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Abstract. We present new careful semantics within Dung's theory of argumentation. Under such careful semantics, two arguments cannot belong to the same extension whenever one of them indirectly attacks a third argument while the other one indirectly defends the third. We argue that our semantics lead to a better handling of controversial arguments than Dung's ones in some settings. We compare the careful inference relations induced by our semantics w.r.t. cautiousness; we also compare them with the inference relations induced by Dung's semantics.

1 Introduction

Argumentation is a general approach to model defeasible reasoning, in which the two main issues are the generation of arguments and their exploitation so as to draw some conclusions based on the way arguments interact (see e.g., [1–4]).

Among the various theories of argumentation pointed out so far (see e.g., [5–16]) is Dung's theory [5]. Dung's theory is quite influential since it encompasses many approaches to nonmonotonic reasoning and logic programming as special cases; as such, it has been refined and extended by several authors, including [17–21]. In Dung's approach, no assumption is made about the nature of an argument. Dung's theory of argumentation is not concerned with the generation of arguments; arguments and the way they interact w.r.t. the attack relation are considered as initial data of any argumentation framework, which can thus be viewed as a labeled digraph.

Several inference relations can be defined within Dung's theory. Usually, inference is defined at the argument level: an argument is considered derivable from an argumentation framework AF when it belongs to one (credulous consequence) (resp. all (skeptical consequence)) extensions of AF under some semantics, where an extension of AF is an admissible set of arguments (i.e., a conflict-free and self-defending set) that is maximal for a given criterion (made precise by the semantics under consideration). While skeptical derivability can be safely extended to the level of sets of arguments, this is not the case for credulous derivability. Indeed, it can be the case that arguments a and b are (individually) derivable from an argumentation framework AF while the set $\{a, b\}$ is not included in any extension of AF. Now, defining derivability for sets

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of arguments as inclusion into some (resp. all) extensions under Dung's semantics does not always lead to expected conclusions.

Consider the following scenario: in a public meeting, a political activist presents the motivations of her policy using arguments and counter-arguments: *"One should really decrease taxes (a); of course, this requires to cut staff in public services (b), but that is not so dramatic: privatizing some activities will lead to better services since free trading is good for it (c); furthermore, I am confident that we should reduce our economical exchanges with other foreign countries (d); this is antagonistic to promoting free trading, but, anyway, the productivity of our public services is definitely bad (e)"*. This sounds quite strange as a political speech since the speaker admits that she is in favour of conflicting arguments; a political opponent could easily point out the presence of such a conflict and concludes that such a policy is just non-sense; in order to convince the audience that a, d and e should be accepted, a more skillful speech would be: *"One should really decrease taxes (a); of course, this requires to cut staff in public services (b), but the productivity of our public services is definitely bad (e); furthermore, I'm confident that we should reduce our economical exchanges with other foreign countries (d)"*.

From an abstract point of view, the scenario can be encoded in Dung's setting using the following argumentation framework:

Example 1.

Let $AF_1 = \langle A, R \rangle$ with $A = \{a, b, c, d, e\}$ and $R = \{(b, a), (e, b), (c, b), (d, c)\}$. The digraph for AF_1 is depicted on Figure 1.

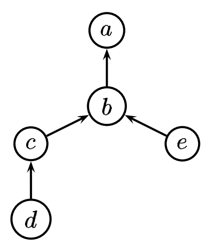

Fig. 1. The digraph for AF_1.

If our clumsy political activist adheres to Dung's semantics, she cannot realize that her first speech must be avoided; indeed, AF_1 has a single extension $\{a, d, e\}$ whatever the semantics among Dung's ones, hence a, d and e are considered jointly derivable, which is just what she wants.

One way to cope with this problem is to ask for more demanding notions of absence of conflicts than the one considered in Dung's theory. In this paper, we define and study new semantics for Dung's framework based on the idea that an admissible set S of arguments should not include controversies, i.e. it should not be the case that an element s_1 of S indirectly attacks another argument s whenever a second element s_2 of

S indirectly defends s. On Example 1, this prevents from deriving the set of arguments $\{a, d, e\}$ as a whole; nevertheless, $\{d\}$ and $\{a, e\}$ remain derivable separately.

The specific case when $s_1 = s_2$ corresponds to the notion of controversial arguments, as introduced by Dung. While some controversial arguments can be inferred using Dung's standard semantics, they are systematically rejected when our careful semantics are considered.

We believe that such prudent semantics can prove helpful to reason with arguments from domains like politics or justice, where a strong notion of "coherence" on the sets of arguments pointed out makes sense.

In the following, we compare the inference relations induced by our new semantics with Dung's ones and show that in many cases one obtains more cautious notions of derivability.

The rest of this paper is organized as follows. We first recall the main definitions and results pertaining to Dung's theory of argumentation. Then, we present our new, careful semantics for argumentation frameworks. In a third section, a comparison of the various notions of acceptability (including Dung's ones) is provided. A final section concludes the paper and gives a few perspectives.

2 Dung's Theory of Argumentation

Let us present some basic definitions at work in Dung's theory of argumentation [5]. We restrict them to finite argumentation frameworks.

Definition 1 (finite argumentation frameworks). *A* finite argumentation framework *is a pair $AF = \langle A, R\rangle$ where A is a finite set of so-called arguments and R is a binary relation over A (a subset of $A \times A$), the attacks relation.*

Clearly enough, the set of finite argumentation frameworks is a proper subset of the set of Dung's finitary argumentation frameworks, where every argument must be attacked by finitely many arguments. The definition above clearly shows that a finite argumentation framework is nothing but a finite, labeled digraph.

The main issue is the inference one, i.e., charactering the sets of arguments which could be reasonably derived from a given argumentation framework. Formally, we shall note $AF \mid\!\sim S$ where $AF = \langle A, R\rangle$ is a finite argumentation framework and $S \subseteq A$, to state that S is a consequence of AF under $\mid\!\sim$. An inference relation $\mid\!\sim$ is typically based on a notion of extension, and an inference principle (credulous or skeptical), so that $AF \mid\!\sim S$ holds if and only if S is included in all (skeptical) or at least one (credulous) extension of AF.

In order to define a notion of extension, a first important notion is the notion of acceptability: an argument a is acceptable w.r.t. a set of arguments whenever it is defended by the set, i.e., every argument which attacks a is attacked by an element of the set.

Definition 2 (acceptable sets). *Let $AF = \langle A, R\rangle$ be a finite argumentation framework. An argument $a \in A$ is* acceptable *w.r.t. a subset S of A if and only if for every $b \in A$ s.t. $(b, a) \in R$, there exists $c \in S$ s.t. $(c, b) \in R$. A set of arguments is acceptable w.r.t. S when each of its elements is acceptable w.r.t. S.*

A second important notion is the notion of absence of conflicts. Intuitively, two arguments should not be considered together whenever one of them attacks the other one.

Definition 3 (conflict-free sets). *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. A subset S of A is* conflict-free *if and only if for every $a, b \in S$, we have $(a, b) \notin R$.*

Requiring the absence of conflicts and the form of autonomy captured by self-acceptability leads to the notion of admissible set.

Definition 4 (admissible sets). *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. A subset S of A is* admissible *if and only if S is conflict-free and acceptable w.r.t. S.*

The significance of the concept of admissible sets is reflected by the fact that every extension of an argumentation framework under the standard semantics introduced by Dung (preferred, stable, complete and grounded extensions) is an admissible set, satisfying some form of optimality:

Definition 5 (extensions). *Let $AF = \langle A, R \rangle$ be a finite argumentation framework.*

- *A subset S of A is* a preferred extension *of AF if and only if it is maximal w.r.t. $\subseteq$ among the set of admissible sets for AF.*
- *A subset S of A is* a stable extension *of AF if and only if it is conflict-free and for every argument a from $A \setminus S$, there exists $b \in S$ s.t. $(b, a) \in R$.*
- *A subset S of A is* a complete extension *of AF if and only if it is admissible and it coincides with the set of arguments acceptable w.r.t. itself.*
- *A subset S of A is* the grounded extension *of AF if and only if it is the least element w.r.t. $\subseteq$ among the complete extensions of AF.*

Dunne and Bench-Capon gave a sufficient condition for the unicity of preferred extensions:

Proposition 1. *Cor. 9 in [22]*
Let $AF = \langle A, R \rangle$ be a finite argumentation framework. If AF has no even-length cycle, then AF has a unique preferred extension.

Example 1 (cont'ed). Let $E = \{a, d, e\}$. E is the grounded extension of AF_1, the unique preferred extension of AF_1, the unique stable extension of AF_1 and the unique complete extension of AF_1.

Formally, complete extensions of AF can be characterized as the fixed points of its characteristic function $\mathcal{F}_{AF}$:

Definition 6 (characteristic functions). *The* characteristic function $\mathcal{F}_{AF}$ *of an argumentation framework $AF = \langle A, R \rangle$ is defined as follows:*

$$\mathcal{F}_{AF} : 2^A \longrightarrow 2^A$$
$$\mathcal{F}_{AF}(S) = \{a \mid a \text{ is acceptable w.r.t. } S\}.$$

Among the complete extensions of AF, the grounded extension of AF is the least element w.r.t. set inclusion [5].

Dung has shown that every argumentation framework AF has a (unique) grounded extension and at least one preferred extension, while it may have zero, one or many stable extensions. These extensions are linked up as follows:

Proposition 2. *Theorem 25 in [5]*
Let AF be an argumentation framework.

1. *Every preferred (resp. stable, complete) extension of AF contains the grounded extension of AF.*
2. *The grounded extension of AF is included in the intersection of all the complete extensions of AF.*

The purest argumentation frameworks AF in Dung's theory are those for which all the notions of acceptability coincide. Dung has provided a sufficient condition for an argumentation framework AF to satisfy this requirement, called the well-foundation of AF; in the finite case, it can be stated as follows:

Definition 7 (well-foundation). *Let $AF = \langle A, R\rangle$ be a finite argumentation framework. AF is* well-founded *if and only if there is no cycle in the digraph $\langle A, R\rangle$.*

Proposition 3. *Theorem 30 in [5]*
A well-founded argumentation framework AF has exactly one complete extension, which is also the unique preferred extension, the unique stable extension and the grounded extension of AF.

Example 1 (cont'ed). AF_1 has no cycle. Hence AF_1 is well-founded.

Dung has also shown that every stable extension is preferred and every preferred extension is complete; however, none of the converse inclusions holds. When all the preferred extensions of an argumentation framework are stable ones, the framework is said to be coherent:

Definition 8 (coherence). *Let $AF = \langle A, R\rangle$ be an argumentation framework. AF is* coherent *if and only if every preferred extension of AF is also stable.*

Coherence is a desirable property. Dung gave a sufficient condition for it based on the notion of controversial argument:

Definition 9 (controversial arguments).
Let $AF = \langle A, R\rangle$ be an argumentation framework.

– *Let $a, b \in A$. a* indirectly attacks *b if and only if there exists an odd-length path from a to b in the digraph for AF.*
– *Let $a, b \in A$. a* indirectly defends *b if and only if there exists an even-length path from a to b in the digraph for AF. The length of this path is not zero.*
– *Let $a, b \in A$. a is* controversial *w.r.t. b if and only if a indirectly attacks b and a indirectly defends b.*

- AF *is* uncontroversial *if and only if there is no pair* a, b *of arguments of* A *such that* a *is controversial w.r.t.* b.
- AF *is* limited controversial *if and only if there is no infinite sequence of arguments* $a_0, \ldots, a_n, \ldots$ *of* A *s.t.* a_{i+1} *is controversial w.r.t.* a_i.

Dung has shown the following theorem:

Proposition 4. *Theorem 33 in [5]*
Every uncontroversial or limited controversial argumentation framework is coherent.

3 Careful Extensions

Let us now present our new semantics for Dung's argumentation frameworks. They are based on the notion of super-controversial pair of arguments:

Definition 10 (super-controversial arguments).
Let $AF = \langle A, R \rangle$ *be a finite argumentation framework and let* $a, b, c \in A$. (a, b) *is* super-controversial *w.r.t.* c *if and only if* a *indirectly attacks* c *and* b *indirectly defends* c*.*

Example 1 (cont'ed). In AF_l, (d, e) is super-controversial w.r.t. a.

Obviously enough, the notion of super-controversial pair of arguments extends the notion of controversial arguments since a is controversial w.r.t. c if and only if (a, a) is super-controversial w.r.t. c.

In order to address Example 1 in a more satisfying way, we need to reinforce Dung's notion of conflict-free set of arguments; we consider in addition the notion of *controversial-free* set of arguments:

Definition 11 (controversial-free sets). *Let* $AF = \langle A, R \rangle$ *be a finite argumentation framework.* $S \subseteq A$ *is* controversial-free *for* AF *if and only if for every* $a, b \in S$ *and every* $c \in A$, (a, b) *is not super-controversial w.r.t.* c*.*

Definition 12 (c-admissible sets). *Let* $AF = \langle A, R \rangle$ *be a finite argumentation framework.* $S \subseteq A$ *is* c(areful)-admissible *for* AF *if and only if every* $a \in S$ *is acceptable w.r.t.* S *and* S *is conflict-free and controversial-free for* AF*.*

Example 1 (cont'ed). $\{d\}$, and $\{a, e\}$ and its subsets except $\{a\}$ are the c-admissible sets for AF_1.

From Definition 12, the next lemma follows immmediately:

Lemma 1. *Let* a, b *be two arguments of a finite argumentation framework* AF*. If* a *is controversial w.r.t.* b*, then* $\{a\}$ *cannot be included in a c-admissible set for* AF*.*

Obviously, the absence of controversial arguments within a set is only necessary to ensure that the set is controversial-free, hence potentially c-admissible (as Example 1 shows, this is not a sufficient condition). Since every argument belonging to an odd-length cycle of AF is controversial w.r.t. any argument of the cycle [23], no such argument can belong to a c-admissible set. In this respect, our approach departs from [18, 19] who consider that odd-length and even-length cycles in an argumentation framework should be handled in the same way.

On this ground, one can define several notions of *careful extensions*, echoing Dung's ones. Let us start with *preferred c-extensions*:

Definition 13 (preferred c-extensions). *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. A c-admissible set $S \subseteq A$ for AF is* a preferred c-extension *of AF if and only if $\nexists S' \subseteq A$ s.t. $S \subset S'$ and S' is c-admissible for AF.*

Example 1 (cont'ed). $\{a, e\}$ and $\{d\}$ are the preferred c-extensions of AF_1.

We have the following easy proposition:

Proposition 5. *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. For every c-admissible set $S \subseteq A$ for AF, there exists at least one preferred c-extension $E \subseteq A$ of AF s.t. $S \subseteq E$.*

Since $\emptyset$ is c-admissible for any AF, we obtain as a corollary:

Corollary 1. *Every finite argumentation framework $AF = \langle A, R \rangle$ has a preferred c-extension.*

What can be found in preferred c-extensions? Though every argument which is not attacked belongs to at least one preferred c-extension of AF, it is not the case (in general) that it belongs to every preferred c-extension of AF (see Example 1). In this respect, c-preferred extensions hardly contrast with preferred extensions.

Let us now consider the notion of *stable c-extension*:

Definition 14 (stable c-extensions).

Let $AF = \langle A, R \rangle$ be a finite argumentation framework. A conflict-free and controversial-free subset S of A is a stable c-extension *of AF if and only if S attacks (in a direct way) every argument from $A \setminus S$.*

Example 1 (cont'ed). AF_1 has no stable c-extension.

Every finite argumentation framework has at least one preferred c-extension, and zero, one or many stable c-extensions.

Finally, as for Dung's extensions, we have:

Lemma 2. *Every stable c-extension of a finite argumentation framework AF also is a preferred c-extension of AF. The converse does not hold.*

Here is a more complex example for illustrating those notions:

Example 2.

Let $AF_2 = \langle A, R \rangle$ with $A = \{a, b, c, d, i, n\}$ and $R = \{(i, n), (n, a), (b, a), (c, a), (d, c), (b, d), (d, b)\}$. The digraph for AF_2 is depicted on Figure 2.

$E_1 = \{i, a, d\}$ and $E_2 = \{i, b, c\}$ are the two preferred (and stable) extensions of AF_2. E_1 is the unique stable c-extension of AF_2. E_1 and $E_3 = \{b, c\}$ are the two preferred c-extensions of AF_2.

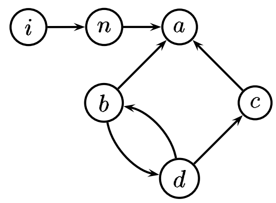

Fig. 2. The digraph for AF_2.

Let us now explain how c-extensions can be characterized using some fixed point construction:

Definition 15 (c-characteristic functions).
The c-characteristic function $\mathcal{F}^c_{AF}$ *of a finite argumentation framework* $AF = \langle A, R \rangle$ *is defined as follows:*

$$\mathcal{F}^c_{AF} : 2^A \longrightarrow 2^A$$
$$\mathcal{F}^c_{AF}(S) = \{a \mid a \textit{ is acceptable w.r.t. } S \textit{ and } S \cup \{a\} \textit{ is conflict-free and controversial-free for } AF\}.$$

We immediately get that:

Lemma 3. *Let* $AF = \langle A, R \rangle$ *be a finite argumentation framework and let* $S \subseteq A$ *be a conflict-free and controversial-free set for* AF*.* S *is c-admissible for* AF *if and only if* $S \subseteq \mathcal{F}^c_{AF}(S)$*.*

Contrariwise to the characteristic function of an argumentation framework, $\mathcal{F}^c_{AF}$ is in general nonmonotonic w.r.t. $\subseteq$ (and this is also the case for its restriction to the set of all c-admissible subsets of A). Accordingly, we cannot define a notion of c-grounded extension corresponding to the grounded one.

Let us now introduce a notion of *complete c-extension*:

Definition 16 (complete c-extensions). *Let* $AF = \langle A, R \rangle$ *be a finite argumentation framework and let* S *be a c-admissible set for* AF*.* S *is a* complete c-extension *of* AF *if and only if every argument* a *which is acceptable w.r.t.* S *and such that* $S \cup \{a\}$ *is conflict-free and controversial-free for* AF *belongs to* S*.*

Example 1 (cont'ed). $\{a, e\}$ is a complete c-extension of AF_1.

From the definition, it comes immediately that:

Lemma 4. *A conflict-free and controversial-free set of arguments* S *is a complete c-extension of* AF *if and only if* $\mathcal{F}^c_{AF}(S) = S$*.*

Let us now define several inference relations based on our careful semantics for argumentation frameworks:

Definition 17 (careful inference relations). $\mid\!\sim_c^{q,s}$ *denotes the* careful inference relation *obtained by considering a careful semantics* s *(where* $s = P(referred)$, $s = S(table)$*) and* q *is an inference principle, either credulous (*$q = \exists$*) or skeptical (*$q = \forall$*).*

For instance, $S \subseteq A$ is a consequence of AF w.r.t. $\mid\!\sim_c^{\forall,P}$, noted $AF \mid\!\sim_c^{\forall,P} S$, indicates that S is included in every preferred c-extension of AF.

We have compared all the careful inference relations induced by the different semantics w.r.t. cautiousness. We have focused on the case of finite argumentation frameworks with a stable c-extension (otherwise, both $\mid\!\sim_c^{\forall,S}$ and $\mid\!\sim_c^{\exists,S}$ trivialize). We have obtained the following results:

Proposition 6. *The cautiousness relations reported in Table 1 hold for every finite argumentation framework which has a stable c-extension (Each time a cell contains a* $\subseteq$*, it means that for every* $AF = \langle A, R\rangle$ *and every* $S \subseteq A$*, if* S *is a consequence of* AF *w.r.t. the inference relation indexing the row, then* S *is a consequence of* AF *w.r.t. the inference relation indexing the column.)*

	$\mid\!\sim_c^{\exists,P}$	$\mid\!\sim_c^{\exists,S}$	$\mid\!\sim_c^{\forall,P}$	$\mid\!\sim_c^{\forall,S}$
$\mid\!\sim_c^{\exists,P}$	$=$	$\not\subseteq$	$\not\subseteq$	$\not\subseteq$
$\mid\!\sim_c^{\exists,S}$	$\subseteq$	$=$	$\not\subseteq$	$\not\subseteq$
$\mid\!\sim_c^{\forall,P}$	$\subseteq$	$\subseteq$	$=$	$\subseteq$
$\mid\!\sim_c^{\forall,S}$	$\subseteq$	$\subseteq$	$\not\subseteq$	$=$

Table 1. Cautiousness links between careful inference relations for AFs with a stable c-extension.

One can note that the cautiousness picture for careful inference relations is similar to the one for the inference relations induced from Dung's semantics (assuming that the argumentation frameworks under consideration have stable extension(s)):

$$\mid\!\sim_c^{\forall,P} \subset \mid\!\sim_c^{\forall,S} \subset \mid\!\sim_c^{\exists,S} \subset \mid\!\sim_c^{\exists,P} .$$

4 Comparisons with Dung's Framework

Let us now compare our careful semantics with Dung's ones. Let us start with a comparison in terms of extensions.

4.1 Comparing extensions

First of all, we immediately obtain the following easy result:

Proposition 7. *Every c-admissible set for a finite argumentation framework* AF *is also admissible for* AF*. The converse does not hold.*

Clearly, this does not imply that every preferred c-extension is a preferred extension which is conflict-free and controversial-free since maximality w.r.t. set inclusion is required among c-admissible sets. Nevertheless, as a consequence of Proposition 7, we have:

Corollary 2. *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. For every preferred c-extension E_c of AF, there exists at least one preferred extension E of AF s.t. $E_c \subseteq E$.*

Example 2 (cont'ed). In AF_2, $E_3 \subset E_2$.

This corollary shows in particular that when AF has a unique preferred extension E (especially, when AF is well-founded or without even-length cycle, or *trivial* – i.e., when the unique preferred extension of it is empty), E includes every preferred c-extension of AF.

However, unlike preferred extensions, a well-founded argumentation framework AF can have more than one preferred c-extension (see Example 1).

It can also be the case that a preferred extension of AF does not include any of the preferred c-extensions of AF. Furthermore, the presence of even-length cycles in AF does not prevent it from having a unique preferred c-extension. Those two points are illustrated by the following example:

Example 3. Let $AF_3 = \langle A, R \rangle$ with $A = \{a, b, c, e, n, i\}$ and $R = \{(b, e), (b, c), (c, e), (b, a), (a, i), (n, i), (i, n)\}$. The digraph for AF_3 is depicted on Figure 3.

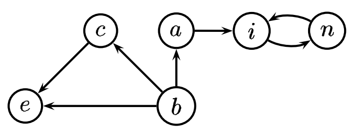

Fig. 3. The digraph for AF_3.

$E_1 = \{b, n\}$ and $E_2 = \{b, i\}$ are the preferred (and stable) extensions of AF_3. $E_3 = \{n\}$ is the unique preferred c-extension of AF_3. We have $E_3 \not\subseteq E_2$. Observe that though the digraph for AF_3 has an even-length cycle, AF_3 has a unique preferred c-extension.

Another easy consequence of Proposition 7 is:

Corollary 3. *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. If AF is trivial, then AF is c-trivial, i.e., the unique preferred c-extension of AF is empty. The converse does not hold.*

Let us now turn to stable c-extensions:

Lemma 5. *Every stable c-extension of a finite argumentation framework AF also is a stable extension of AF. The converse does not hold.*

As a direct consequence, we obtain that every stable c-extension of a finite argumentation framework AF also is a preferred extension of AF. However, the converse does not hold.

While every well-founded argumentation framework has a stable extension, it is not the case that every well-founded argumentation framework has a stable c-extension; furthermore, it is also not the case that every argumentation framework which is uncontroversial has a stable c-extension. Example 1 is a counter-example for both cases. In the same vein, a finite argumentation framework AF that is both well-founded and uncontroversial is not always c-coherent (i.e., such that every preferred c-extension of AF is a stable c-extension of AF). In particular, it is not the case that a coherent finite argumentation framework AF is always c-coherent as well (see Example 1).

It turns out that argumentation frameworks AF with a stable c-extension are particularly interesting. Indeed, whenever AF has a stable c-extension, we have:

Proposition 8. *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. If AF has a stable c-extension, the grounded extension of AF and the intersection of all preferred extensions of AF coincide (i.e., AF is relatively grounded).*

Proposition 9. *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. If AF has a stable c-extension, the intersection of all preferred c-extensions of AF is included in the grounded extension of AF.*

Subsequently, if AF has a stable c-extension, the intersection of all preferred c-extensions of AF is included in the intersection of all preferred extensions of AF; hence, it is also included in the intersection of all stable extensions of AF.

Now, what's about controversies? We have the two following lemmata:

Lemma 6. *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. AF is limited controversial if and only if AF has no odd-length cycle.*

Lemma 7. *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. If AF has a stable c-extension, then AF is limited controversial.*

Thanks to Proposition 4, we obtain the following corollary:

Lemma 8. *Let $AF = \langle A, R \rangle$ be a finite argumentation framework. If AF has a stable c-extension, then AF is coherent.*

We also have:

Proposition 10. *Let $AF = \langle A, R \rangle$ be a finite argumentation framework s.t. AF has a stable c-extension. For every preferred c-extension E_c of AF, there exists at least one stable extension S of AF s.t. $E_c \subseteq S$.*

Finally, we can show that the set of complete extensions of AF and the set of complete c-extensions of AF are not comparable w.r.t. $\subseteq$.

4.2 Comparing inference relations

We have compared our careful inference relations with the ones induced by Dung's semantics w.r.t. cautiousness. Let $\mid\!\sim^{q,s}$ denote the inference relation obtained by considering Dung's semantics s (where $s = P(referred)$, $s = S(table)$ or $s = G(rounded)$) and q is an inference principle, either credulous ($q = \exists$) or skeptical ($q = \forall$).

Since the presence of a stable c-extension changes the picture, we have first considered this specific case, then the general case. We have obtained the following results:

Proposition 11. *The cautiousness relations reported in Table 2 hold for every finite argumentation framework which has a stable c-extension.*

	$\mid\!\sim_c^{\exists,P}$	$\mid\!\sim_c^{\exists,S}$	$\mid\!\sim_c^{\forall,P}$	$\mid\!\sim_c^{\forall,S}$
$\mid\!\sim^{\exists,P}$	$\not\subseteq, \supseteq$	$\not\subseteq, \supseteq$	$\not\subseteq, \supseteq$	$\not\subseteq, \supseteq$
$\mid\!\sim^{\exists,S}$	$\not\subseteq, \supseteq$	$\not\subseteq, \supseteq$	$\not\subseteq, \supseteq$	$\not\subseteq, \supseteq$
$\mid\!\sim^{\forall,P}$	$\subseteq, \not\supseteq$	$\subseteq, \not\supseteq$	$\not\subseteq, \supseteq$	$\subseteq, \not\supseteq$
$\mid\!\sim^{\forall,S}$	$\subseteq, \not\supseteq$	$\subseteq, \not\supseteq$	$\not\subseteq, \supseteq$	$\subseteq, \not\supseteq$
$\mid\!\sim^{\cdot,G}$	$\subseteq, \not\supseteq$	$\subseteq, \not\supseteq$	$\not\subseteq, \supseteq$	$\subseteq, \not\supseteq$

Table 2. Cautiousness links between careful relations and Dung's ones for AFs with a stable c-extension.

Tables 1 and 2 are summarized on Figure 4 (Each arrow can be read as "strictly more cautious than").

$$\begin{tikzcd}
\mid\!\sim^{\cdot,G} = \mid\!\sim^{\forall,P} = \mid\!\sim^{\forall,S} \arrow[dr] & & & \mid\!\sim^{\exists,S} = \mid\!\sim^{\exists,P} \\
\mid\!\sim_c^{\forall,P} \arrow[u] & \mid\!\sim_c^{\forall,S} \arrow[r] & \mid\!\sim_c^{\exists,S} \arrow[r] & \mid\!\sim_c^{\exists,P} \arrow[u]
\end{tikzcd}$$

Fig. 4. Cautiousness links between inference relations for AFs with a stable c-extension.

In the light of Figure 4, one can observe that the most cautious inference relation among those considered here is $\mid\!\sim_c^{\forall,P}$ and the least cautious ones are $\mid\!\sim^{\exists,S} = \mid\!\sim^{\exists,P}$. Furthermore, strict cautiousness is a complete ordering over the set of inference relations considered in this paper.

Let us now turn to the general case, i.e., argumentation frameworks which do not have necessarily a stable c-extension, or even a stable extension. We do not put potentially trivial relations into the picture (i.e., $\mid\!\sim_c^{\forall,S}$, $\mid\!\sim_c^{\exists,S}$, $\mid\!\sim^{\forall,S}$, $\mid\!\sim^{\exists,S}$ are not considered hereafter):

Proposition 12. *The cautiousness relations reported in Tables 3 and 4 hold for every finite argumentation framework.*

	$\mid\!\sim_c^{\exists,P}$	$\mid\!\sim_c^{\forall,P}$
$\mid\!\sim_c^{\exists,P}$	$=$	$\not\subseteq$
$\mid\!\sim_c^{\forall,P}$	$\subseteq$	$=$

Table 3. Cautiousness links between careful relations.

	$\mid\!\sim_c^{\exists,P}$	$\mid\!\sim_c^{\forall,P}$
$\mid\!\sim^{\exists,P}$	$\not\subseteq, \supseteq$	$\not\subseteq, \supseteq$
$\mid\!\sim^{\forall,P}$	$\not\subseteq, \not\supseteq$	$\not\subseteq, \not\supseteq$
$\mid\!\sim^{\cdot,G}$	$\not\subseteq, \not\supseteq$	$\not\subseteq, \not\supseteq$

Table 4. Cautiousness links between careful relations and Dung's ones.

Tables 3 and 4 are summarized on Figure 5.

Proposition 12 shows that the lack of a stable c-extension does not question the way $\mid\!\sim_c^{\exists,P}$ and $\mid\!\sim_c^{\forall,P}$ are linked up (see Proposition 6). Contrastingly, when the existence of a stable c-extension is not guaranteed, the cautiousness links between our careful relations and Dung's ones are heavily modified. Compared with the results reported on Figure 4, one can observe that in the general case, it is not guaranteed that $\mid\!\sim^{\cdot,G}$ and $\mid\!\sim^{\forall,P}$ coincide, that $\mid\!\sim_c^{\forall,P}$ is more cautious than $\mid\!\sim^{\cdot,G}$ and that $\mid\!\sim^{\forall,P}$ is more cautious than $\mid\!\sim_c^{\exists,P}$.

$$\mid\!\sim^{\cdot,G} \longrightarrow \mid\!\sim^{\forall,P} \longrightarrow \mid\!\sim^{\exists,P} \longleftarrow \mid\!\sim_c^{\exists,P} \longleftarrow \mid\!\sim_c^{\forall,P}$$

Fig. 5. Cautiousness links between inference relations.

5 Some Complexity Issues

Before concluding the paper, let us consider some complexity issues. Indeed, in an AI perspective, it is important to determine how hard are the new inference relations we pointed out w.r.t. the computational point of view. We assume the reader acquainted with basic notions of complexity theory, especially the complexity classes P, NP, coNP and the polynomial hierarchy (see e.g. [24]).

We have shown in a previous paper [25] that considering sets of arguments (instead of single arguments) as input queries for the inference problem does not lead to a complexity shift when Dung's inference relations are considered (the purpose is to determine whether such sets are derivable from a given finite argumentation framework AF). As to the careful inference relations, the same conclusion can be drawn.

First of all, it is easy to show that, given a finite argumentation framework AF, deciding whether a given argument indirectly attacks (resp. indirectly defends) a given argument is in P, and deciding whether a set of arguments is controversial-free is in P. Accordingly, deciding whether a given set of arguments is c-admissible for AF is in P. As a consequence, deciding whether a given set of arguments is a stable c-extension of AF is in P as well. Therefore, deciding whether a given set of arguments S is included in every stable c-extension of AF is in coNP (in order to show that the complementary problem is in NP, it is sufficient to guess a set $E \subseteq A$ and to check in polynomial time that E is a stable c-extension of AF and that S is not included in E).

Besides, deciding whether a set of arguments S is a preferred c-extension of AF is in coNP (in order to show that the complementary problem is in NP, it is sufficient to guess a proper superset S' of S and to check in polynomial time that S' is c-admissible for AF). As a consequence, deciding whether a given set of arguments S is included in every preferred c-extension of AF is in Π_2^p (in order to show that the complementary problem is in Σ_2^p, it is sufficient to guess a set $E \subseteq A$ and to check in polynomial time using an NP oracle that E is a preferred c-extension of AF and that S is not included in E).

Finally, deciding whether a given set of arguments is included in a preferred c-extension (resp. a stable c-extension) of AF is in NP. To be more precise:

Definition 18. C-CA(AF, S) *is defined as follows:*
$\star$ **Input** $AF = \langle A, R \rangle$ *a finite argumentation framework, and* $S \subseteq A$.
$\star$ **Quest** *Is* S *included in a preferred c-extension of* AF*?*

Proposition 13. C-CA(AF, S) *is* NP-*complete.*

Definition 19. IN-C-STAB(AF, S) *is defined as follows:*
$\star$ **Input** $AF = \langle A, R \rangle$ *a finite argumentation framework, and* $S \subseteq A$.
$\star$ **Quest** *Is* S *included in a stable c-extension of* AF*?*

Proposition 14. IN-C-STAB(AF, S) *is* NP-*complete.*

Accordingly, our careful inference relations are not computationally more complex that the corresponding ones based on Dung's semantics (see [26, 27]).

6 Conclusion

We have presented new careful semantics within Dung's theory of argumentation. Under such careful semantics, two arguments cannot belong to the same extension whenever one of them indirectly attacks a third argument, while the second one indirectly defends it. In particular, controversial arguments are always rejected. This seems to be highly desirable in domains where controversies can be interpreted as contradictions, as we exemplified it. We have also compared our careful inference relations with Dung's ones and considered some complexity issues, showing that our inference relations are not more complex than the corresponding ones based on Dung's semantics.

Our work calls for some perspectives. A first perspective consists in developing specific algorithms for computing careful extensions, based on algorithms for computing extensions like those described in [28–30]. A second perspective consists in combining the notion of safe extension introduced recently [31] with the notion of careful extension.

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