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#### La conception analytique des propriétés viscoélastiques optimales des materiaux pour les sandwichs et les composites

### Analytical Formulation of Optimal Viscoelastic Designer Material Properties for Sandwich / Composites

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#### ABSTRACT

Viscoelastic materials are known for their ability to dissipate energy. This property has been successfully used by the author and his colleagues to produce effective passive structural control for column and plate creep buckling, various vibratory modes, and aero-viscoelastic phenomena, such as torsional divergence, lifting surface and panel flutter, and attenuation of aerodynamic noise in panels. In self-excited systems the application of increased dissipation may stabilize or destabilize such systems depending on the influence of damping and all other forces on phase relations. Conventional design and analysis formulations call for use of the best available "off the shelf" materials. On the other hand, the optimum designer material protocols based on calculus of variation principles developed in [1] are formulated to determine the global best elastic or viscoelastic properties for specified service conditions. It has been previously established in [2] that in isotropic and anisotropic viscoelastic materials the shape of the relaxation curve is a major contributor to the material;s response performance. In particular, it has been shown that Region C and the ratio  $E_0 \ / \ E_\infty$  of the relaxation modulus, as seen in Fig.1, are the most influential in dictating material dissipation rates. Consequently, such relaxation modulus functions are tailored through prescriptions of appropriate functionally graded viscoelastic materials to produce the desired designer material performance. Relaxation moduli are, of course, highly temperature sensitive and performances must be evaluated relative to operational demands. In this paper, an analytical study presents optimal sandwich combinations of high shear modulus auxetic [3] webs with composite faceplates of proper number of stacking sequences and fibers as well as their orientations, and their viscoelastic material properties. The constraints that can be imposed consist of one or more selected from weight, dimensions, cost, deformations, failure probabilities, survival / life-times, etc. Some preliminary results are presented. The delamination failure analyses are based on uniaxial viscoelastic experimental data found in [4] and the theoretical stochastic failure criteria developed in [5]. For the same structural weight, the optimized designer viscoelastic sandwich composite plate clearly shows substantial longer survival times and orders of magnitude smaller probabilities of delamination. Extensions of these analyses to multi-element structures, i. e. entire structures, are also presented.

Key Words : aero-servo-viscoelasticity, designer viscoelastic materials, functionally graded materials, failure probability, servo controls, structural control, survival times, torsional creep divergence, viscoelasticity

#### INTRODUCTION

Viscoelastic mechanical properties of existing materials, such as metals, concrete, polymers, cheeses and human and animal tissue, are extensively described in [6 - 10] while relations between their chemistry and such properties are only now starting to be understood [11]. Current conventional wisdom and practice in structural analysis of air, space,

naval and land vehicles call for use of the best "off the shelf" available materials. Novel and unconventional analytical direct and inverse protocols have been previously successfully formulated [1], [12 - 14] through computer simulations in a number of pilot studies seeking to determine the best designer viscoelastic material properties and their spatial non-homogeneous distributions (viscoelastic functionally graded materials, i.e. VFGM) throughout structural members tailored and engineered to specific tasks, such as beam bending, column and plate buckling, thermal stresses, structural survival times, aero-viscoelastic flutter and aero-acoustics, to mention a few.

The term "composites" is generically used to denote any substance made of two or more materials with distinct properties. Notable examples are textiles, reinforced concrete, polymer or ceramic matrices with polymer or metal fibers [15 – 27]. Another example is a plasma sprayed five-layer functionally graded  $ZrO_2$  / Ni Co Cr AlY thermal barrier coating of a  $\mu$ -meter scale thickness described in [28] and shown in Fig. 2.

#### ANALYSIS

#### **Viscoelastic Constitutive Relations**

Consider an anisotropic, isothermal, composite, rectangular plate with dimensions  $a \otimes b \otimes h$  in a Cartesian coordinate system  $x = \{x_i\}$  with i = 1, 2, 3, with  $x_3$  normal to the plane of the plate. All materials, i.e. fibers and matrix, obey linear viscoelastic constitutive relations of the type [29 – 31]

$$\sigma_{ij}(x,t) = \int_{-\infty}^{t} E_{ijkl} \left[ \underbrace{t-t', x, \mathcal{F}(x)}_{= \Xi(x,t-t')} \right] \epsilon_{kl}(x,t') dt'$$
(1)

where  $\mathcal{F}(x)$  represents the relation describing the distribution of fiber orientation  $\theta_{or}(x)$ , number of plies  $N_{pl}(x)$ , number of fibers per ply  $N_{fp}(x)$ , volume fractions  $v_{fr}(x)$  and stacking sequences st(x), or

$$\mathcal{F}(x) = \mathcal{F}\left[\theta_{or}(x), N_{pl}(x), N_{fp}(x), v_{fr}(x), st(x)\right]$$
(2)

This function may also be thought of as defining functionally graded material (FGM) properties. The various roles of these functions in relation to optimum material performance (designer materials) will be analyzed in subsequent sections. Due to the complexity and number of variables some *a priori* specifications of these functions may be necessary and practical. For instance, some variables may not be independent of each other. If the stacking sequence is to be optimized or prescribed because of manufacturing dictates, then only one of the  $\theta_{or}$ s can be an independent variable. The  $N_{pl}$  and  $N_{fp}$  determine the  $v_{fr}$ . As a simple example one can consider a configuration where the the number of fibers is constant and the stacking sequence is repeated such that ply angles are  $\theta_n = \theta_1 + (n-1)\Delta\theta$  for  $n = 1, 2, ..., N_{pl}$  and

$$\mathcal{F}(x) = \mathcal{F}(\theta_1, \Delta \theta, N_{pl}, N_{fp}) \tag{3}$$

The integrals always reduce to convolution ones with E = E(x, t - t') whenever the  $\mathcal{F}$  is time independent and no transient temperatures are present to modify viscoelastic material properties. The spatially non-homogeneous anisotropic relaxation modulus functions are definable in terms of Prony series [32], such that

$$E_{ijkl}(x,t) = E_{ijkl}^{\infty} [\Xi_1(x)] + \sum_{n=1}^{N_{ijkl}} E_{\underline{ijkln}} [\Xi_1(x)] \exp\left(-\frac{t}{\tau_{\underline{ijkln}} [\Xi_1(x)]}\right) =$$
(4)

where the instantaneous elastic modulus  $E^0_{ijkl} \mbox{ is given by }$ 

$$E_{ijkl}^{0}(x) = E_{ijkl}(x,0) = E_{ijkl}^{\infty}[\Xi_{1}(x)] + \sum_{n=1}^{N_{ijkl}} E_{ijkln}[\Xi_{1}(x)]$$
(5)

The underscored indices indicate no summations.

#### Governing Relations, Constraints and Designer Composite Materials

In general terms the problem can be stated in the following manner. The viscoelastic composite plate obeys depending on loading conditions (tractions, moments, etc.) one or up to  $\mathcal{L}_{\mathcal{L}}$  number of differential-integral relations, which can be expressed in general terms as

$$\mathcal{L}_{\ell}\left(x, t, \underbrace{\mathcal{F}(x), E_{ijkln}\left[\Xi_{1}(x)\right], \tau_{ijkln}\left[\Xi_{1}(x)\right]}_{= \Xi_{\mathcal{L}}(x)}\right) = 0 \qquad \ell = 1, 2, \dots, \mathcal{L}_{\mathcal{L}}$$
(6)

subject to  $\mathcal{P}_\wp$  number of boundary conditions (BC)

$$\mathcal{BC}_{\wp}\left(x,t,\Xi_{\mathcal{L}}(x)\right) = 0 \qquad \wp = 1,2,\dots,\mathcal{P}_{\wp}$$

$$\tag{7}$$

with a pre-determined constraint function C based on weight  $\rho$ , maximum deflections  $w_{max}$ , structural lifetime  $t_{LF}$ , failure probability  $P_F$ , cost COST, etc., given by

$$\mathcal{C}\left(x,t,\underbrace{\mathcal{F}(x),E_{ijkln}\left[\Xi_{1}(x)\right],\tau_{ijkln}\left[\Xi_{1}(x)\right],\rho,w_{max},\mathcal{COST},t_{LF},P_{F},\ldots}_{=\Xi_{\mathcal{C}}(x)}\right) = 0$$
(8)

A number of protocols for determining the various parameters are presented and analyzed in [1]. Let  $S_p$  be the general designation for each individual parameter in Eqs. (6) to (8) with a total number  $\kappa$ . Selecting, for instance, a Lagrangian multiplier ( $\lambda_\ell$ ) approach one obtains after applying Galerkin's method to eliminate all *x*-dependencies

$$\frac{\partial}{\partial S_p} \left\{ \mathcal{L}_{\ell}(t, \Xi_{\mathcal{L}}^*) + \lambda_{\ell} \mathcal{C}(t, \Xi_{\mathcal{C}}^*) \right\} = 0$$

$$\ell = 1, 2, \dots, \mathcal{L}_{\mathcal{L}} \quad \text{and} \quad p = 1, 2, \dots, \kappa + \mathcal{L}_{\mathcal{L}}$$
(9)

The time dependence in (9) is removed through the use of the least squares method with at least  $25(\kappa + \mathcal{L}_{\mathcal{L}})$  time values.

Specifically, a plate under aerodynamic loading has a governing relation for the mid-plane deflection w(x,t) which

reads

$$\mathcal{L}_{1}\left\{w(x_{1}, x_{2}, t)\right\} = \int_{-\infty}^{t} \frac{\partial^{2}}{\partial x_{1}^{2}} \left(D_{1}(x, t - t') \frac{\partial^{2}w(x, t')}{\partial x_{1}^{2}}\right) dt'$$

$$+ \int_{-\infty}^{t} \left[2 \frac{\partial^{2}}{\partial x_{1} \partial x_{2}} \left(D_{2}(x, t - t') \frac{\partial^{2}w(x, t')}{\partial x_{1} \partial x_{2}}\right) + \frac{\partial^{2}}{\partial x_{2}^{2}} \left(D_{3}(x, t - t') \frac{\partial^{2}w(x, t')}{\partial x_{2}^{2}}\right)\right] dt'$$

$$- \left[\int_{-\infty}^{t} \int_{0}^{a} D_{4}(\overline{x}_{1}, x_{2}, t - t') \left(\frac{\partial w(\overline{x}_{1}, x_{2}, t')}{\partial \overline{x}_{1}}\right)^{2} d\overline{x}_{1} dt' + \underbrace{\mathcal{N}_{11}^{\mathbf{EX}}(x_{2}, t)}_{\text{external force}}\right] \frac{\partial^{2}w}{\partial x_{1}^{2}} + \underbrace{m_{p} \frac{\partial^{2}w(x, t)}{\partial t^{2}}}_{\text{inertia effects}}$$

$$+ \left[\underbrace{\int_{-\infty}^{t} \int_{0}^{b} D_{5}(x_{1}, \overline{x}_{2}, t - t') \left(\frac{\partial w(x_{1}, \overline{x}_{2}, t')}{\partial \overline{x}_{2}}\right)^{2} d\overline{x}_{2} dt'}_{\text{in plane force due to length change in x_{1} direction (T_{5})}\right] \frac{\partial^{2}w}{d\overline{x}_{2}^{2}}$$

$$- \left[ \int_{-\infty}^{t} \int_{0}^{a} D_{0}(\overline{x}_{1}, x_{2}, t - t') \frac{\partial w(\overline{x}_{1}, x_{2}, t')}{\partial \overline{x}_{1}} \frac{\partial w(\overline{x}_{1}, x_{2}, t')}{\partial x_{2}} \frac{\partial \overline{x}_{1} dt'}{\partial x_{2}} + \int_{external force}^{Ex} (t) \int_{external force}^{b} \frac{\partial^{2} w}{\partial x_{1} \partial x_{2}} \right] \frac{\partial^{2} w}{\partial x_{1} \partial x_{2}}$$

$$- \left[ \int_{-\infty}^{t} \int_{0}^{a} D_{7}(x_{1}, \overline{x}_{2}, t - t') \frac{\partial w(x_{1}, \overline{x}_{2}, t')}{\partial x_{1}} \frac{\partial w(x_{1}, \overline{x}_{2}, t')}{\partial \overline{x}_{2}} \frac{\partial \overline{x}_{2} dt'}{\partial \overline{x}_{2}} + \int_{external force}^{Ex} (t) \int_{external force}^{b} \frac{\partial^{2} w}{\partial x_{1} \partial x_{2}} \right] \frac{\partial^{2} w}{\partial x_{1} \partial x_{2}}$$

$$+ a_{0} q \sin \left\{ \frac{\pi}{2 \alpha_{ST}} \left[ \underbrace{\bigvee_{external force}^{wig contribution} \frac{panel contribution}{\int_{ext} (11)} \frac{panel contribution}{\int_{ext} (11)} \frac{panel contribution}{\int_{ext} (11)} \frac{panel contribution}{\int_{ext} (11)} \right] \right\}$$

$$+ \underbrace{\mathcal{N}_{11}^{P}(x, t) \frac{\partial^{2} w}{\partial x_{1}^{2}}}_{x_{1} piezo force} (T_{12}) \sum_{x_{2} piezo force} (T_{13}) x_{1} thermal force (T_{14}) x_{2} thermal force (T_{15})$$

$$+ \underbrace{2 \mathcal{N}_{12}^{T}(x, t) \frac{\partial^{2} w}{\partial x_{1} \partial x_{2}}}_{thermal force} + \underbrace{\partial M_{11}^{T}}_{x_{1}(17)} \underbrace{\partial M_{21}^{2}}_{x_{2}(18)} \underbrace{\partial M_{22}^{2}}_{x_{3} vad due} \underbrace{\partial M_{11}^{T}}_{x_{3} vad due} \underbrace{\partial M_{12}^{T}}_{x_{3} vad due} \underbrace{\partial M_{1$$

where the plate  $w = w(x,t) = w(x_1,x_2,t)$  and the wing  $W = W(x_2,t)$  unless otherwise indicated. In the absence of chordwise bending the panel effective angle of attack due to wing contributions is

$$\mathcal{A}_{W}(\alpha,\theta,W) = \overbrace{\alpha_{r}(x_{2}) - \alpha_{0}(x_{2})}^{f_{\alpha}(x_{2}) = \text{built in}}_{\text{rigid angles}} + \overbrace{\alpha(x_{2})}^{\text{angle of attack}}_{\text{attack}} + \underbrace{\underbrace{\Theta(x_{2},t)}_{\text{of twist}}}_{\text{wing argle of twist}} + \underbrace{\operatorname{arctan}\left(\frac{1}{U_{\infty}}\frac{\partial W(x_{2},t)}{\partial t}\right)}_{\text{wing argle of twist}}$$
(11)

For elastic plates based on Eqs. (10) similar relations can be derived without the time integrals and with

$$D_m^E(x) \equiv D_m(x, t, t') \tag{12}$$

If the elastic or viscoelastic material is isotropic then in terms  $T_1$ , the bending rigidities become

$$D_1 = D_2 = D_3 \equiv D \tag{13}$$

and the  ${\cal T}_1$  terms reduce to

Term 
$$T_1$$
 viscoelastic  $\Longrightarrow \int_{-\infty}^t D(x, t - t') \nabla w(x, t') dt'$  or  $\int_{-\infty}^t D(x, t, t') \nabla w(x, t') dt'$  (14)

Term 
$$T_1$$
 elastic  $\implies D^E(x) \nabla w^E(x,t)$  (15)

with

$$\nabla = \frac{\partial^4}{\partial x_1^4} + 2 \frac{\partial^4}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4}{\partial x_2^4}$$
(16)

It is to be noted that in corresponding elastic isotropic cases, the modulus  $D^E$  is defined as

$$D^{E} = \frac{E^{E} h^{3}}{12 \left[1 - (\nu^{E})^{2}\right]}$$
(17)

whereas for equivalent isotropic viscoelastic materials the use of Poisson's ratio is inappropriate and counter-indicated [33, 34], since there is no viscoelastic counterpart for the elastic expression (17), except under severely restricted conditions. In all other more general cases, viscoelastic Poisson ratios are time, loading history and stress dependent [33], [34]. For viscoelastic materials, therefore, only relaxation moduli or functions can be used to define bending rigidities, i. e.

$$D_{ijkl}(t) = \frac{E_{ijkl}(t)h^3}{12}$$
(18)

The linearity or nonlinearity of terms in the governing relation of Eq. (10) is independently influenced by geometry (size of deflections), material property characterizations. boundary conditions and failure conditions. It should be noted that if a single term of the governing relations exhibits nonlinear properties, then integral transforms, such as Laplace and Fourier, are inapplicable and solutions to the governing DEs or integral-differential relations must be obtained by other means.

For nonhomogeneous materials Eqs. (18) must be returned to their fundamental roots and become

$$D_{ijkl}(x_1, x_2, t) = \int_{-h/2}^{h/2} E_{ijkl}(x_1, x_2, x_3, t) x_3^2 dx_3$$
(19)

rather than the simple moment and product of inertia expressions associated with homogeneous elastic and/or viscoelastic material plates. For auxetic materials, quantities associated with shear, such as  $D_2$ ,  $D_6$ . $D_7$ , can be orders of magnitude larger than the corresponding bending stiffnesses, i.e.  $D_2 \gg D_1 \simeq D_3$ , etc., [1], [3].

#### **Stability and Failure Considerations**

Viscoelastic plates are subject to two separate and unrelated failure modes : (1) creep buckling - stability criteria in bending and/or twisting and (2) failures due to de-bonding and/or internal core cracks - material property induced failures due to combined bending, compressive and shear stresses. In addition if they are subjected to aerodynamic loading such as lift and/or aerodynamic noise they will subject to further modes of instability, such as panel flutter.

The classical elastic and creep buckling definitions are

elastic 
$$\Rightarrow \lim_{\sigma_{11}\to\sigma_{11}^{cr}} \left\{ w^{E}(x,q) \right\} \to \infty \text{ or } \lim_{\sigma_{11}\to\sigma_{11}^{cr}} \left\{ \frac{\partial w^{E}(x,q)}{\partial \sigma_{11}} \right\} \to \infty$$
  

$$\sigma_{11}^{cr} = \frac{\pi^{2} k_{c} E^{0}}{12(1-\nu_{E}^{2})} \left( \frac{h}{b} \right)^{2}$$
viscoelastic  $\Rightarrow \lim_{t\to t_{cr}} \left\{ w(x,t) \right\} \to \infty \text{ or } \lim_{t\to t_{cr}^{*}} \left\{ \frac{\partial w(x,t)}{\partial t} \right\} \to \infty$ 

$$\sigma_{11} < \sigma_{11}^{cr}$$
(20)
(20)
(21)

However, in [35 - 37] it has been shown that small deflection linear viscoelastic analysis without any twisting results in finite deflections for  $0 < t_{cr}$  and  $t_{cr}^* < \infty$ . This proof can be readily extended to the present coupled deformations. Consequently,

alternate creep buckling definitions must be formulated. Two distinct types based on (1) strain reversal in time and on (2) time dependent material failure criteria have been offered.

In [38] it has been proposed and successfully experimentally demonstrated that for elastic plates the buckling load can be established by analyzing graphs of outer plate fiber strains where compressive strains due loads and tensile strains due to bending take place. The elastic in-plane buckling load  $N_{cr}$  is then defined as

$$\lim_{N \to N_{cr}} \frac{\partial \left[ \epsilon_c^E(N) + \epsilon_t^E(N) \right]}{\partial N} \to 0 \qquad t = 0$$
(22)

This definition has been modified in [40] for creep buckling of columns and can now be applied to plates in the form

$$\lim_{t \to t_{cr}^{\#}} \frac{\partial \left[\epsilon_c(t) + \epsilon_t(t)\right]}{\partial t} \to 0 \qquad 0 \le N_{11} < N_{11}^{cr} \quad \text{and} \quad 0 < t_{cr}^{\#} < \infty$$
(23)

If thermal loads are present, then one needs to change the domain to  $0 \le N_{11}(t) + N_{11}^T(t) < N_{11}^{cr}$  in order to incorporate in-plane thermal load  $N_{\alpha\beta}^T$  effects.

Viscoelastic failure criteria, such as ultimate stresses, degrade in time independently of relaxation moduli and failures may occur before or after any creep buckling instabilities manifest themselves. These are material failures which are independent of creep buckling and define the life time of the structure designated as  $t_{LF}$ . Consequently,  $t_{cr}$  or  $t_{cr}^{\#}$  or  $t_{cr}^{\#}$  may be greater, smaller or equal than  $t_{LF}$  [39], [40]. Indeed, in Refs. [35], [41] and [42] the Shanley & Ryder [43] interaction or stress ratio curve approach has been used to estimate plate failures under combined inelastic deterministic stresses.

Some failure mechanisms observed in composites are substantially different from those seen in metals as for example delamination which is a phenomenon unique to composites [4], [44] [45]. From an aerospace design analysis point of view, one needs only to consider delamination onset because at that stage a structure has for all practical purposes failed, particularly if it is a light weight flight structure. In [4] an expression has been formulated for the temperature, moisture and time dependency of uniaxial composite failure stresses. An extensive review of available experimental composite failure data is presented in [45] where such data is used to formulated deterministic and stochastic delamination failure analyses. Experimental results indicate that uniaxial deterministic delamination onset stresses in tension and shear obey laws of the type

$$\sigma_{ij}^{F}(t) = \begin{cases} \sigma_{ij0}^{F} & -\infty \le t \le t_{2}^{F} \\ \sigma_{ij0}^{F} \left[ 1 - \left| \frac{\log(t/t_{4}^{F})}{\log(t_{3}^{F}/t_{4}^{F})} \right| \right] & 0 < t_{2}^{F} \le t \le t_{3}^{F} \\ 0 & t \ge t_{3}^{F} \end{cases}$$
(24)

where all parameters are material, temperature, moisture and uni-axial load (compression, shear, bending, etc.) dependent (Fig. 3).

#### **DISCUSSION and CONCLUSIONS**

In Eq. (10) the input forces are Terms  $T_{11}$  (lift) and  $T_{20}$  (aerodynamic noise). Each force in its own right creates a self-excited system and can result in time dependent viscoelastic panel instabilities, such as creep buckling and creep flutter. Additionally and independently failures due to delamination, aging, cracks, etc., can take place. These failure modes are illustrated in Figs. 4 and 5. The first plots indicate the importance of aerodynamic noise at lower flight velocities, while flutter generally predominates at higher values. Fig. 5 shows the relative contributions to maximum panel deflections due to aerodynamic noise, creep buckling and creep flutter. Each instability mode in turn provides limits on panel lifetimes or survival times, unless preceded by delamination or other material failures, which have their own survival times.

As can be seen from the results displayed in the above Analysis Section, the number of candidate optimization variables defining the designer material can be large. In Ref. [11], protocols for manufacturing polymers to order, i.e. to

produce prescribed relaxation moduli, failure conditions, weight, cost, etc., are presented and it is shown that they still are in their early development phase. Consequently, at this stage designer material properties serve as a guide for manufacturers to plan for what material properties are needed to be engineered now and in the near future. In the meantime, results obtained for designer materials serve to improve the design selection by helping to identify, based on their properties, the "best" currently available materials to be used for particular service conditions.

Fig. 6 presents a typical example of the merits of the viscoelastic composite designer material protocols by selecting proper values for the variables that produce lower delamination probabilities and longer life times than conventional procedures based on catalogue available materials. Similar charts can be generated for optimal designer lifetimes governing creep buckling and flutter conditions. Weibull failure probability distribution functions [46] were used in the analysis.

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Fig. 4. Aerodynamic noise and flutter boundaries

1.2



Fig. 2. Five-layer functionally graded ZrO<sub>2</sub>/Ni Co Cr AlY coating [28]



Fig. 5. Aerodynamic noise, panel flutter and buckling



Fig. 3. Delamination stresses [4]



Fig. 6. Probability of delamination