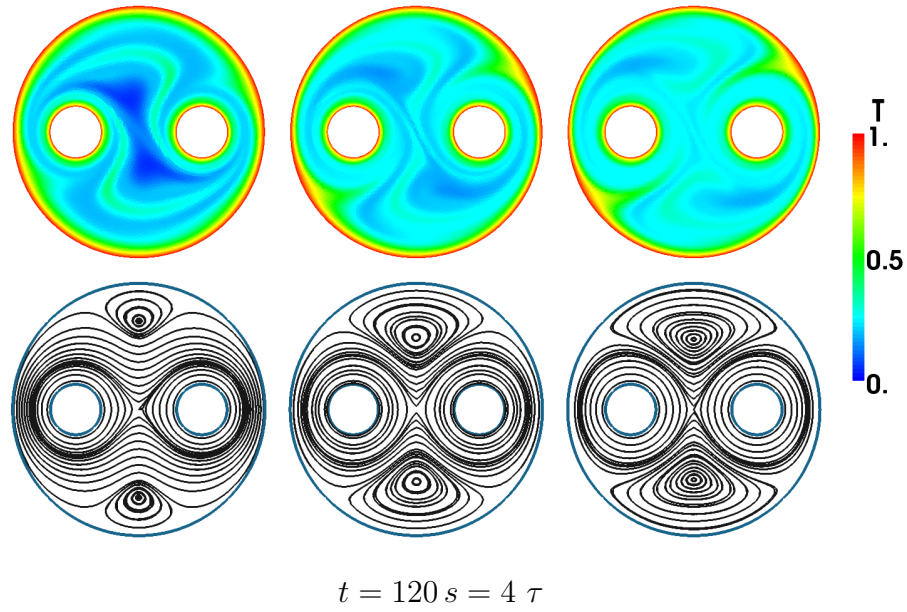


Figure 8: For the case of alternately-modulated stirring, the flow patterns (temperature fields and streamlines) at two different instants ($t = 120 s$, top half, and $t = 135 s$, bottom half) are presented for the three rheological fluid behaviors: shear-thinning fluid (left), Newtonian fluid (middle) and shear-thickening fluid (right). The period of modulation is $\tau = 30s$.

the lower part corresponds to the instant $t = 135 \text{ s} = 4.5\tau$. At $t = 120 \text{ s}$, the two rods are rotating at their maximum angular velocity while the velocity of the tank is at its minimum value (see Fig. 2). By examining the streamlines, we notice that the size of the zone of influence of the rods, *i.e.* the zone of the driven fluid, depends on the fluid's nature: it is confined for the case of the shear-thinning fluid and is larger for both the Newtonian and the shear-thickening fluids. For the latter, this zone is the largest. The effect on the temperature field is the opposite: for Newtonian and shear-thickening fluids, the streamlines that are parallel to the surface of the rods form large zones and act as insulation, which restrains the convective heat transfer from the rods. On the other hand, in the case of the shear-thinning fluid, the narrowness of this zone induces fluid streams between the surfaces of the two rods and promotes heat transfer from them to the center of the mixer.

Later at $t = 135 \text{ s}$, the tank is rotating at its maximum velocity while the rods are at their minimum. A fluid recirculation zone appears between the rods. It is larger when the fluid is less viscous because the rods have narrower zones of driven fluid. Thus, for the shear-thinning fluid, this recirculation blends the heated fluid, which was extracted from the rods earlier. As will be established below by a statistical analysis of the flow, for the continuously-modulated stirring protocol, the best mixing results are achieved for the case of the shear-thinning fluid.

In the case of alternating rotations, Figure 8 is the equivalent of Fig. 7. Hence, the upper part of the figure ($t = 4\tau$) corresponds to the tank at rest for $\tau/2$ while the rods are rotating. By examining the streamlines, we can observe that the more the fluid is viscous the more the effect of the rods extends towards the tank to form four parabolic points on its boundary. Also, two symmetrical vortices are created near this boundary, their size is very small for the shear-thinning fluid. As a consequence, these vortices cause two hot fluid streams to be extracted from the tank wall and have their origin at two of the four parabolic points. In the case of the shear-thickening fluid, these streams almost reach the center of the tank.

At $t = 4.5\tau$ (the lower part of the Fig. 8), the tank rotation is at its maximum velocity while the rods are at rest. We observe two recirculation zones in the center of the tank with a size that increases with increasing fluid viscosity. Moreover, two parabolic points form on each rod and hot fluid streams releases from these points. We can clearly see that the temperature field contains unmixed cold zones for the case of the shear-thinning fluid and that the best homogenization is obtained for the shear-thickening fluid.

In comparison with Fig. 7, we can observe that the alternated modulation produces a hotter fluid and gives a better homogenization for the three fluids. These visual observations will be confirmed statistically in the following section.

4.2. Temporal evolutions of the temperature statistics indicators

For the three rheological fluid behaviors, the temporal evolutions of the mean value of the dimensionless temperature, T_m^* , and its standard deviation, σ , are shown in Fig. 9 for the following stirring protocols: (a) non-modulated, (b) continuously modulated and (c) alternately modulated. The fixed-wall temperature that is imposed on all of the boundaries acts as a source for the evolution of the scalar; thus, the mean temperature will not be constant with time but will evolve asymptotically from $T_m^* = 0$ to $T_m^* = 1$. The behavior of both the Newtonian and shear-thickening fluids is exactly the same as the non-modulated and continuously-modulated stirring protocols. They both give a poorer mixing efficiency than the shear-thinning fluid (Fig. 9(a) and (b)). However, this situation is reversed for the non-continuously-modulated stirring protocol for which the shear-thickening fluid gives a more efficient thermal mixing. When a non-continuous modulation of the wall velocity is considered (Fig. 9(c)), the exponential decay rates of the standard deviation of the temperature fields are always higher than for the two other types of stirring protocols that have been studied (Fig. 9 (a) and (b)). This last result is understandable when we observe the temperature fields in Fig. 8 and is corroborated by the study of the probability distribution functions of the dimensionless temperature, T_m^* , in section 4.4.

The flow mechanisms that are involved in the three modulation modes are quite different. In the non-modulated and continuously-modulated stirring processes of a shear-thinning fluid, a supplementary recirculation zone forms at the center of the mixer due to the lower viscosity of the fluid (lower part of Fig. 7). This recirculation increases the heat transfer from the two rods.

4.3. The effect of the modulation's period size

Until now the period size was fixed at the value $\tau = 30$ s. In this section, we investigate the effect of increasing or decreasing this period size in the case of alternating modulation. For both shear-thinning and shear-thickening fluids, Fig. 10 presents the results that have been obtained for mixing processes with periods of 15 s, 30 s and 60 s.

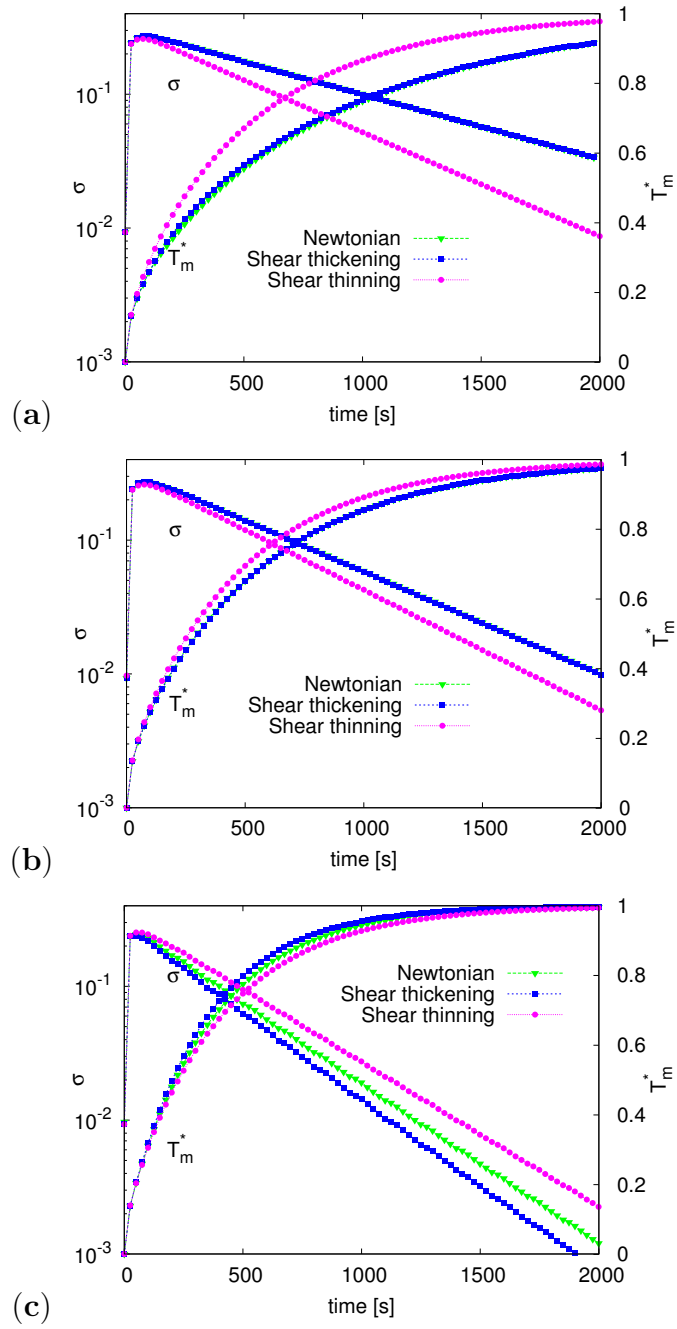


Figure 9: For the three rheological fluid behaviors, the temporal evolution of the mean value of the dimensionless temperature and its standard deviation is presented for (a) non-modulated, (b) continuously-modulated and (c) non-continuously-modulated stirring protocols.

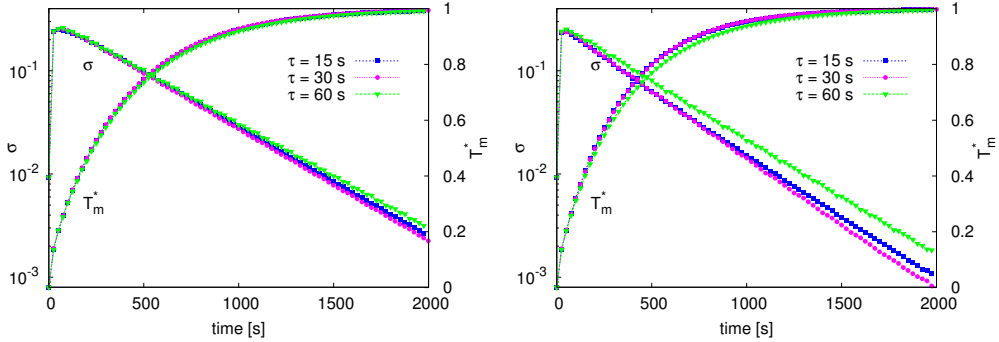


Figure 10: Temporal evolution of the mean value of the dimensionless temperature and its standard deviation for three different period sizes. Left: shear-thinning fluid, right: shear-thickening fluid.

For the two fluids, the size of the period has a slight effect on the evolution of the mean temperature. For the standard deviation, the mixing of the shear-thickening fluid is impacted by the change of this parameter while the mixing of the shear-thinning fluid presents little difference. For both fluids, the period size of $\tau = 30$ s seems to give the best results. In our previous studies [11, 12], we have found similar conclusions for Newtonian fluids. An explanation can be advanced by observing the evolution of the temperature field in the mixer (as the one in Fig. 8) during several periods of the mixing process. It is found that when the period is shorter ($\tau = 15$ s) the hot fluid streams that are extracted from the walls are smaller; thus, they are returned back to the boundary during the next movement of the wall. On the other hand, when the period is longer ($\tau = 60$ s), the mixing process loses some of its temporal diversification or its unsteady character because a pseudo-steady state is reached during each of the wall's movements. The value $\tau = 30$ s is not the optimal value but we think that it is around it.

4.4. Probability distribution functions of the temperature scalars

We focus here on the probability distribution functions (PDFs) of the dimensionless temperature, T^* , for the whole mixer section that is filled by the fluid. The PDFs of T^* fields are shown in Fig. 11 for the three stirring protocols at two different times. In each plot, the three fluid behaviors are compared.

In general and in all of the figures, we notice the presence of a significant

peak (which corresponds to the most probable temperature in the fluid) that is located on the left. To the right of this peak, for the non-modulated stirring protocol (Fig. 11 (a)), we observe some small modulations that indicate the persistence of the scalar temperature gradients within the fluid. Then, with time, as the peak moves towards the high temperatures, these modulations compress towards $T^* = 1$. In this case, the shear-thinning fluid gives the best thermal mixing efficiency; thus, the results that have been found in section 4.2 are then confirmed. For the continuously-modulated stirring protocol (Fig. 11(b)), we observe the same general features as in Fig. 11 (a); the PDFs are just translated a little towards $T^* = 1$. The difference is now considerably reduced between the T^* location of the PDF peak that is observed in shear-thinning fluids and the two other fluids. The PDFs that correspond to the non-continuously-modulated stirring protocol (Fig. 11 (c)) are completely different; the peak of the most probable temperature is wider and its tail is reduced towards $T^* = 1$ and has smaller modulations. Contrary to the results that have been obtained for the two other stirring protocols, the shear-thinning fluid gives the poorest thermal mixing efficiency. In this case, the T^* peak conserves a left tail towards the cold temperatures, which indicates the persistence of cold, poorly-mixed zones within the fluid; these zones can be seen in Fig. 8 (left). With Fig. 11 (c), we can conclude that the chaotic flow that is encountered for the non-continuously-modulated stirring protocol is particularly interesting for the thermal mixing of shear-thickening fluids. Generally, non-continuously-modulated stirring protocols have a better efficiency than the two other stirring protocols.

An important feature of this thermal chaotic flow can be shown for the case of the shear-thickening fluid in Fig. 12 by the PDFs of the rescaled, dimensionless temperature:

$$X = \frac{T^* - T_m}{\sigma} \quad (13)$$

These PDFs are superimposed when they are plotted for different times during the mixing process but at the same phase of the period (see Fig. 12 (a)). This is the signature of a strange eigenmode [9, 12], which is characterized by the production of persistent patterns in the flow. These patterns arise from a combination of stretching, folding and thermal diffusion. In Fig. 12 (b), the PDFs are plotted for different times during a period. From these PDF evolutions, we can observe that the temperature distributions evolve

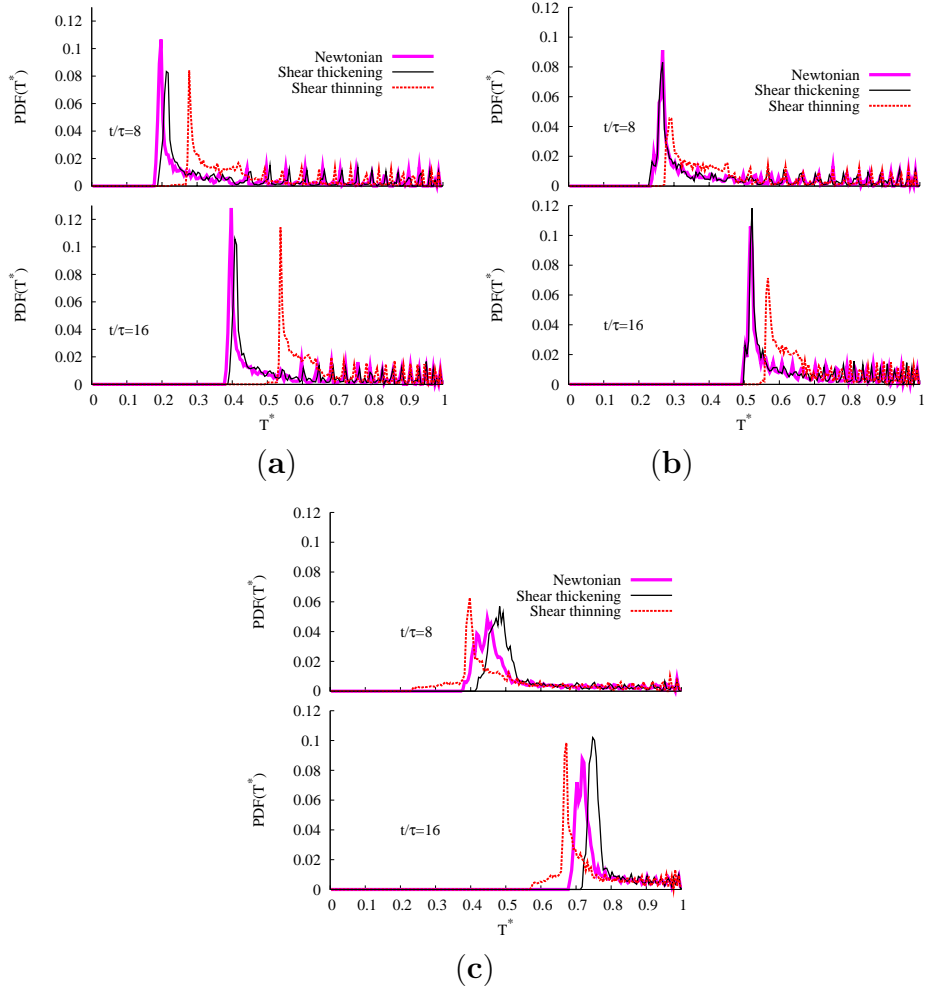


Figure 11: Probability distribution functions (PDFs) of T^* for the three rheological fluid behaviors at two different times and for the (a) non-modulated, (b) continuously-modulated and (c) non-continuously-modulated stirring protocols.

within the fluid during a period but that it regains its previous form from one period earlier.

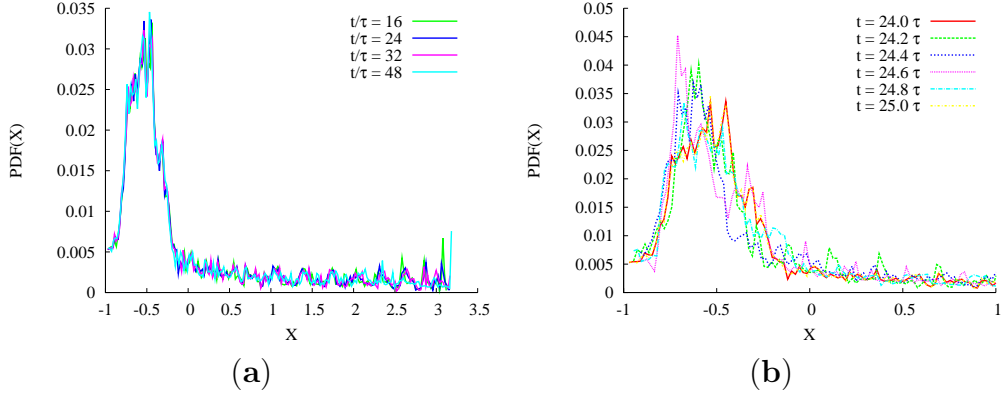


Figure 12: PDFs of the rescaled dimensionless temperature, X , for the shear-thickening fluid and the non-continuously-modulated stirring protocol **(a)** at different period times ($\tau = 30$ s) and **(b)** at different times during one period.

5. Conclusion

Numerical simulations of the coupled mixing and heating performances that are induced by chaotic advection in a 2D two-rod mixer for non-Newtonian power-law fluids were performed in this study. Three different stirring protocols were chosen: non-modulated, continuous and alternating (non-continuous). The last two were able to give chaotic flow trajectories. In order to study the thermal mixing enhancement mechanism within the fluids, different mixing and energy indicators and statistical tools were used. According to the wall boundary condition that was considered (constant wall temperature), the following main conclusions can be made based on the obtained results:

- for the non-chaotic and partially chaotic flows, i.e., non-modulated and continuously-modulated stirring, the thermal mixing is more effective for the shear-thinning fluid,
- for all of the rheological fluid behaviors, a high degree of thermal mixing is obtained for the alternating-stirring protocol (fully-chaotic flow),

- for the alternating-stirring protocol, the most effective thermal mixing is obtained for the shear-thickening fluid,
- for the alternating-stirring protocol, cold, poorly-mixed zones exist when a shear-thinning fluid is considered.

Further studies will explore the influence of the fluid's yield stress and also the effect of the thermodependence of the complex fluids on the efficiency of thermal chaotic mixing. An additional mid-term objective is to be able to model the flow behavior of the concentrated emulsions [26, 27].

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