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AN OPTIMAL CONTROL PROBLEM: BOND GRAPH REPRESENTATION AND SOLVER IMPLEMENTATION

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Abstract. The paper deals with an optimal control problem, namely a bond graph approach, from the problem definition up to the simulation stage. The main role of this article is to supply an alternative approach for solving optimal control problems, both at the representation level and the numerical resolution level.

The work presented in this paper, although it concerns optimal control, applies this effort at different levels. The framework of this research belongs to a more general task of dimensioning mechatronic systems. The optimal control implementation constitutes one of the phases that has to be overcome, in order to advance in the direction of the virtual prototyping of these systems.

The class of the problem concerns linear time invariant single input/ single output systems, where the performance index corresponds to dissipative energy. The control variable is taken into account as dissipative loss also, using a weighting factor.

1. Introduction

The importance of energy loss minimization in a real industrial environment is substantial, especially nowadays, where every small loss is critical. Also, the multidisciplinary character of the problem addressed makes the use of traditional methods tedious, so the technological level of modeling cannot satisfy the needs. Unfortunately, the mathematical level does not suit either, so the use of a physical level modeling language is advantageous. Bond graph language has proved itself a very efficient tool in modeling, analysis and the design of mechatronic systems, especially from energy and dynamic point of view [11], [2].

This research subject is the next step in our general task focusing the dimensioning of mechatronic systems. Optimal control implementation constitutes one of the barriers that have to be overcome to advance in the direction of the virtual prototyping of these systems. It relies on the work of Fotsu Ngwompo [3], [4], [5], [6]. A first approach to this problem also was within the framework of the thesis of Xia (2002) [15], [16] with the introduction of concept of Bond Graph model of the adjoint system.

The virtual prototyping of the mechatronic systems can be exploited profitably within the framework of bond graph modelling because of the separate representation of the physical structure and the inherent calculation structure corresponding to this representation. Taking into account the multi-field character and the energetic point of view constitutes an additional argument for the choice of this representation. Also, energy minimization provides a conclusive value in the industrial applications. In a design phase for example, it provides criteria to dimension of the actuating components of the actuating chain, by avoiding both useless oversizing and dangerous undersizing, usually the result of steady state analysis.

This class of the problems concerns linear time invariant single input/ single output systems, where the performance index corresponds to dissipative energy, and the control variable is taken into account also as dissipative loss by means of a weighting factor.

The paper is organized into 4 sections. The first two deal with the bond graph representation of an optimal control problem. In the first section, a systematic procedure aims at automatically generating the bond graph representation of an optimal control problem having the bond graph representation of the system under study as its starting point. The steps are listed, and it should be noted that this procedure only takes place at a graphic level, skipping the symbolic developments usually implied by the use of Pontryagin methods.

The second section familiarizes the reader with the use of the procedure shown in the first section. A rather simple (a second order linear system) example is used. The model follows the procedure described in the first section, which provides, as output, the bond graph model corresponding to the specified optimal control problem. The analytical developments are then carried out just to confirm that the result obtained by the proposed bond graph approach corresponds to the one obtained by applying the classical Pontryagin procedure.

The second part of the paper deals with the numerical issues. First, as the Pontryagin principle leads to a particular case of boundary value problems (TPVBP) [1], [12] a brief study of the suitable techniques for solving them is carried out. The first subsection includes the theoretical developments necessary to explain the motivation for the practical method implemented here. This section also contains a small paragraph to show the research framework and the basis for choosing the method. This section also reveals the practical means for solving the two point boundary value problems.

Software was created especially to address the problem and used on the same example as the one treated in the second section.

Finally, the conclusions are pooled. The limitations of the procedure and perspectives for future research are then discussed.

2. Bond graph representation of an optimal control problem

Before we specify the different steps to pursue, it is appropriate to present description of the problem. The procedure provided here aims at determining the optimal control solution that is supposed to alter a dynamic system state, from a known initial state at time t_0 to a fixed final state at a fixed final time t_f , and minimising an integral type performance index, more precisely an energy loss, quadratic in terms of state variables and control variable.

A systematic procedure that aims at automatically generating the bond graph representation of an optimal control problem, given the bond graph representation, was established at the Laboratoire d'Automatique Industrielle [9]. The procedure has, as a starting point, the bond graph model of the original LTI SISO system for which the optimal control is required to minimize a dissipative and control based quadratic cost function. The steps to follow are enumerated here and for further information the reader is referred to the bibliography. For the moment, the procedure is given for bond graph representations containing only 1-port elements.

1. Establish the partial adjoint bond graph representation by taking the junction structure, the I , C and R elements of the original bond graph representation, and reversing the R element parameter signs. At this stage, the energy source element associated to the control disappears from this bond graph representation.
2. For each dissipative phenomenon involved in the given cost function, replace both corresponding 1-port R elements respectively in the original system and the adjoint bond graphs by a coupling 2-port R element with the constitutive matrix (1)

$$\begin{pmatrix} e_{orig} \\ e_{adj} \end{pmatrix} = \begin{pmatrix} R & 0 \\ 2R & -R \end{pmatrix} \cdot \begin{pmatrix} f_{orig} \\ f_{adj} \end{pmatrix} \quad (1)$$

Here R is the parameter characterizing the original dissipative phenomenon, the first row and column are associated to the power conjugate variables on the original system bond graph side port, and the second row and column are associated to the power conjugate variables on the adjoint bond graph side port. At this stage, it is important to have had at the beginning taken into account of the R element corresponding to the energy supply device dissipation.

3. Replace the source element in the original system bond graph by a double detector and mirror it with a double source at the same place on the optimising bond graph. The double source imposes both null effort and flow.
4. Assign bicausality to the obtained bond graph. The analytical exploitation of the bicausal bond graph representation obtained provides the system of equations and the optimal control solutions to the given initial problem.

3. Example

A rather academic example is chosen in order to familiarize the reader with the procedure described above. The example consists of a second order passive low-pass filter, connected to a current generator that imposes the control variable. The function to be minimised is a dissipative cost function, which takes into account the power losses on R_2 and R_4 and a control weighting factor, implemented here for practical grounds as a resistor R_1 in series with the power supply. The electrical diagram and the bond graph model are shown in figures 1 and 2 respectively.

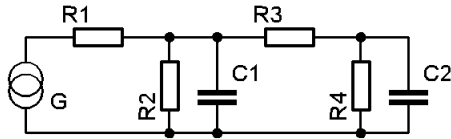


Fig1. Electrical diagram

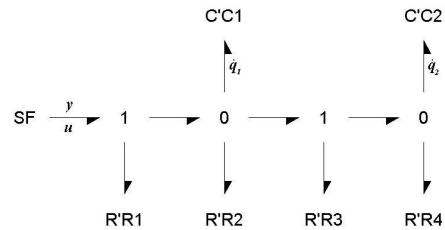


Fig2. Bond graph representation

The control, which is a current in this configuration, is supposed to alter the state vector from an initial state x_i at time t_i to a final state x_f at t_f , while minimising the performance index specified below (2).

$$V = \int_{t_i}^{t_f} (P_{R_1} + P_{R_2} + P_{R_4}) dt \quad (2)$$

By applying the procedure shown in the previous section, the adjoint system can be obtained from the original system, by replacing the sources, then the two models are assembled using the cost function. Bicausality is assigned afterwards [7], [8]:

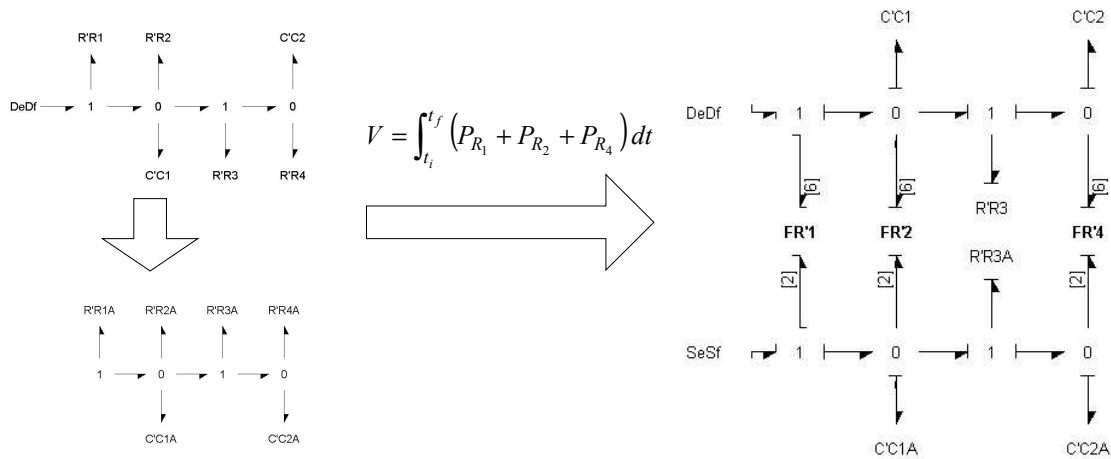


Figure 3. Bond graph representation of the optimal control problem

In the figure 3, the MS1 convention is used for resistive multiport elements $FR1$, $FR2$ and $FR4$. Although the integral causality assignment imposes an effort-input causality for the $FR2$ and $FR4$ elements and a mixed one on $FR1$, their constitutive relation is supplied in the flow-input form, because the Esacap makes use of the implicit form of the constitutive relation.

$$\begin{cases} \text{FR1:} \begin{pmatrix} e_2 \\ e_6 \end{pmatrix} = \begin{pmatrix} R_1 & 0 \\ 2R_1 & -R_1 \end{pmatrix} \cdot \begin{pmatrix} f_2 \\ f_6 \end{pmatrix} \\ \text{FR2:} \begin{pmatrix} e_6 \\ e_2 \end{pmatrix} = \begin{pmatrix} R_2 & 0 \\ 2R_2 & -R_2 \end{pmatrix} \cdot \begin{pmatrix} f_6 \\ f_2 \end{pmatrix} \\ \text{FR4:} \begin{pmatrix} e_6 \\ e_2 \end{pmatrix} = \begin{pmatrix} R_4 & 0 \\ 2R_4 & -R_4 \end{pmatrix} \cdot \begin{pmatrix} f_6 \\ f_2 \end{pmatrix} \end{cases} \quad (3)$$

The space-state model produced from the bond graph representation of the optimal control problem is shown in (4).

$$\begin{cases} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_{\lambda_1} \\ \dot{q}_{\lambda_2} \end{pmatrix} = \begin{pmatrix} -\frac{R_2 + R_3}{C_1 R_2 R_3} & \frac{1}{C_2 R_3} & -\frac{1}{2R_1 C_1} & 0 \\ \frac{1}{R_3 C_1} & -\frac{R_3 + R_4}{C_2 R_3 R_4} & 0 & 0 \\ -\frac{2}{R_2 C_1} & 0 & \frac{R_2 + R_3}{C_1 R_2 R_3} & -\frac{1}{R_3 C_2} \\ 0 & -\frac{2}{R_4 C_2} & -\frac{1}{C_1 R_3} & \frac{R_3 + R_4}{C_2 R_3 R_4} \end{pmatrix} \cdot \begin{pmatrix} q_1 \\ q_2 \\ q_{\lambda_1} \\ q_{\lambda_2} \end{pmatrix} \\ u^0 = -\frac{q_{\lambda_1}}{2R_1 C_1} \end{cases} \quad (4)$$

The classical developments made in the following subsection were carried out with the sole purpose of illustrating, by the means of this example, the correctness of the procedure presented in the second section. As this step only employs classical developments [13], [14], [17], only some key points are exposed.

First of all, we exploit the integral causality assignment on the bond graph model, shown in fig 2, as it directly provides an energy-based state space model. Since this step is not a key issue in the present paper developments, we do not detail how this equation system is obtained and the reader is referred to the bibliography [2], [11].

$$\begin{cases} \dot{q}_1 = -q_1 \frac{R_2 + R_3}{C_1 R_2 R_3} + q_2 \frac{1}{C_2 R_3} + u \\ \dot{q}_2 = q_1 \frac{1}{R_3 C_1} - q_2 \frac{R_3 + R_4}{C_2 R_3 R_4} \\ y = q_1 \frac{1}{C_1} + R_1 u \end{cases} \quad (5)$$

Using the cost function (2), the conventional Pontryagin principle application yields:

$$\begin{cases} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \end{pmatrix} = \begin{pmatrix} -\frac{R_2 + R_3}{C_1 R_2 R_3} & \frac{1}{C_2 R_3} & -\frac{1}{2R_1} & 0 \\ \frac{1}{R_3 C_1} & -\frac{R_3 + R_4}{C_2 R_3 R_4} & 0 & 0 \\ -\frac{2}{R_2 C_1^2} & 0 & \frac{R_2 + R_3}{C_1 R_2 R_3} & -\frac{1}{R_3 C_1} \\ 0 & -\frac{2}{R_4 C_2^2} & -\frac{1}{C_2 R_3} & \frac{R_3 + R_4}{C_2 R_3 R_4} \end{pmatrix} \cdot \begin{pmatrix} q_1 \\ q_2 \\ \lambda_1 \\ \lambda_2 \end{pmatrix} \\ u^0 = -\frac{\lambda_1}{2R_1} \end{cases} \quad (6)$$

The state variables of the adjoint system are in fact coenergy variables. The reader can verify that the system (6) is equivalent to the system (4) when using the variable mapping (7). It is important to perform this variable mapping in order to achieve the homogeneity of a bond graph model.

$$\begin{cases} q_{\lambda 1} = C_1 \cdot \lambda_1 \\ q_{\lambda 2} = C_2 \cdot \lambda_2 \end{cases} \quad (7)$$

4. Implementing the solver

This research is part of a more generalised methodology for sizing mechatronic systems developed at LAI. Optimal control is required only at a virtual level, such as the specified output is to be fed to the system under sizing study. The algorithm is implemented only to prove the feasibility, so the demands for the solver are not as severe and rigorous as for an industrial implementation. The main limitation is to preserve as much as possible of the original problem framework, displayed as a flowchart in fig 4.

As stated in the previous section, by applying the procedure depicted in the second section, to the original dynamic system, the optimal control problem leads to a system of differential equations having twice the size of the original system. The control is supposed to alter the initial known state x_i at t_i to a fixed final state x_f at a fixed final time t_f . So, we have two n -sized sets of boundary conditions to satisfy (8). These conditions, which point to the same set of state variables (the state variables of the initial system), are known as Two Point Boundary-Value Problems, involving here ordinary differential-equations (TPBVP-ODE).

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_1 \mathbf{x} + \mathbf{A}_2 \mathbf{x}_\lambda \\ \dot{\mathbf{x}}_\lambda = \mathbf{A}_3 \mathbf{x} + \mathbf{A}_4 \mathbf{x}_\lambda \\ \mathbf{x}(t_i) = \mathbf{x}_i \\ \mathbf{x}(t_f) = \mathbf{x}_f \end{cases} \quad (8)$$

There are different traditional approaches for solving dynamic optimisation problems. These are commonly classified as direct and indirect solutions. Generally the direct solutions involve the parameterisation of the control and state functions via a piecewise polynomial approximation, thus transforming the problem into a static parameter optimisation problem [1]. The second class of approaches especially answers the TPBVP-DAE problems. Several techniques are available, by example shooting methods, finite difference or collocation methods, each with its advantages and drawbacks, which are mostly apparent when dealing with nonlinear systems [1], [12].

Based on the fact that the system in focus is linear, we have implemented a simpler method for solving the problem of finding a costate initial condition from the initial set of boundary conditions. This method is described below [1], [18]:

By writing x_a as the vector constructed using x and x_λ ,

$$\mathbf{x}_a = \begin{pmatrix} \mathbf{x} \\ \mathbf{x}_\lambda \end{pmatrix} \quad (9)$$

We can consider now the whole system as homogenous dynamic system:

$$\dot{\mathbf{x}}_a = \mathbf{A}_a \mathbf{x}_a \quad (10)$$

For a free linear system, the solution is given by the equation (11). As final time is fixed, we can consider the exponent matrix as constant.

$$\mathbf{x}_a(t_f) = e^{\mathbf{A}_a(t_f-t_i)} \mathbf{x}_a(t_i) = \mathbf{M}_a \mathbf{x}_a(t_i) \quad (11)$$

At this time, it is appropriate to disassemble the x_a vector:

$$\begin{pmatrix} \mathbf{x}(t_f) \\ \mathbf{x}_\lambda(t_f) \end{pmatrix} = \mathbf{M}_a \cdot \begin{pmatrix} \mathbf{x}(t_i) \\ \mathbf{x}_\lambda(t_i) \end{pmatrix} = \begin{pmatrix} \mathbf{M} & \mathbf{N} \\ \mathbf{P} & \mathbf{R} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{x}(t_i) \\ \mathbf{x}_\lambda(t_i) \end{pmatrix} \quad (12)$$

Here, only the x_f vector deserves our attention, and we can write it down in a more detailed manner, to show how the initial costate condition can be obtained:

$$\mathbf{x}(t_f) = \mathbf{M} \mathbf{x}(t_i) + \mathbf{N} \mathbf{x}_\lambda(t_i) \Leftrightarrow \mathbf{x}_\lambda(t_i) = \mathbf{N}^{-1}(\mathbf{x}(t_f) - \mathbf{M} \mathbf{x}(t_i)) \quad (13)$$

Numerical implementation:

Concerning (13) there are several methods of finding the M matrix. The most direct involve the direct calculus of $e^{A(t_f-t_i)}$ matrix, using the Padé or Taylor series approximations. Due to its simplicity, only the Taylor series matrix exponent approximation was implemented at this time.

We implemented the high-level algorithmic part in Visual Basic, by foreseeing further developments that might exclude the use of conventional simulation environments. The main motivation for that choice is the previous experience in VB. The flowcharts depicted below reveal the initial framework and the resolution using this algorithm. The shaded-box operations are completely transparent for the user.

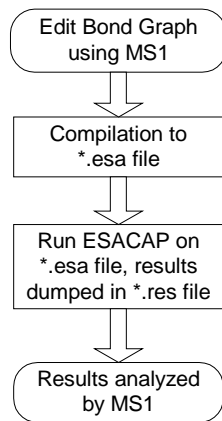


Figure4. Normal operation flowchart

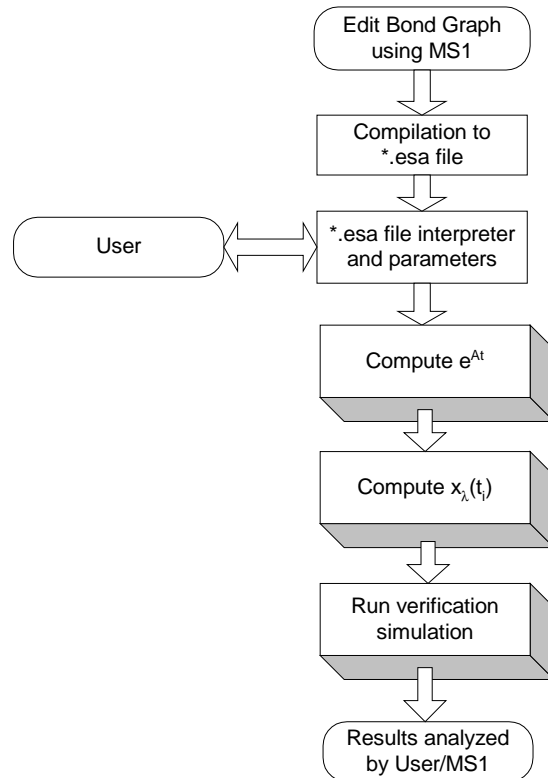


Figure5. Modified operation flowchart

5. Numerical results based on the example

By following the protocol depicted in figure 5, firstly the bond graph model is assembled in an MS1 environment. Once the bicausality assigned, the model is compiled into a ESACAP-format file. This file is recovered using a standard GUI, and the interactive interpretation of the file begins. Once this stage finished, the user must choose, among other simulation parameters, the final state values, as these values are required for the computation of the initial costate values, using (13).

More precisely, for the system presented as an example in the second section, by choosing as system parameters $C_1 = 1.0\text{F}$, $C_2 = 1.0\text{F}$, $R_1 = 10.0\Omega$, $R_2 = 10.0\Omega$, $R_3 = 1.0\Omega$, $R_4 = 1.0\Omega$, initial states $q_{1i} = 0.33\text{C}$ and $q_{2i} = 0.45\text{C}$ at initial time $t_i = 0$, final imposed states $q_{1f} = 3.0\text{C}$ and $q_{2f} = 10.0\text{C}$ at final simulation time of $t_f = 3.0\text{s}$, using the Taylor series matrix exponent approximation, the algorithm finds as initial costates the values $x_{\lambda 1i} = -391.0045\text{E}+02\text{C}$ and $x_{\lambda 2i} = -4.047704\text{E}+02\text{C}$, values that, supplied as initial costate values, yield the final time state at $q_{1f} = 3.000314\text{C}$ and $q_{2f} = 1.000005\text{C}$. The obtained precision is suitable for the initial purposes. The control variable evolution is shown in figure 6, and the state evolution in time is displayed in figure 7.

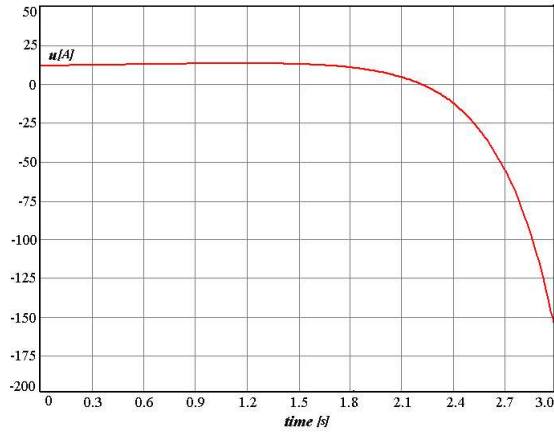


Fig6. Optimal control

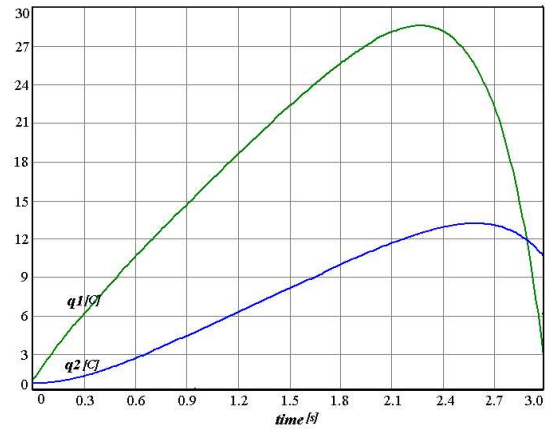


Fig7. Optimal state trajectories

6. Conclusions and perspectives

This paper presents an alternative approach for solving an optimal control problem. The first stage of this approach is provided at the problem formulation level. On this level we use the advantages provided by the bond graph language, so the bond graph representation of the system under study is used as a starting point for an entirely graphic, intuitive and systematic procedure. By following the steps provided in second section, an augmented bond graph representation is obtained, whose bicausal exploitation then provides the optimal system and the control solution, at a symbolic level. We consider that this graphic procedure provides a straightforward solution for obtaining the optimal control system, compared to the classical symbolic developments.

The second stage of the approach concerns the numerical issues. Due to the research framework particularities, a more simplified technique for solving TPBV problems has been implemented, which obtains the state transition matrix for the augmented system by matrix exponent calculus. The practical numerical implementation is then validated by the numerical results.

As exposed in the first section, at this stage, the cost function is defined at a physical level, as a dissipative type. Even though this formulation is suitable for the present industrial environment, there are problems which have different cost functions and require consideration. Stored energy or mixed-energy optimisation criteria provide perspectives for future studies, i.e. extending the bond graph generation procedure for these problems.

At this point, only non-bounded linear systems have been treated, and the next step of study will focus on the nonlinear systems. Some additional terms in the Pontryagin function will have to be taken into account, these will add some extra elements in the bond graph model.

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