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# Sky coverage estimation for multiconjugate adaptive optics systems: strategies and results

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## ABSTRACT

We propose an improvement of the sky coverage estimation for multiconjugate adaptive optics (MCAO) systems. A new algorithm is presented which allows us to account for the real corrected field-of-view surface corrected by an MCAO system [depending on guide star (GS) positions and system characteristics] as well as the type of strategy (star-oriented or layer-oriented) considered for the wavefront sensing. An application to the European Southern Observatory MCAO demonstrator (MAD) system is considered. In the context of this particular application, the importance of parameters such as the GS geometry, the generalized isoplanatic angle and the magnitude difference between GSs is highlighted.

**Key words:** instrumentation: adaptive optics – techniques: high angular resolution.

## 1 INTRODUCTION

Adaptive optics (AO) is a powerful technique to correct for the degradation induced by atmospheric turbulence and to reach the diffraction limit of large ground-based telescopes. Because of anisoplanatic effects, the correction is efficient only in a limited field of view (FoV) around the wavefront sensor (WFS) guide star (GS). These effects originate from the fact that the turbulence is distributed in the volume above the telescope. Then, wavefronts from different directions in the sky have different aberrations. For typical atmospheric conditions and at near-infrared wavelengths, the isoplanatic domain is only a few tens of arcseconds (Fried 1982). In addition, AO systems need to have relatively bright GSs [typically lower than magnitude 16–17 (see Rousset et al. 2002)] to produce a significant correction. The combination of these two points dramatically limits the portion of the sky accessible with classical AO. In order to extend the FoV, a modification of the classical AO is required, leading to the concept of multiconjugate AO (MCAO) (Dicke 1975; Beckers 1988; Ellerbroek & Rigaut 2000). Using a GS asterism to measure the wavefront in several directions in the FoV and several deformable mirrors (DMs) conjugated at different selected altitudes allows the MCAO system to correct for turbulence over a field larger than the isoplanatic patch.

When the GSs are natural ones, a key point of AO is related to the part of the sky accessible for a given system performance. This is provided by a global sky coverage study. Classically, sky coverage provides the fraction of the sky that contains stars meeting given

conditions on their fluxes in order to achieve a given system performance. The generalization to MCAO implies being able to account for the number, magnitude and magnitude difference of the natural GSs (NGSs). Several algorithms have been proposed to deal with all these parameters (Le Louarn et al. 1998; Marchetti, Raggazzoni & Diolaiti 2002). Nevertheless, they do not take into account the relative positions of the stars in the FoV; yet, this parameter is essential to quantify the final performance of a MCAO system (Fusco et al. 2000). We propose in this article an extension of the ‘classical’ notion of sky coverage (Le Louarn et al. 1998) for MCAO by introducing additional parameters in the sky coverage algorithm. These parameters are related to the observing conditions [GS geometry and magnitude difference between GSs (Raggazzoni et al. 2002), isoplanatic angle] and to the system itself (WFS concept).

The article is structured as follows. We first recall, in Section 2, the different approaches already proposed for sky coverage in MCAO. In Section 3, the wavefront sensing and the reconstruction concepts in MCAO are briefly described. We particularly highlight the difference between star- and layer-oriented concepts. In Sections 4 & 5, definitions and descriptions are proposed for the various types of FoV that must be considered in a MCAO system and which are mandatory for an accurate sky coverage estimation. In Section 6 a description of our new approach for sky coverage computation in MCAO is proposed. Improvements with respect to existing algorithms are highlighted. All of these points are gathered in a new sky coverage algorithm called ‘surface sky coverage’ (SSC). This new algorithm uses, as a basic input, a statistical model of the stellar population, called the Besançon model [see Robin et al. (2003) for more details] which is described in Section 6.4. Finally, examples of application are proposed in Section 7 for the MCAO demonstrator

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(MAD) developed at the European Southern Observatory (ESO). It allows us to compare classical sky coverage estimation with our new approach and to highlight all the possibilities of the new sky coverage algorithm.

## 2 SKY COVERAGE IN MCAO

The notion of sky coverage is rather simple for classical AO. Indeed, in this case, the system performance can be directly related to the GS magnitude and its separation from the object of interest (for given atmospheric conditions). The AO sky coverage is therefore nothing but a simple count of GSs in a given sky region. Its value is given by the number of GSs (of magnitude lower than a given limit) multiplied by the isoplanatic angle over the global size of the sky region. Unfortunately, the generalization to MCAO systems is not obvious. Instead of one single parameter, the final system performance is given by a combination of multiple factors in interaction.

Two different kinds of studies can be performed to derive sky coverage information from MCAO design and performance requirements.

The first one, which is considered in this article, consists of a high-level study and, in fact, a generalization of the classical AO sky coverage estimation. The goal of our approach is rapidly to derive a first estimation of the sky coverage related to a given MCAO concept and WFS scheme. The idea here is to link the MCAO global performance to physical information on guide stars (number, magnitudes and positions) and on the system (diameter, FoV, WFS approaches and WFS devices). A first system analysis allows observational constraints to be determined (magnitude ranges, number of GSs, maximum distance between GSs, maximum magnitude difference between GSs) as a function of system characteristics (FoV, telescope diameter, system throughput, reconstruction approaches, WFS device, number of corrected modes . . .) and expected performance (correction level and uniformity in the FoV). It is clear that such a kind of approach will only give tendencies, but it is essential to explore a large domain of parameters and rapidly to obtain tendencies on general behaviours as well as orders of magnitude for sky coverage (as a function of galactic position, for instance) for various

kinds of MCAO systems. It allows us to obtain the first trade-offs concerning the system design and to adjust the scientific requirements. In particular, it may help us to select the systems for which laser guide stars are mandatory. It would also help us to provide first inputs for MCAO systems on future extremely large telescopes.

The second kind of study consists of a complete simulation of a given MCAO system for a large number of GS configurations obtained from real star fields or statistical models (Arcidiacono et al. 2004; Assemat 2004). This approach is complex and time-consuming (since a lot of simulations have to be performed to reach a good statistical convergence). Nevertheless, it allows us to account for complex system characteristics and provide accurate and exhaustive results. It has to be performed at a final stage of a system design in order to confirm and refine the global system performance and the system design.

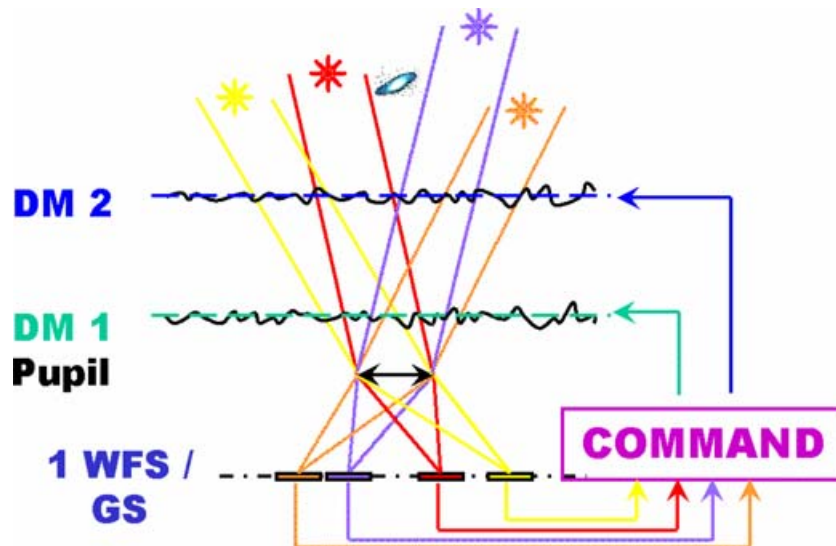
These two approaches complement each other, and have to be considered at different stages of the study of a MCAO system. In any case, information is required on the stellar distribution in the sky (for various spectral bandwidths). This information can be provided by:

- (i) the direct use of real star fields given by cross-correlation of catalogues;
- (ii) a statistical approach which provides synthesized star fields.

In the following, we have adopted the statistical approach using the so-called Besançon model (Robin et al. 2003) which will be briefly presented in Section 6.4.

## 3 WAVEFRONT SENSING AND RECONSTRUCTION CONCEPTS IN MCAO

Wavefront sensing and reconstruction processes are two of the main issues of any MCAO system. Indeed, an accurate wavefront sensing concept is essential to obtain pertinent information on the turbulent volume for which the MCAO has to correct. In addition, these WFS measurements have to be coupled with an optimal reconstruction process (see Ellerbroek 1994; Fusco et al. 2001; Le Roux et al. 2004) in order to find the best command to apply on each DM.



**Figure 1.** SO concept in MCAO. In this WFS concept, one WFS per GS is used and the measurements from all the sensors are combined to control several DMs.

Two approaches have been proposed so far to measure and correct for the turbulent volume in MCAO: the star-oriented (SO) and the layer-oriented (LO) schemes. They are briefly described below.

### 3.1 The SO concept

In the SO case (see Fig. 1) the wavefront  $[\Phi_{\text{SO}}^{\alpha_k}(\mathbf{r})]$  is measured in each GS direction ( $\alpha_k$ ) by a dedicated WFS as shown in equation (1):

$$\{\Phi_{\text{SO}}^{\alpha_k}(\mathbf{r})\}_{k=1}^{\mathcal{K}_*} = \left\{ \sum_{l=1}^{n_l} \varphi_{\text{true}}^l(\mathbf{r} - \alpha_k h_l) + \text{Noise}(\bar{N}_k) \right\}_{k=1}^{\mathcal{K}_*}, \quad (1)$$

where  $\varphi_{\text{true}}^l$  represents the true turbulent wavefront in the layer  $l$ ,  $\bar{N}_k$  is the flux per GS direction,  $\mathcal{K}_*$  is the number of GS directions,  $n_l$  is the number of turbulent layers used to estimate the turbulent volume and  $h_l$  is the altitude of the  $l$ th layer.  $\mathbf{r}$  stands for the pupil coordinates. The Noise (.) function depends on the WFS characteristics.

The reconstruction process consists of the computation of the turbulent volume from the wavefronts measured in the GS directions followed by a projection on to the DMs in order to obtain the best correction for a specified field of interest [see Fusco et al. (2001) for a complete description]. Hence the quality of this process is directly related to the noisy data of each WFS, i.e. to the magnitude of each GS.

From a photometric point of view, the only constraint for the SO concept consists of a limiting magnitude ( $m_{\text{lim}}$  per GS).  $m_{\text{lim}}$  is defined as follows: considering a GS distribution in the technical FoV and a given set of system characteristics (sampling frequency, sub-aperture number, detector noise, transition...), the MCAO performance is achieved within the corrected FoV when all the GSs have a magnitude lower than or equal to  $m_{\text{lim}}$ .

### 3.2 The LO concept

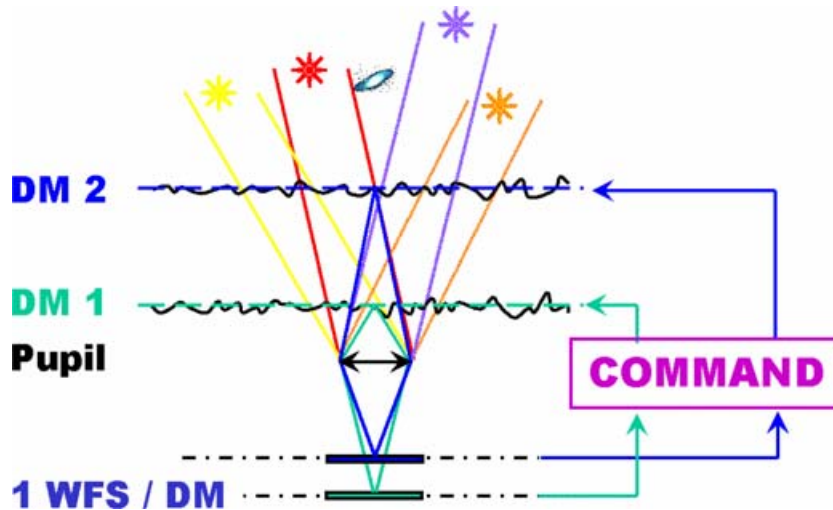
In the LO case (see Fig. 2), the phase measurement is performed using one WFS per DM. The WFS device is optically conjugated to a given altitude  $h_j$ . In that case, the measured phase  $[\varphi_{\text{LO}}^{\text{WFS layer } j}(\mathbf{r})]$  is much more complex when compared with the SO case as shown

in equation (2):

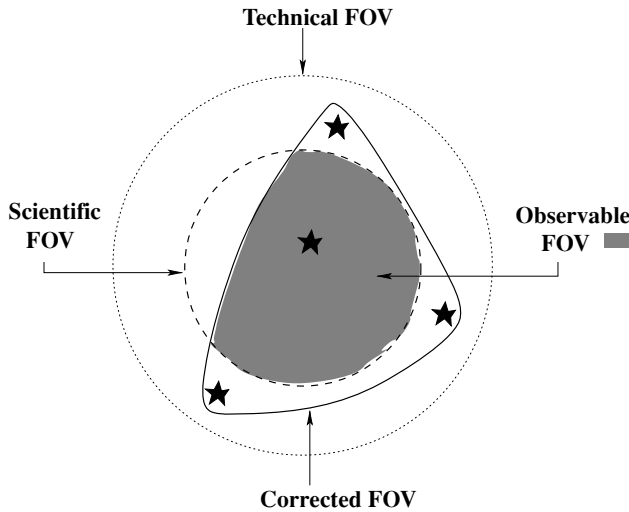
$$\begin{aligned} & \left\{ \varphi_{\text{LO}}^{\text{WFS layer } j}(\mathbf{r}) \right\}_{j=1}^{n_{\text{WFS}}} \\ &= \left\{ \frac{\sum_{k=1}^{\mathcal{K}_*} [\eta_k \bar{N}_k \sum_{l=1}^{n_l} \varphi_{\text{true}}^l(\mathbf{r} - \alpha_k(h_j - h_l))] + \frac{1}{\gamma_j} \text{Noise}(\gamma_j \sum_{k=1}^{\mathcal{K}_*} \eta_k \bar{N}_k [\text{Pup}(\mathbf{r} - \alpha_k h_j)])}{\sum_{k=1}^{\mathcal{K}_*} [\eta_k \bar{N}_k \text{Pup}(\mathbf{r} - \alpha_k h_j)]} \right\}_{j=1}^{n_{\text{WFS}}}, \quad (2) \end{aligned}$$

where Pup stands for the pupil function, and  $\gamma_j$  represents the flux separation between the WFS ( $\sum_{j=1}^{n_{\text{WFS}}} \gamma_j = 1$ ) and  $\eta_k$  the optical attenuation in each GS direction before the flux co-addition on to the WFS. The main interest of such an approach is the light co-addition before the detection, which increases the signal-to-noise ratio per WFS especially when noisy CCDs are considered (Bello et al. 2003). There is no reconstruction process needed in that case since the measured phase  $\varphi_{\text{LO}}^{\text{WFS layer } j}(\mathbf{r})$  can be used to control the DMs directly. Nevertheless, this co-addition has a drawback, the wavefronts coming from different directions in the FoV being mixed and weighted by the GS flux [leading to an information loss and a possible phase estimation problem (Bello et al. 2003; Nicolle et al. 2006)]. As shown in equation (2), the wavefronts coming from the brighter stars are dominant in the phase measurement process. A way to deal with GS flux differences is optically to attenuate the flux of the brighter stars before the detection [ $\eta_k$  coefficients in equation (2), see Nicolle et al. (2006)]. From a photometric point of view, the relevant parameters in the LO case are the integrated flux over all the GSs, the magnitude difference between GSs and the values of the attenuation coefficients  $\eta_k$  ( $0 \leq \eta_k \leq 1$ ). Using these coefficients, one reduces the effects of magnitude difference on the LO measurement (Nicolle et al. 2005), but it leads to a reduction of the global available flux and then to an increase of the noise measurements.

To summarize, if a SO approach is considered, the critical parameter would be the GS magnitude, while two parameters would be used in the LO case: the integrated flux per WFS (or layer) and the magnitude difference between GSs (including a possible attenuation coefficient per GS direction).



**Figure 2.** LO concept in MCAO. In this case, there is one WFS per DM (conjugate at the same altitude). The fluxes coming from all the GSs are co-added before the detection on each WFS.



**Figure 3.** Definition of the various notions of FoV in MCAO: the technical FoV (Tech-FoV) (largest one) in which the GSs have to be found; the scientific FoV (Sci-FoV) ( $\leq$  the technical FoV) in which the required MCAO performance should be achieved; the corrected FoV (Corr-FoV) in which, for a given GS configuration, the MCAO performance is actually achieved; and the observable FoV (Obs-FoV) which represents the part of the scientific FoV actually corrected for a given GS configuration.

#### 4 VARIETIES OF FOV IN MCAO

Whatever the chosen WFS concept, the use of several bright GSs is mandatory in order to sense the turbulent volume. These GSs have to be found in a FoV large enough to ensure a good measurement of the turbulent volume and thus a good correction in the directions of interest.

According to the scientific need, the FoV in which GSs have to be found may be different from the one in which the MCAO performance has to be achieved.

More generally, in MCAO, four notions of FoV can be defined to describe the system requirements and final performance fully. This clearly increases the complexity of the sky coverage estimation. These four FoV are described below and illustrated in Fig. 3.

##### 4.1 The technical FoV

The technical FoV (Tech-FoV) is the field in which GSs have to be found in order to sense the turbulent volume. Several constraints govern the final choice of this field.

(i) Technical constraints related to the telescope and system design.

(ii) Scientific constraints – that is, the average value and the evolution of the system performance in the field (Strehl ratio, encircled energy ...). These constraints are linked to the telescope and system design as well as to the atmospheric parameters (seeing,  $C_n^2$  profile ...). In particular, the size of the technical field directly impacts on the performance of a MCAO system since an increase of the technical FoV implies a larger volume of turbulence to be sensed and thus a possible increase of tomographic reconstruction error. It is interesting to mention here that larger telescope diameters may allow us to deal with larger technical FoVs (Ragazzoni 1999; Fusco et al. 2000) which should be potentially interesting for a MCAO system based on NGSs for extremely large telescopes.

##### 4.2 The scientific FoV

The scientific FoV (Sci-FoV) corresponds to the field in which the scientific targets have to be located and thus in which the system performance has to be achieved. The Sci-FoV can be smaller than the Tech-FoV depending on the telescope design. The performance to be achieved in the Sci-FoV depends on the GS separation and magnitudes. For instance, in terms of the Strehl ratio, the performance  $SR_\alpha$  ( $\alpha \subseteq$  Sci-FoV) can be expressed as follows:

$$SR_\alpha \simeq e^{-\sigma_\alpha^2}, \quad (3)$$

with (see Fusco et al. 2000)

$$\sigma_\alpha^2 = \mathbf{M}_\alpha \mathcal{C} \{ \sigma_{\text{noise},k}^2 \}_k. \quad (4)$$

Here,  $\mathbf{M}_\alpha$  is a projection matrix from the reconstructed phase in the DM on to a given direction  $\alpha$  of the Sci-FoV.  $\mathbf{M}_\alpha$  depends on the Sci-FoV size, the  $C_n^2$  and its sampling by the DMs.  $\mathcal{C} \{ \sigma_{\text{noise},k}^2 \}_k$  is the noise propagation from the WFS measurements to the tomographic reconstruction of the volumic phase.  $\mathcal{C} \{ \sigma_{\text{noise},k}^2 \}_k$  depends on the GS flux, the WFS characteristics and the GS relative positions.  $k$  stands for the  $k$ th GS direction.

From equation (4), it is clear that the MCAO performance in the Sci-FoV will depend on the following parameters:

- (i) the GS relative positions in the Tech-FoV but also with respect to the Sci-FoV (see Sections 4.3 and 4.4) which allow us to compute  $\mathbf{M}_\alpha$ ;
- (ii) the GS limiting magnitudes and GS magnitude difference in the case of the LO WFS concept.

These parameters should be taken into account in the MCAO sky coverage in order to provide accurate results. They will be used to provide conditions on GSs which allow us to ensure that  $\sigma_\alpha^2$  is small (or uniform) enough to fulfil the performance requirement in the whole Sci-FoV.

##### 4.3 The corrected FoV

The corrected FoV (Corr-FoV) represents, for a given GS configuration, the part of the Tech-FoV actually corrected by the MCAO system.

The Corr-FoV (see Fig. 3) is equal to the convolution of the surface defined by the stars within the Tech-FoV with the isoplanatic field. Note that we have assumed here that the MCAO system is able to interpolate the wavefront inside the surface defined by the GSs. Such an approximation is directly linked to the telescope diameter, the number of corrected modes and the  $C_n^2$  profile.

##### 4.4 The observable FoV

The observable FoV (Obs-FoV) is the last but also the really important FoV (from an astronomer's point of view). It corresponds to the part of the Sci-FoV actually corrected by the MCAO system for a given GS configuration. It is nothing but the intersection of the Corr-FoV with the Sci-FoV:

$$\text{Obs-FoV} = \text{Corr-FoV} \cap \text{Sci-FoV}. \quad (5)$$

It is interesting to note here that when the Tech-FoV is equal to the Sci-FoV, it automatically implies an equality between the Corr-FoV and Obs-FoV.

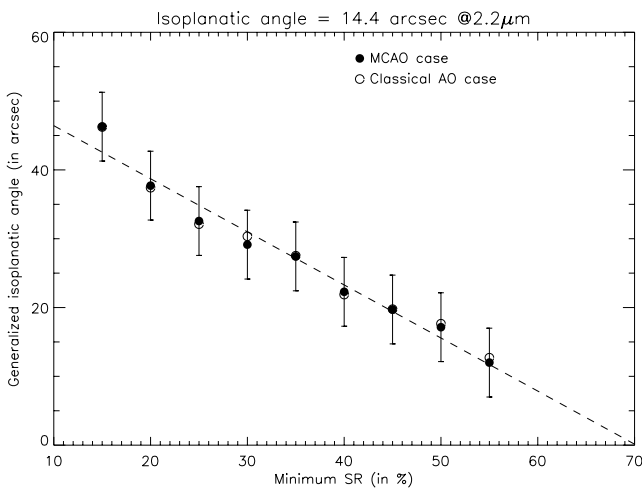
## 5 ISOPLANATIC ANGLE

In classical AO, the isoplanatic angle is defined as a distance from the GS position where the AO performance is better than a minimum requirement, this limit value being usually expressed as a residual variance or a Strehl ratio. It seems clear that, generally speaking, this isoplanatic angle should depend on the required performance (limit value), the system characteristics (actuator number, temporal sampling frequency . . .), the GS characteristics (flux) and of course the atmospheric conditions ( $C_n^2$  profile). A particular case is the  $\theta_0$  angle defined by Fried (Fried 1982), which represents the isoplanatic angle obtained in the case of a perfect AO system (infinite number of actuators, infinite sampling frequency, no noise) and for a residual phase smaller than  $1 \text{ rad}^2$  in the FoV. Under this assumption, Fried gives a simple expression of  $\theta_0$  which only depends on atmospheric conditions but remains pessimistic for realistic AO systems.

The generalization of the classical AO isoplanatic angle to MCAO systems is not straightforward, and two points have to be distinguished.

(i) The system capability to interpolate and correct for the turbulent volume within the field defined by the GS positions. This is characterized by the number and the positions of the GSs for the wavefront interpolation (Fusco et al. 2000) and by the number of DMs for the turbulent volume correction (Tokovinin et al. 2000). In this paper, we will assume that the MCAO system is well-dimensioned, i.e. the required performance is achieved in the field surface delimited by the GS positions. In other words, the choice of the Tech-FoV is done to ensure an efficient interpolation between GSs.

(ii) The degradation of the correction *outside* the corrected area due to angular decorrelation of the wavefront with respect to the DM correction. In a first approximation, this decorrelation should be the same in MCAO and classical AO and barely depends on the size of the corrected area (large for a MCAO system and equal to a single point for classical AO). In order to validate this assumption, an estimation of the isoplanatic angle is obtained on simulation (using the 100 GS realizations) as a function of a minimum Strehl ratio



**Figure 4.** Evolution of the generalized isoplanatic angle with respect to the specification in terms of minimum performance to achieve in the FoV. Average and rms values (filled circles) obtained for 100 GS configurations (randomly determined) are considered. The same results (open circles) are plotted in the case of a classical AO system (using the same parameters for the turbulence profile). The dashed line represents a linear fit of the measured isoplanatic values.

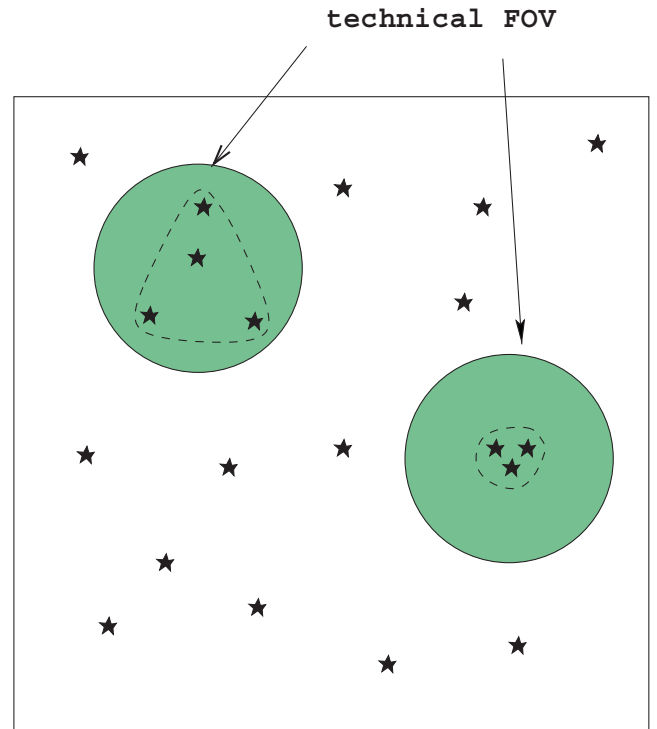
specification for a classical AO system and a MCAO one (a MAD-like system as defined in Section 7.1). The results are presented in Fig. 4. The isoplanatic angles on classical AO and MCAO are nearly identical. For this particular system and for a typical Paranal profile, the evolution of the isoplanatic angle with the minimum Strehl ratio can be fitted by a linear law. Depending on the chosen criterion, the isoplanatic angle goes from 50 down to 10 arcsec. This has to be compared with Fried’s  $\theta_0$  of 14.4 arcsec (which is known to be a pessimistic value) for the considered atmospheric conditions and imaging wavelengths ( $2.2 \mu\text{m}$ ). These results allow us to consider simple AO data to estimate the generalized isoplanatic angle to be used in the SSC algorithm and thus avoid a complex MCAO simulation.

## 6 SKY COVERAGE IN MCAO

### 6.1 The conventional definition

The performance of a MCAO system depends on the quantity and quality of the wavefront measurements that can be obtained in the Tech-FoV. Thus the knowledge of star number and magnitude within a given region is a key point to study: the conventional MCAO sky coverage (Le Louarn et al. 1998; Marchetti et al. 2002) provides this information. It is based on the computation of the probability of finding stars that meet a set of conditions within one Tech-FoV, in the region of the sky considered. The classical sky coverage corresponds to the average number (in per cent) of Tech-FoVs in which the conditions on stars (number and/or magnitude) are met (Fig. 5) without any information on GS distributions in the FoV.

Assuming that the stellar distribution follows Poisson statistics, one can compute the probability  $P$  of finding at least one star within



**Figure 5.** Conventional MCAO sky coverage. The solid line represents the Tech-FoV, the dashed line the actual Corr-FoV given the GS configuration. In the classical MCAO sky coverage definition, the Tech-FoV is assumed to be corrected whatever the GS configuration.

a given radius  $r$  (Le Louarn et al. 1998):

$$P_{N_{\text{stars}} > 0}(m, r) = 1 - \exp\left[-\frac{\pi r^2 \nu(m)}{3600^2}\right], \quad (6)$$

where  $\nu(m)$  is the density of stars brighter than the magnitude  $m$  (per square degree) in a given galactic position (provided by the Besançon model). The probability of finding more than  $X$  stars is obtained from equation (6):

$$P_{N_{\text{stars}} > X}(m, r) = 1 - \sum_{i=0}^X P_{N_{\text{stars}} = X-i}(m, r). \quad (7)$$

This relation does not allow us to introduce specific conditions on stars such as limiting magnitude difference, etc. For this reason, a simulated process proposed by Marchetti et al. (2002) is considered.

To compute the sky coverage for different conditions (Galactic coordinates, limiting magnitude, FoV, etc.), and in order to account for GS positions in the FoV, star fields are generated using the following process. We first define conditions on stars and on the Tech-FoV:

- (i) a 1-deg<sup>2</sup> field is simulated on 512 × 512 pixels;
- (ii) the number of stars is generated using a Poisson distribution with the mean equal to the stellar density provided by the Galactic model at this magnitude (Besançon model);
- (iii) for each magnitude, the positions of the stars are defined by a random deviate drawn from a uniform distribution.

The sky coverage is computed using these simulated stellar fields. We search all of the Tech-FoV (defined as proposed in Section 4.1) within the 1-deg<sup>2</sup> star field in which the searched conditions on stars are met. The process is repeated typically 500 times and the sky coverage is obtained by averaging these results.

In the classical sky coverage approach, it is implicitly assumed that all of the Sci-FoV is corrected if the right number of GSs is found in the Tech-FoV. In other words, it does not account for the relative position between GSs within the Tech-FoV. Fig. 5 illustrates this approximation: the two grey Tech-FoVs (equal here to the scientific one) contain the searched GSs. In the classical sky coverage, they have the same weight in the computation, whereas the parts of the Tech-FoV really sensed by these two stellar configurations are very different. The results of the classical sky coverage in MCAO are thus optimistic.

Hence this sky coverage definition needs to be improved. In particular, accounting for the GS geometry is essential to describe well the system performance in a given FoV. This leads us to introduce some new parameters in the sky coverage computation to be as close as possible to the real description of the MCAO performance in the FoV and in particular the four FoV definitions (technical, scientific, corrected and observable) presented in Section 4.

## 6.2 The ‘surface’ sky coverage definition

In order to refine the sky coverage estimation, the classical approach defined in the previous section has to be corrected from the relative position of GSs within the Tech-FoV with respect to the Sci-FoV. This process is detailed below.

### 6.2.1 Refinement of the Obs-FoV computation

The computation of the Obs-FoV is obtained using the GS distribution in each Tech-FoV, the turbulence parameters (definition of an isoplanatic angle which accounts for the turbulence profile and

system expected performance) and the system characteristics (see Section 5):

- (i) the limiting magnitude per GS;
- (ii) the integrated magnitude;
- (iii) the  $\Delta_m$  (difference of magnitudes) between GSs;
- (iv) the minimum number of WFSs ( $N_{\text{WFS}, \text{min}}$ ) required to obtain a correct wavefront reconstruction (minimum number of GSs); and
- (v) the maximum number of WFSs ( $N_{\text{WFS}, \text{max}}$ ).

From these data, one can compute the corrected surface from the available GSs that *fulfil flux conditions* (limiting GS magnitude per WFS and/or limit difference of magnitude between GSs) and the Sci-FoV. Three cases have to be considered for each Tech-FoV.

- (i) The number of GSs is smaller than  $N_{\text{WFS}, \text{min}}$ . In that case the surface value is set to 0.
- (ii) The number of GSs is larger than  $N_{\text{WFS}, \text{min}}$  but smaller than  $N_{\text{WFS}, \text{max}}$ . In that case we compute the largest possible surface including all the GSs.
- (iii) The number of GSs is larger than  $N_{\text{WFS}, \text{max}}$ . In that case we compute all the possible surfaces including the maximum number of GSs and we define the global surface as the union of all the computed surfaces.

For each case, the surface is obtained by the convolution of the polygon formed by the GS position with a disc of diameter equal to the isoplanatic angle (the higher the performance, the smaller the isoplanatic angle as illustrated in Fig. 4).

The surface optimization is rather simple when only conditions on individual GS magnitudes are considered (limiting magnitude per GS). When a condition on the magnitude difference between GSs is added (including the possibility of dimming the brightest star as presented in Section 3.2), however, the optimization process becomes more complex. Indeed, the maximization of the Obs-FoV has to be performed under the constraint of a maximum flux difference between GSs in addition to the constraint on GS flux. The surface maximization algorithm accounts for this kind of attenuation and allows us to find the optimal observable surface for a given maximum flux difference between GSs (with the possibility of an attenuation of the brightest star) and a given integrated flux.

## 6.3 The ‘surface’ sky coverage computation

The observable surface is introduced in the sky coverage computation by introducing the ratio between the surface of the Obs-FoV and the surface of the Sci-FoV in the sky coverage computation. This new sky coverage definition is called the ‘surface’ sky coverage (SSC). For each GS geometry, the fraction of the Sci-FoV really sensed by the GSs is defined by

$$\mu_s = \frac{S_{\text{obs-fov}}}{S_{\text{sci-fov}}}; \quad (8)$$

$\mu_s$  is called the weighted coefficient  $\in [0, 1]$ .

The smaller the Sci-FoV is in comparison with the Tech-FoV, the closer to 1  $\mu_s$  should be. The SSC defines the percentage of the sky that can be observed at a given galactic coordinate for

- (i) a given Tech-FoV,
- (ii) a given Sci-FoV
- (iii) a given system performance:

$$P_{N_{\text{stars}} > X}^{\text{Surface}}(m, r) = \langle \eta \mu_s \rangle, \quad (9)$$

where  $\eta$  is a Boolean number which is set to 0 when conditions on GSs in the FoV are not fulfilled and set to 1 otherwise.  $\langle \cdot \rangle$  stands for a statistical average on random realizations obtained using a statistical model of the stellar population (see Section 6.4).

It is interesting to note here that if  $\mu_s$  is always set to 1 then,  $P_{N_{\text{stars}} > X}^{\text{Surface}}(m, r)$  is nothing but the classical sky coverage for MCAO.

#### 6.4 Model of stellar population

The sky coverage algorithm requires a statistical knowledge of the stellar distribution in the Galaxy. In the following, we have considered the Besançon model (Robin et al. 2003). This model reproduces the stellar content of the Galaxy using some physical assumptions and a scenario of star formation and evolution.

### 7 SKY COVERAGE ESTIMATIONS

#### 7.1 System assumptions

In order to demonstrate the relevance of our approach for sky coverage estimation in MCAO, and to highlight the importance of the new parameters integrated in the algorithm (surface defined by the GS distributions, isoplanatic angle, magnitude difference between GSs in the case of LO WFSs), we have considered two MCAO system configurations.

The first one is based on a SO scheme based on the MAD design (see Section 7.2) with three WFSs and two DMs. In that case,

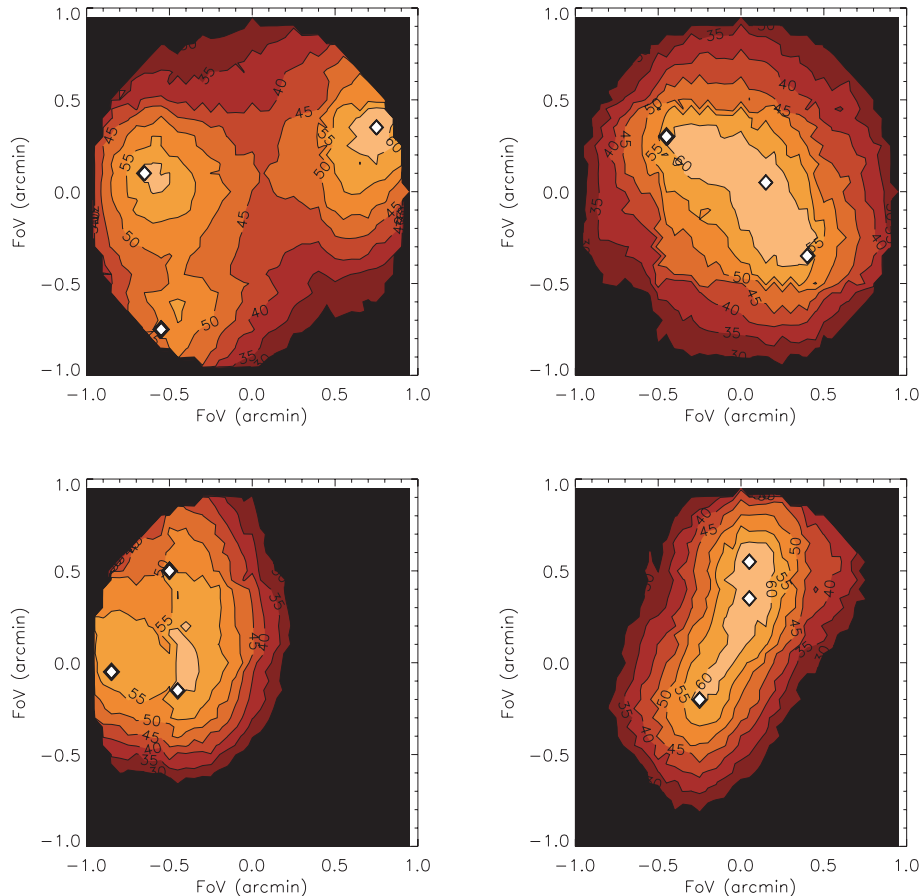
the important parameter is the limiting magnitude by WFS, that is the minimum number of photons required to obtain an accurate wavefront reconstruction per direction.

The second configuration is based on a LO scheme (see Section 7.3) with two measured layers (i.e two WFSs) and two DMs. Between three and eight GSs are co-added on each WFS and a 50/50 beam splitting between the two measured planes. The upper limit (eight GSs) is fixed by the number of devices used to collect the light from each star and to co-add this light on to a single detector. The lower limit (three GSs) is fixed by the need for an average of the wavefronts coming from several sky directions in order to isolate the contribution of the layer to be sensed by the WFS. In addition to a minimum number of GSs, a limiting magnitude difference  $\Delta_m$  between GSs is also mandatory to ensure a good estimation of the phase in the turbulent layers (Raggazzoni et al. 2002; Nicolle et al. 2005) as presented in Section 3.2. For the LO WFS, the important parameter is the limiting integrated magnitude (that is the equivalent magnitude of all the GSs co-added on each WFS) defined as

$$\text{mag}_{\text{int,lim}} = 2.5 \log_{10} \left( 0.5 \sum_{i=1}^{N_{\text{gs}}} 10^{-0.4 \text{mag}_i} \right), \quad (10)$$

where the 0.5 factor comes from the beam splitting between layers (that is between WFSs). It is interesting to note that the larger the number of measured layers, the smaller the flux separation coefficient.

Both SO and LO WFS performance depend on a limiting magnitude (or integrated magnitude in the LO case). It is clear that such



**Figure 6.** Strehl ratio evolution in the Tech-FoV (2 arcmin diameter field) for four GS configurations. Iso-Strehl ratios are plotted. The diamond symbols represent the GS positions.



a value depends both on the required performance of the MCAO system and on the telescope and system characteristics:

- (i) global throughput, imaging and analysis wavelengths, sampling frequency;
- (ii) WFS type – Shack–Hartmann, pyramid, curvature . . . ;
- (iii) detector characteristics – read-out noise, quantum efficiency.

In the following we will rely on the MAD (MCAO demonstrator, currently being developed at ESO) to set the main system characteristics even if several parameters will be modified in order to study their impact on the system sky coverage (in particular the CCD characteristics). The MAD bench is composed of two DMs of 66 actuators each (bimorph technology) respectively conjugated at 0 and 7 km from the telescope pupil. We have chosen a MAD-like configuration because this system already includes a SO (based on Shack–Hartmann devices) and a LO (based on pyramid devices) WFS mode.

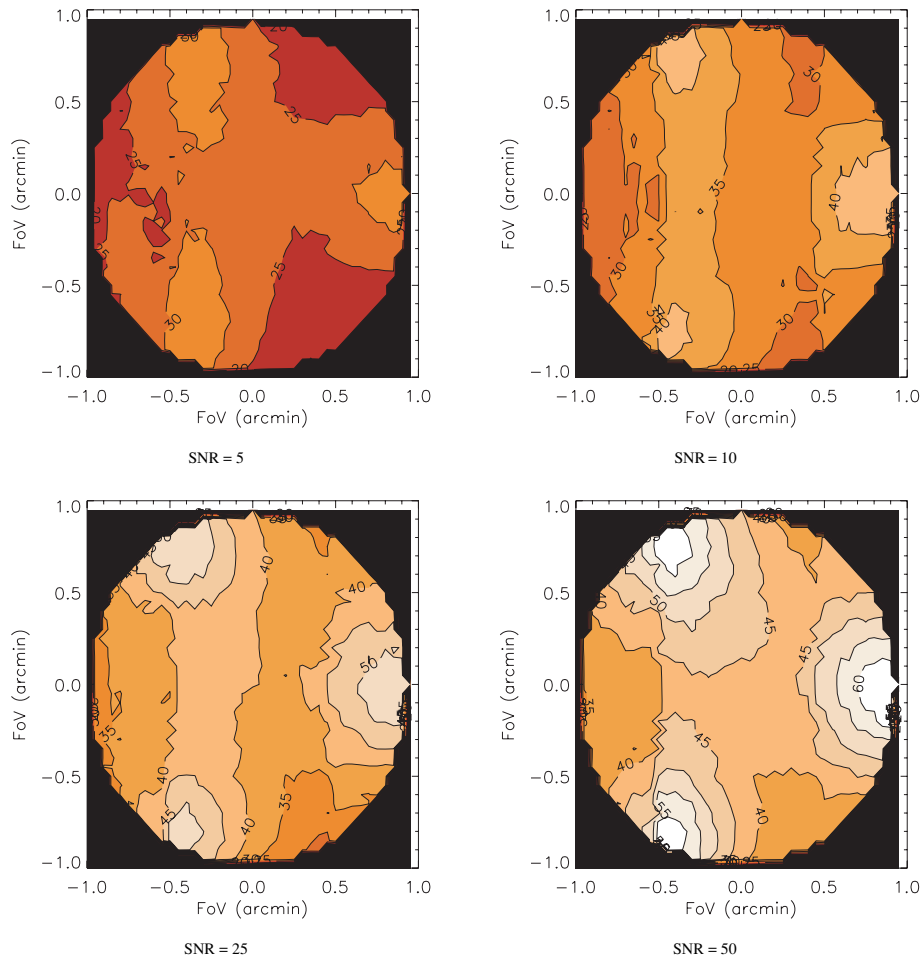
## 7.2 The SO mode

Let us first consider the case of SO measurements. As presented before, three WFSs (that is three GSs) are considered. The Tech-FoV is a disc of 2 arcmin diameter and the Sci-FoV is equal to the Tech-FoV. For each WFS a Shack–Hartmann device is considered with  $8 \times 8$  sub-apertures ( $8 \times 8$  pixels per sub-aperture) with a 2.4 FoV diaphragm. Each Shack–Hartmann device can pick up a GS in the entire FoV.

Considering first that the GS magnitude is faint enough to ensure a good wavefront measurement per GS direction, we plot in Fig. 6 the Strehl ratio evolution in the FoV for four GS configurations (randomly positioned in the FoV). It is clear that the definition of a corrected surface depends on a criterion on the system performance. In the following we have chosen to consider a minimum Strehl ratio (equal to 30 per cent) as a limit value defining the corrected FoV. This figure highlights the importance of the introduction of a surface parameter in the sky coverage computation.

As explained before, the SSC algorithm uses an isoplanatic angle (see Section 5) to account for the different corrected surfaces associated with various performance criteria (the only assumption is that the required performance is achieved within the GS configuration). The surface obtained from the GS positions is convolved with an isoplanatic angle to provide the estimated corrected surface for the given GS positions. An estimation of the isoplanatic angle is obtained as a function of minimum Strehl ratio specifications using Fig. 4.

The last, but essential, parameter to consider is of course the GS limiting magnitude. This limiting magnitude depends on a lot of parameters, from expected system performances to WFS and detector characteristics, system throughput and atmospheric conditions. The first point to study is the behaviour of the system performance with respect to WFS noise for typical atmospheric conditions. In order to limit the number of variables, we will consider a reduced parameter on each WFS to characterize its performance. This parameter is a signal-to-noise ratio (SNR) term which can be defined as the ratio

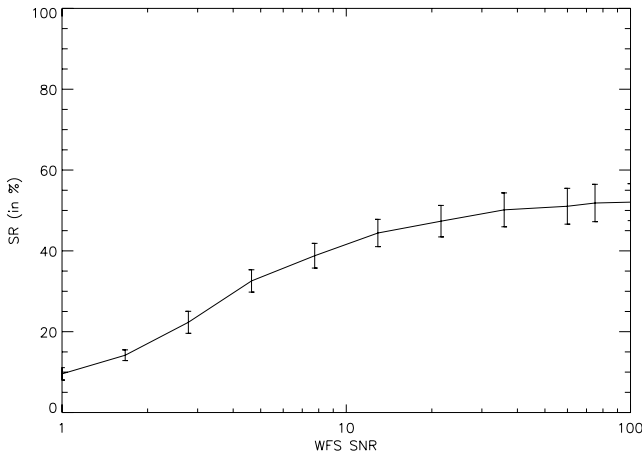


**Figure 7.** Strehl ratio evolution in the Tech-FoV (2 arcmin diameter field) for four SNR on GSs (same SNR per GS). Iso-Strehl ratios are plotted.

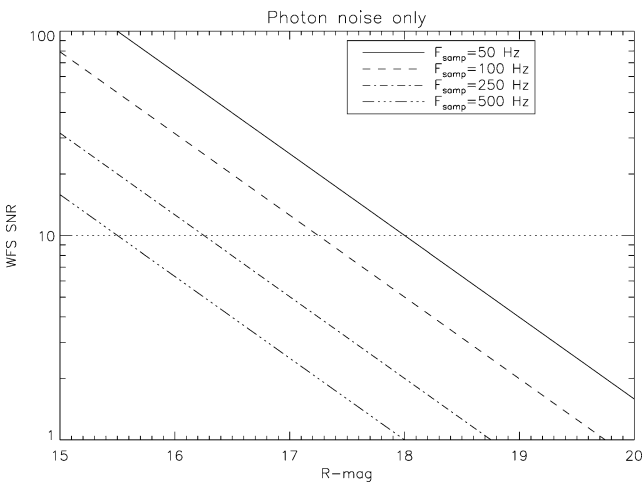
of the signal to be measured by the WFS (angle of arrival in the case of a Shack–Hartmann WFS) to the measurement noise (photon and detector noise):

$$\text{SNR} = \frac{\sigma_{\text{aoa}}^2}{\sigma_{\text{ph}}^2 + \sigma_{\text{detec}}^2}. \quad (11)$$

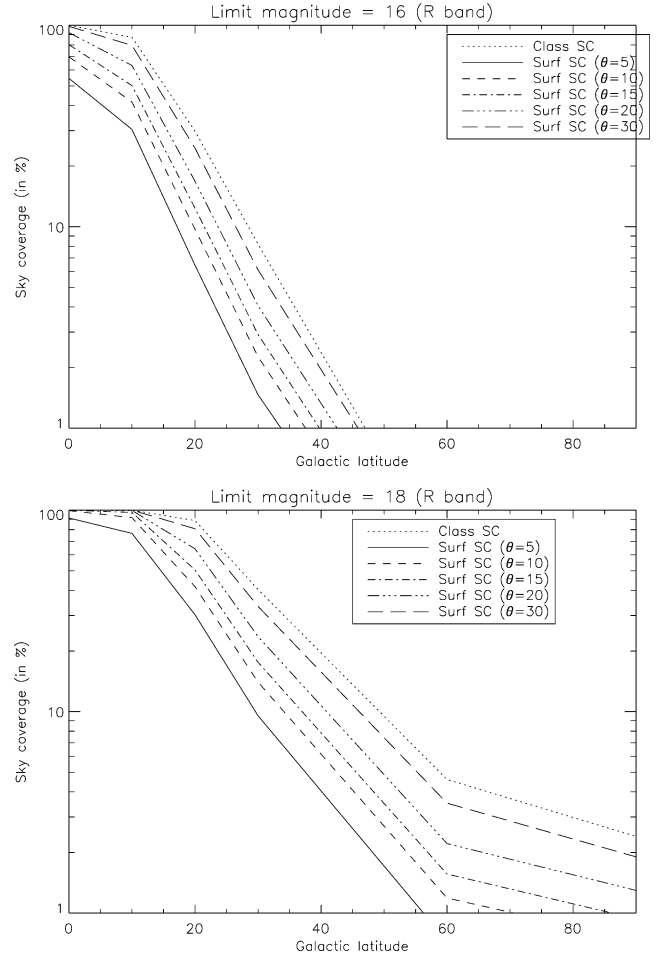
SNR depends on atmospheric conditions through  $\sigma_{\text{aoa}}^2$  and system and GS characteristics through  $\sigma_{\text{ph}}^2 + \sigma_{\text{detec}}^2$ . As an example, in Fig. 7 we plot the two-dimensional map performance obtained for various SNRs (5, 10, 25, 50) and assuming an equilateral triangular GS geometry. The three WFSs are assumed to have the same SNR. A more complete description of the SNR influence is proposed in Fig. 8 where the averaged Strehl ratio (and its rms fluctuation in the Tech-FoV) is plotted as a function of the SNR (the same for each GS). The GS geometry is the same as the one considered in Fig. 7. It shows that an average Strehl ratio greater than 40 per cent is reached for a SNR greater than 10. A plateau is achieved for SNRs larger than 50. In the following we will consider that a SNR of 10 is a limit to ensure a pertinent wavefront reconstruction. Fig. 9 connects the WFS SNR to the GS magnitudes for various system sampling frequencies and assuming a given system throughput (here  $2 \times$



**Figure 8.** Average and rms values of the Strehl ratio in the Tech-FoV for various SNRs on the three WFSs.



**Figure 9.** Evolution of the WFS SNR as a function of the GS magnitude (R band) and for various sampling frequencies (from 50 to 500 Hz). A photon noise limited detector is considered.



**Figure 10.** Comparison between classical and SSC estimations for various galactic latitudes and various generalized isoplanatic angles. Two cases have been considered in terms of limiting GS magnitude per WFS: 16 (upper panel) and 18 (lower panel). The SO MAD system has been considered.

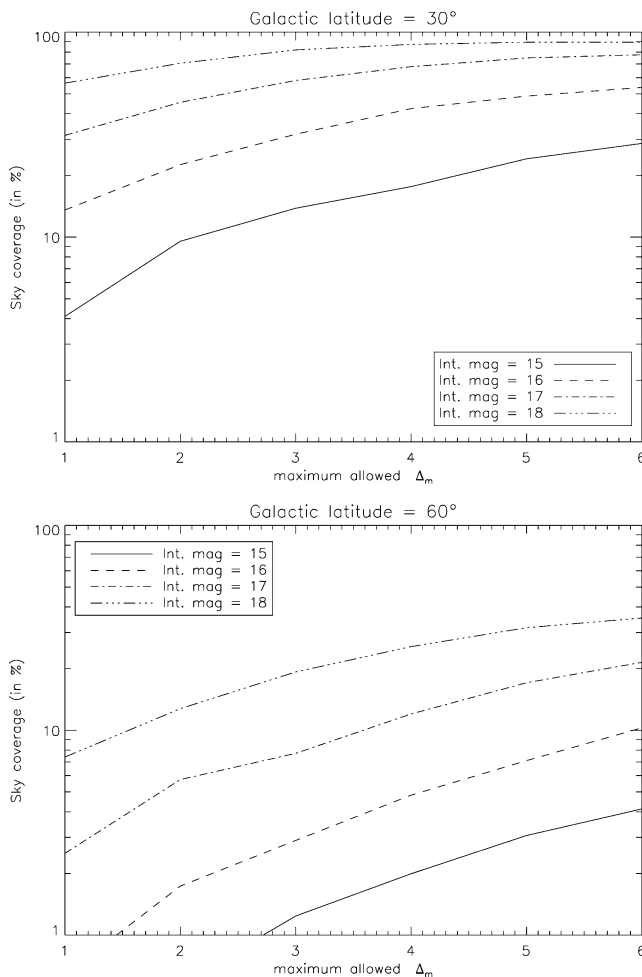
$10^{10}$  photon  $\text{m}^{-2}$   $\text{s}^{-1}$  for a 0-mag GS) and detector characteristics (Electron Multiplication CCD with zero read-out noise)

From Figs 4, 8 and 9, it is now possible to use our SSC algorithm in order to determine the real sky coverage of a MCAO system as a function of the required performance in the field (that is the generalized isoplanatic angle derived from Fig. 4) and as a function of the galactic latitude. Because the GS limiting magnitude is extremely dependent on the system characteristics, we have performed the sky coverage for two limiting GS magnitudes (16 and 18). The different plots in Fig. 10 show the overestimation of a classical MCAO sky coverage with respect to our new SSC algorithm. Of course this overestimation depends on the generalized isoplanatic angle or, in other words, on the tolerated performance in the FoV. The lower the limit performance, the smaller the overestimation should be.

### 7.3 The LO mode

In addition to its SO mode, the MAD MCAO system has a LO WFS developed by INAF (Istituto Nazionale di Astro Fisica). In this mode, eight star-enlarger devices (see Raggazzoni et al. 2002) are used to pick up the flux coming from eight GSs and to recombine the beams of these eight GSs on to two pyramid WFSs respectively conjugated to 0 and 7 km.

As explained in Section 3.2, the main difference between SO and LO from a photometric point of view is that the limiting GS magnitude per WFS of the SO approach is replaced by the integrated magnitude per WFS (sum of the flux coming from all the GSs after separation between the two WFSs) and a magnitude difference per GS. This limiting magnitude difference per GS [essential to ensure a good WFS measurement and reconstruction in a LO scheme as shown in Nicolle et al. (2006)] can be ensured using optical densities for the brighter GS direction before the beam co-addition. A complete study of the effect of GS magnitude difference on MAD performance is beyond the scope of this article, but it is interesting to study the behaviour of the SSC with respect to this parameter. As explained in Section 6.3, our algorithm is able to find the set of GSs maximizing the observable surface under constraints on (i) minimum integrated magnitude, and (ii) maximum magnitude difference. Another constraint is to consider at least three GSs (minimum value) up to eight GSs (maximum number of star-enlarger devices). The optimization of the surface defined by the GS position under the photometric constraints and GS number (3–8) leads to a complex multi-variable iterative process. It is nevertheless possible to find the optimal GS configuration that fulfils all the photometric requirements and ensures the largest corrected surface.



**Figure 11.** SSC as a function of the magnitude gap between GSs for the LO concept. Upper panel: galactic latitude =  $30^\circ$ ; lower panel: galactic latitude =  $60^\circ$ . For each plot, four different integrated magnitudes per WFS are considered.

Fig. 11 presents the sky coverage estimates obtained with our algorithm in the LO case and for various maximum differences of magnitude between GSs. The case of  $30^\circ$  (upper plot) and  $60^\circ$  (lower plot) galactic latitude are considered. For each galactic latitude four integrated magnitudes per WFS are considered: 15, 16, 17 and 18.

As expected, increasing the magnitude gap between GSs leads to significant improvements in term of sky coverage. Indeed, it allows us to consider a large number of GSs for a given integrated magnitude, or in other words to increase the number of photons for a given GS configuration, because the brightest stars are less (or even no more) optically attenuated before the detection. Fig. 11 highlights the importance of the authorized gap of magnitude between GSs in terms of sky coverage. A compromise has to be found between requirements in terms of wavefront measurement accuracy (which is degraded by the magnitude difference between GSs in a LO scheme) and the expected sky coverage of the MCAO system. Such a kind of study will be proposed in a future paper.

## 8 CONCLUSION

We have proposed an improvement of the conventional sky coverage for MCAO. The new algorithm accounts for different conditions on GSs (number and flux conditions) as well as (which is new) the observed surface defined by the GSs. To represent well the diversity of the MCAO in terms of both concept and expected performance, four different notions of FoV have been introduced. The new sky coverage algorithm proposed in this article allows us to deal with SO and LO WFS concepts, with GS geometry in the FoV, with photometric issues (GS magnitude, integrated magnitude and magnitude difference between GSs in the case of the LO concept), turbulence characteristics and system expected performance (through the use of a generalized isoplanatic angle). An application to the MAD system developed at ESO has been considered. In the framework of this particular application, the importance of parameters such as the GS geometry, the generalized isoplanatic angle and the difference magnitude between GSs has been highlighted.

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