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A PERTURBATION FINITE ELEMENT METHOD FOR EFFICIENTLY CALCULATING SKIN AND PROXIMITY EFFECTS IN ANY CONDUCTIVE MATERIAL

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Abstract: A perturbation finite element method for solving eddy current problems in two separate steps is developed for considering conductive and magnetic materials subject to strong skin and proximity effects. The proposed method allows to efficiently and accurately determine the current density distribution and ensuing Joule losses in conductors of any shape in the time domain or for repetitive solutions (e.g., parameterized or nonlinear analyses). A limit problem is first solved by considering the perfect conductive or magnetic nature of the materials, via appropriate boundary conditions. Its solution gives the source for eddy current perturbation sub-problems in each conductor, each one requiring its own mesh.

Keywords: Eddy currents, perturbation method, skin and proximity effects.

1. Introduction

A precise consideration of the skin and proximity effects in conductors is important for an accurate calculation of the ensuing Joule losses. Calculating these effects with the classical application of the finite element (FE) method usually presents difficulties. The mesh to be generated must be fine enough with respect to the skin depth in all the materials, which then leads to a heavy system of equations.

Impedance boundary conditions (BCs) [1] defined on the conductor boundaries are an alternative to avoid meshing their interior. Such conditions are nevertheless generally based on analytical solutions of ideal problems and are therefore only valid in practice far from any geometrical discontinuities, e.g., edges and corners. They are also generally limited to frequency domain and linear analyses.

In this contribution, a method is developed to overcome the limitations of impedance BCs, allowing conductors of any shape to be considered not only in the frequency domain but also in the time domain. The magnetic vector potential FE magnetodynamic formulation is used. The developed method extends the one in [7] by considering the magnetic properties of the conductive materials. It will also give a more detailed description of the treatment of active conductors. It rests on a coupling of limit and perturbation solutions, each of these being calculated in distinct meshes. A limit eddy current FE problem is first solved by considering either perfect conductive or magnetic properties, via appropriate conditions on the conductor boundaries. The solution of the limit problem then gives the source for FE perturbation sub-problems in each conductor then considered with a finite conductivity or permeability.

2. From perfect to non-perfect materials – The eddy current formulations

The equations and relations governing the magnetodynamic (eddy current) problem in Ω are

$$\begin{aligned} \text{curl } \mathbf{h} &= \mathbf{j}, \quad \text{curl } \mathbf{e} = -\partial_t \mathbf{b}, \quad \text{div } \mathbf{b} = 0, & (1a-b-c) \\ \mathbf{b} &= \mu \mathbf{h}, \quad \mathbf{j} = \sigma \mathbf{e}, & (2a-b) \end{aligned}$$

where \mathbf{h} is the magnetic field, \mathbf{b} is the magnetic flux density, \mathbf{e} is the electric field, \mathbf{j} is the electric current density (including source and eddy currents), μ is the magnetic permeability and σ is the electric conductivity. The eddy current conducting part of Ω is denoted Ω_c and the non-conducting one Ω_c^C , with $\Omega = \Omega_c \cup \Omega_c^C$. Massive conductors belong to Ω_c . In the following, the subscripts u and p will refer to unperturbed and perturbed quantities, respectively.

Instead of directly solving the eddy current problem with the actual conductivity or permeability for all the materials, a so-called unperturbed or limit problem is first defined in Ω by considering some conductors $\Omega_{c,i}$ (i is the conductor index) with either an infinite conductivity or permeability. This results in a zero skin depth and thus in surface currents. The interior of the conductor regions $\Omega_{c,i}$ can thus be extracted from the studied domain Ω in (1) and treated via a BC fixing on their boundaries $\partial\Omega_{c,i}$ either a zero normal magnetic flux density or a zero tangential magnetic field.

The consideration of the actual conductivity or permeability of the concerned conductors, these defining the perturbing region $\Omega_{c,i} \subset \Omega_c$, will further lead to field distortions. The perturbed eddy current problem focuses thus on $\Omega_{c,i}$ and its neighborhood, their union Ω_p will serve as the studied domain. The perturbation given by the change of properties of the conducting region $\Omega_{c,i}$ alters the distribution of the eddy current density and the magnetic field. The fields in these conductors are not surface fields anymore but penetrate them.

Particularizing (1) and (2) for both the unperturbed and perturbed quantities, and subtracting the unperturbed equations from the perturbed ones, a perturbation problem (defined as the difference between perturbed and unperturbed problems) is obtained in Ω_p (initially in Ω) [2], [3]. Keeping the equations in terms of the distortions $\mathbf{h} = \mathbf{h}_p - \mathbf{h}_u$ and $\mathbf{e} = \mathbf{e}_p - \mathbf{e}_u$, one gets

$$\begin{aligned} \text{curl } \mathbf{h} &= \sigma_p \mathbf{e} + \mathbf{j}_s, \quad \text{curl } \mathbf{e} = -\mu_p \partial_t \mathbf{h} - \mathbf{k}_s, & (3-4) \\ \mathbf{n} \times \mathbf{h} \Big|_{\partial\Omega_p} &= 0 \quad \text{or} \quad \mathbf{n} \times \mathbf{e} \Big|_{\partial\Omega_p} = 0, & (5a-b) \end{aligned}$$

with the sources \mathbf{j}_s and \mathbf{k}_s defined only in $\Omega_{c,i}$ and given by the unperturbed solution, i.e.

$$\mathbf{j}_s = (\sigma_p - \sigma_u) \mathbf{e}_u \text{ in } \Omega_{c,i}, \quad (6)$$

$$\mathbf{k}_s = (\mu_p - \mu_u) \partial_t \mathbf{h}_u \text{ in } \Omega_{c,i}. \quad (7)$$

The perturbation problem (3)-(7) is actually rigorously defined in the whole studied domain Ω , taking account of the geometrical and material details of the initial unperturbed problem. The conditions (5a) or (5b) neglect the distortion at a certain distance from $\Omega_{c,i}$, which is actually only correct at infinity (for Ω_p extended to the whole space). For convenience, an approximation neglecting some of these initial details will be made. The so-modified studied domain Ω_p can be a portion or not of Ω , with or without inclusion of initial materials, these being possibly simplified. At the discrete level, the meshes of both unperturbed and perturbed problems can then be significantly simplified, each problem asking for mesh refinement of different regions.

The sources \mathbf{j}_s and \mathbf{k}_s (6)-(7) act as sources reduced to $\Omega_{c,i}$ for the perturbation equations (3)-(4). This is an interesting consequence of the use of the distortions \mathbf{h} and \mathbf{e} as unknowns instead of the perturbed fields \mathbf{h}_p and \mathbf{e}_p directly. Another implication is the homogeneous nature of the boundary conditions (5a) or (5b).

The perturbed problem, with the unknown fields \mathbf{h}_p and \mathbf{e}_p , would require non-homogeneous conditions, involving the unperturbed fields as surface sources to be projected on the perturbed mesh boundary $\partial\Omega_p$. However, such BCs can only be applied if the domain Ω_p is bounded. Indeed a boundary at infinity would support a zero source, with consequently no information at all for the perturbed problem. The unperturbed field \mathbf{h}_u could alternatively be used as a volume source field in the whole Ω_p , but with the disadvantage of necessitating its evaluation and projection on the whole domain. These drawbacks justify the use of the sources \mathbf{j}_s and \mathbf{k}_s (6)-(7), the reduced support of which noticeably limits the evaluation and projection operations.

For the perfect conductor limit case ($\sigma_u \rightarrow \infty$, $\mathbf{e}_u \rightarrow 0$ and σ_p finite in $\Omega_{c,i}$), the terms defining the source current density \mathbf{j}_s (6) are, on the one hand, $\sigma_u \mathbf{e}_u$ being at the limit the surface current density on $\partial\Omega_{c,i}$ and, on the other hand, $\sigma_p \mathbf{e}_u$ being null in $\Omega_{c,i}$ because \mathbf{e}_u tends to zero. Consequently, one has to consider \mathbf{j}_s as a surface source current density, i.e. $\mathbf{j}_s = -\sigma_u \mathbf{e}_u$ on $\partial\Omega_{c,i}$. The source \mathbf{k}_s (7) is zero because the permeability in $\Omega_{c,i}$ is not altered.

For the perfect magnetic material ($\mu_u \rightarrow \infty$, $\mathbf{h}_u \rightarrow 0$ and μ_p finite in $\Omega_{c,i}$), the source \mathbf{k}_s (7) is reduced to $-\mu_u \partial_t \mathbf{h}_u$. The source current density \mathbf{j}_s (6) is zero, the conductivity in $\Omega_{c,i}$ remaining unchanged.

The weak forms of the perturbation problems are to be written coherently depending on the place of the sources in the equations (3) and (4), which will be detailed in the extended paper. Because the magnetic vector potential formulation is based on the weak form of (3), the source \mathbf{j}_s can be directly involved in one of its integral terms, whereas the source \mathbf{k}_s asks to be involved in extended strong definitions of the electric field ($\mathbf{e} = -\partial_t \mathbf{a} - \text{grad } v + \partial_t \mathbf{a}_u$) and the magnetic vector potential ($\mathbf{b} = \text{curl } \mathbf{a} + \mathbf{b}_u$). A clear distinction between weak and strong forms is of importance for correctly managing

the integral terms with the surface sources. Consequences on the circuit relations, relating the total current and the voltage of each conductor region, will be detailed as well.

3. Application

The developed technique will be validated on application examples with passive and active conductors and its domain of validity will be determined. Its main advantages versus the impedance-type boundary technique will be pointed out. An illustration of results is given in Fig. 1.

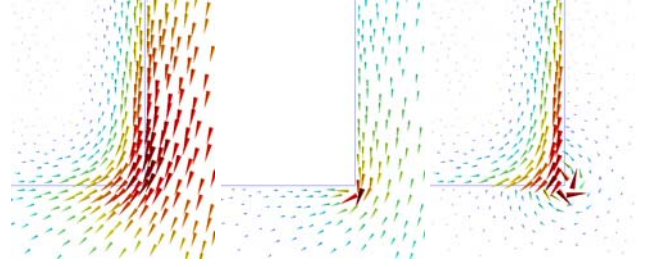


Fig. 1. Magnetic flux density for the classical FE solution (left), the unperturbed or limit solution with perfect conductor (middle) and the perturbation solution (right).

4. Conclusions

The developed perturbation method offers a way to uncouple FE regions in eddy current frequency and time domain analyses with high frequency excitations, allowing the solution process to be lightened. The skin and proximity effects in both active and passive conductors with magnetic properties can be accurately determined in a wide frequency range, allowing precise losses calculations in inductors as well as in external conducting pieces, in particular in inductively heated pieces.

Once calculated, the source limit solution for perfect conductors or magnetic materials can be used in each sub-problem not only for a single high frequency signal but for several signals. This allows efficient parameterized analyses on the signal form and the electric and magnetic characteristics of the conductors in a wide range, i.e. on all the parameters affecting the skin depth. Nonlinear analyses will then clearly benefit from this.

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