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# A simple model for turbulence intermittencies.

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## Abstract

Whether turbulence intermittencies shall be described by a log-Poisson, a log-stable pdf or other distributions is still debated nowadays. In this paper, a bridge between polymer physics, self-avoiding walk and random vortex stretching is established which may help in getting a new insight on this topics. Actually a very simple relationship between stability index of the stable law and the well known Flory exponent stemming from polymer physics is established. Moreover the scaling of turbulence intermittencies with Reynolds number is also obtained and the overall picture is very close to Tennekes' simple model for the fine scale structure of turbulence [Phys. Fluids, **11**, 3 (1968)] : vortex tubes of Kolmogorov length width are bend by bigger vortices of Taylor length scale. This thus results in both a simple and sound model with no fitting parameter needed.

## 1. Introduction

Since Discovery of intermittencies in turbulence by Batchelor and Townsend in 1949 (see Frisch, 1995, for a review) there have been numerous debates on how to model properly this phenomenon. One of the first modelling is due to Oboukhov and Kolmogorov who predicted a lognormal law for the distribution of the dissipation of turbulence (Kolmogorov, 1962) (Actually Kolmogorov, 1941, predicted this law in studies on pulverization/fragmentation and Oboukhov, 1961, used it in turbulence modelling). The next important step has been the definition of "scale similarity" by Novikov (1966). This has led to a lot of interesting works on multifractal modelling of turbulence (again Frisch is a good reference; see also Evertz and Mandelbrot, 1992, for the definition of a multifractal measure) which ultimately resulted in multiscale analysis (for instance resorting to wavelets theory cf. Farge, 1992). However this has not led to any simple law. On the other side, in the wake of Lovejoy and Mandelbrot (1985) work on multifractal analysis in raindrop distribution (where some reference to Lévy law of stability parameters  $5/3$  can be found), Kida (1991a, 1991b) generalized Kolmogorov-Oboukhov results to Log-stable law with a stability index  $\alpha = 1.65$  and an intermittency parameter  $\mu = 0.21$ .

The purpose of this paper is to show that Kida's empirical analysis can be put on a more solid ground on the condition of changing the value of the stability parameter from 1.65 to  $1/\nu_F$  where  $\nu_F$  stands for the Flory exponent well known in polymer physics (this leads to the value 1.70). Actually In section 2, a bridge between Kuo's effective vortex stretching mechanism (Kuo, 1986) and (linear) polymer growth is drawn leading to a mapping of one mechanism onto the other. Scaling properties of three dimensional self-avoiding walks are then used to obtain the results. In section 3, it is shown how this model can be successfully applied to a given set of experimental and numerical results. Lastly, the number of stretching and bending events is related to the ratio between Taylor micro scale and Kolmogorov scale leading to quite simple picture, very close to some insight of Tennekes (1966), describing the worm-like structure of turbulent vortices.

## 2. Random Vortex Stretching

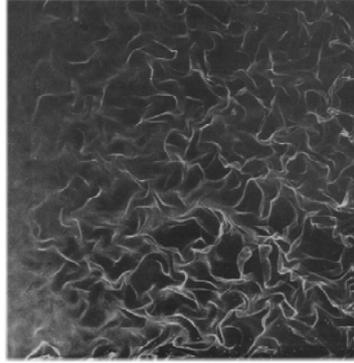


Figure 1: Wrinkling of a fluid surface in isotropic turbulence: Karweit, John Hopkins Univ. 1968 (Van Dyke, 1982). Here, it is material lines that are stretched, but a clear analogy with vortex tubes can be made, since they can be considered as materially linked to the fluid (Lamb, 1932).

Partly following Kuo (1986), a very simple way of modelling Random Vortex Stretching can be obtained by simplifying the vorticity equation:

$$\frac{\partial \vec{\Omega}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{\Omega} = \vec{\Omega} \cdot \vec{\nabla} \vec{v} + \nu \vec{\nabla}^2 \vec{\Omega} - \vec{\Omega} \vec{\nabla} \cdot \vec{v} - \vec{\nabla} \frac{1}{\rho} \times \vec{\nabla} p \quad (1)$$

Where  $\vec{\Omega}$  is defined by  $\frac{1}{2} \vec{\nabla} \times \vec{v}$ . The first term on the second hand stands for vortex stretching, second term for viscous diffusion, third term for vorticity increase related to vortex thinning and the last term is the baroclinic creation of vorticity. Neglecting the second and fourth term (i.e. viscosity effects and baroclinic generation of vorticity) and setting

$$B(t) = \vec{\nabla} \vec{v} \cdot \frac{\vec{\Omega}}{\|\vec{\Omega}\|} - \vec{\nabla} \cdot \vec{v} \quad (2)$$

as the “effective stretching”, one gets

$$\frac{D\vec{\Omega}}{Dt} = B(t) \vec{\Omega} . \quad (3)$$

Where  $D/Dt$  stands for the material derivative. According to Kuo, the random nature of the effective vortex stretching ensures that the vorticity shall evolve toward a log-normal distribution. This shall be examined further in the following sections; it will actually be shown that this hypothesis is not compatible with the topological constraints that affect a vortex line.

### 2.1. Angular acceleration and stretching

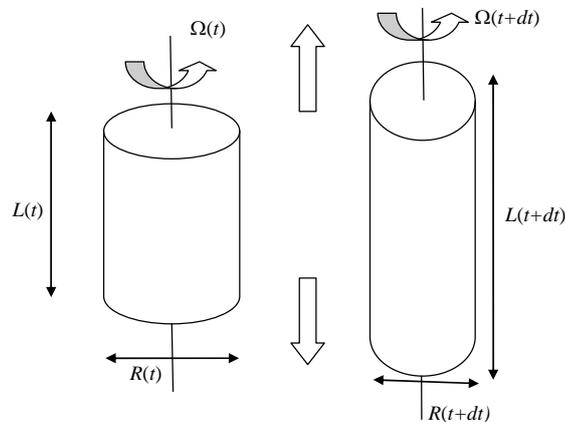


Figure 2: axial stretching of a portion of a vortex tube

Considering a part of a vortex tube, of length  $L(t)$ , of radius  $R(t)$  and angular velocity  $\Omega(t)$ , submitted to an axial stretching (cf. Figure 2). In non viscous flow, vorticity is conserved and materially linked to the fluid particles. Thus the following simple conservation equation can be written:

Mass conservation:

$$R^2 L = \text{constant}. \quad (4)$$

Angular Momentum conservation:

$$R^2 \Omega = \text{constant}. \quad (5)$$

Angular velocity increase can thus be thought to be proportional to the vortex length increment. Note that this analysis is similar to the pioneering works of Lagrange, Helmholtz, Kelvin, Cauchy,...etc. (see Lamb, 1932, for a more rigorous approach and a review. More details on the present, simpler, analysis can be found in Vanyo, 1993). Using (3) this can be summed up into a continuous time stochastic multiplicative process of the form:

$$DL_t = B_L(t) L_t Dt. \quad (6)$$

Or using the logarithmic derivative, one gets:

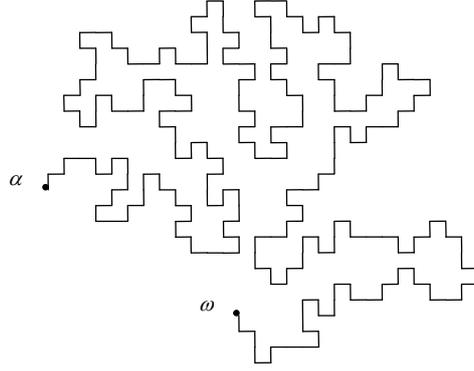
$$D \ln L_t = B_L(t) Dt. \quad (7)$$

The effective stretching  $B$  is *a priori* an unknown random variable (and is *a priori* different in (3) and (7) since in this process the vector nature of equation (3) is lost). The next section will determine which scaling constraint the random variables  $B$  shall obey to.

## 2.2. Topological constraint on a vortex tube and Self-avoiding Walk

To obtain the simple equation (7), several simplifications and hypotheses have been made; indeed viscosity has been neglected. This hypothesis withhold as long as the finer scale of turbulence known as the Kolmogorov scale  $\eta$  has not been reached. At this scale viscosity and inertial term balance each other so that the former can turn the latter into heat. In this paper the following assumption is made: the fluid possesses a null viscosity till the Kolmogorov scale is reached where the viscosity is applied. Neglecting viscosity ensures that Helmholtz theorem on vorticity conservation can be applied (Lamb, 1932). As a sequel to this theorem, vortex line cannot intersect each other. Therefore a vortex tube shall obey (3) or (7) under the constraint of non intersection.

Thought Vortex are mainly found in turbulent flows in the form of vortex tubes a.k.a. “worms” or sometimes vortex sheet, the assumption of simple vortex lines will be maintained in the following for the sake of simplicity (yet a vortex sheet can be considered as a set of vortex line as a material surface can be considered as a set of material line: this is what Figure 1 suggests: each material line seems to be compelled to self avoidance. Note also that vortex line cannot *stricto sensu* be submitted to stretching: only vortex tubes can be stretched cf. Vanyo, 1993). Let us look at the ways in which a vortex line can fold itself under the combined constraint of effective stretching and self avoidance. Letting the dynamics aspect of vortex line folding apart, it is clear that the kinematics aspects can be fully translated in the world of linear polymer growth or equivalently of self-avoiding walk.



**Figure 3: Example of a 2D SAW**

According to Flory pioneering work in polymer physics (see De Gennes, 1979 for instance), some scaling properties concerning the equivalent radius of a polymer chain of  $N$  monomers can be established. Let  $N_N(\mathbf{r})$  be the number of chains of  $N$  monomers of equal size going from the origin to the point of  $\mathbf{r}$  coordinate. Let

$$p_N(\mathbf{r}) = \frac{N_N(\mathbf{r})}{\sum_{\mathbf{r}} N_N(\mathbf{r})} \quad (8)$$

be the probability distribution of chain of length  $N$  and let the gyration radius be defined by:

$$R = \left( \int_0^\infty \mathbf{r}^2 p_N(r) dr \right)^{1/2} \quad (9)$$

Then it is known that:

$$R \propto N^\nu \quad (10)$$

Where  $\nu$  is a universal scaling exponent called Flory's exponent such that its value is appreciatively  $3/5$  (.588 according to numerical simulations cf. Sokal, 1994) in a three dimensional space. Figure 3 gives the simple picture of a 2D SAW (which will be used for the sake of simplicity though the overall picture is three dimensional). The question which still has to be answered is how the flow maps a piece of vortex tube into a SAW similar to that of Figure 3.

### 2.3. SAW and Lévy-stable stretching

Actually the answer is quite straightforward: the vortex must be strained and bent so as to map onto the SAW. But differing from figure 1 where each step is of equal size, there is no a priori reason that the average step size or standard deviation thereof shall be defined and finite. We shall yet make the hypothesis that the average size is defined and finite and we will consider that the actual number of steps is related to the number of bending events in a manner quite similar to how, according to popular belief, the first Arabic number ideograms were defined in the IX<sup>th</sup> century (cf. Figure 4).



**Figure 4: the first Arabic number ideograms are based on the number of (bending) angles**

Each bending event is associated to a stretching event and introducing the Hencky strain  $H = \ln L$ , the latter can be considered to be a good approximation to the number of steps  $N \approx H$ .

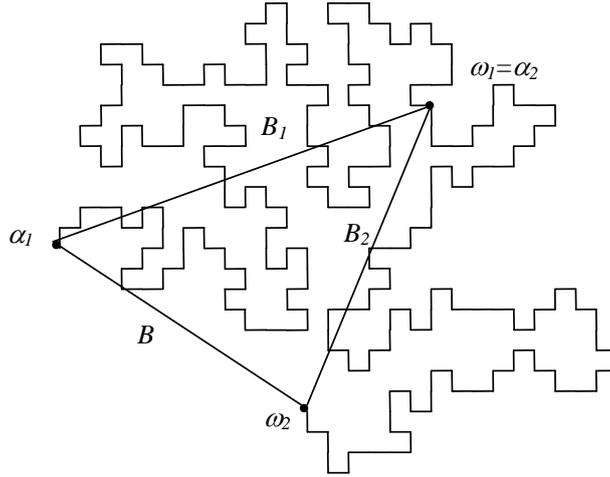
Since the average deviation of the size steps may eventually not be defined, let us define  $\alpha$  as the greatest number such that

$$\int_0^{\infty} \mathbf{r}^{\alpha} p_1(r) dr < \infty \quad (11)$$

Then the following definition

$$\sigma = \left( \int_0^{\infty} \mathbf{r}^{\alpha} p_H(r) dr \right)^{1/\alpha} \quad (12)$$

leads to a finite integral so that (12) can supersede (9) and  $\sigma$  can be considered as a new gyration radius. From the preceding hypotheses, one gets that  $1 < \alpha \leq 2$ .



**Figure 5: scaling property of vortex stretching**

Now, let us consider the scaling properties of this model. Consider that the number of steps is high enough and that the  $N$  steps SAW created by the random effective stretching  $B$  can be divided into two SAW,  $B_1$  and  $B_2$  each having  $H_1$  and  $H_2$  steps. The following equation is quite obvious:

$$H = H_1 + H_2 \quad (13)$$

And assuming  $H = N$  and that equation (10) is still verified, this leads to:

$$\sigma^{1/\nu} = \sigma_1^{1/\nu} + \sigma_2^{1/\nu} \quad (14)$$

$\sigma$  can be qualitatively thought as the intensity scale of the effective random stretching  $B$  and equation (13) indicate that  $H$  considered as a random variable shall belong to the family of Lévy stable law (cf. Feller, 1966). If this is the case, considering (12),  $\sigma$  can be considered as the scaling parameter of the law and (13) leads to  $\sigma^{\alpha} = \sigma_1^{\alpha} + \sigma_2^{\alpha}$  suggesting that the stability index  $\alpha$  shall be taken equal to  $1/\nu$  so as to recover (14). Note that the (self-intersecting) case  $\nu = 1/2$  leads to Kuo's lognormal model. Thus an analogy can be made between polymer growth and vortex stretching. This is summarized in Table 1.

**Table 1: analogy between polymer growth and vortex stretching**

|          | Number of elements | Radius   | Relation              |
|----------|--------------------|----------|-----------------------|
| Polymers | $N$                | $R$      | $R = N^{\nu}$         |
| Vortices | $H$                | $\sigma$ | $H = \sigma^{\alpha}$ |

## 2.4. Interpretation of some previous experimental results

Following Novikov (1966,1994), the spatial distribution of the rate of turbulent energy dissipation  $\varepsilon = dk/dt$  is often, as  $\varepsilon$  represent the death of bigger eddies, thought to give a proper picture of Richardson turbulent cascade. Actually  $\varepsilon$  is never directly measured and, as described by Frisch (1995), a bridge between intermittencies model based on dissipation and intermittencies model based on velocity increment is mostly used. According to Kolmogorov refined similarity hypothesis (Kolmogorov, 1962) the following relation can be obtained

$$|u(x+r) - u(x)|^3 = -\frac{4}{5} \varepsilon_r r \quad (16)$$

where  $\varepsilon_r$  stands for the dissipation averaged on a sphere of radius  $r$ . Then following Tennekes and Lumley (1972), one gets:

$$\varepsilon = \nu \overline{\omega^2} \quad (17)$$

Thus the law of  $\ln(|v(r)|)$  or  $\ln(\varepsilon)$  shall follow the law of  $\ln(|\omega|)$  (or even  $\ln(|\partial u / \partial x|)$ ) up to a scaling factor and a translation. Anyway these laws shall be Lévy stable with stable parameter  $1/\nu_F$  and an asymmetry parameter  $\beta$ , equal to minus one. The last condition stems from the fact that it implies that every positive moments of the log-lévy law are defined and finite.

### 1.1.1. Comparison to Kida previous results

Figure 6 shows the fitting of such a Lévy stable law to Stewart *et al.* (1970) atmospheric turbulence data. The result gives  $\alpha = 1.684$  which is closer to  $1/\nu_F$  than the value 1.65 previously used by Kida (1991b) without directly fitting the pdf. As can be seen on Figure 7, this value of 1.65, as been obtained by fitting the scaling parameter of the moments of the distribution. The following set of definition is used in this description: the  $p$  order hyper-flatness of the velocity increment is defined as

$$H_p(r) = \frac{\langle |u(x+r) - u(x)|^p \rangle}{\langle |u(x+r) - u(x)|^2 \rangle^{p/2}} \quad (17)$$

using Kolmogorov four-fifths law and defining the scaling parameter  $\xi_p$  as :

$$H_p = \exp(\xi_p) \quad (18)$$

If  $\varepsilon_r$  is Log-stably distributed with  $\beta = -1$ , this yields

$$\langle \varepsilon_r^q \rangle = \exp \left[ q m_{\ln \varepsilon_r} - \frac{\sigma^\alpha}{\cos\left(\frac{\pi\alpha}{2}\right)} q^\alpha \right] \quad (19)$$

And then

$$\xi_p = -\frac{\sigma^\alpha}{\cos\left(\frac{\pi\alpha}{2}\right)} \left\{ \left(\frac{p}{3}\right)^\alpha - \frac{p}{2} \left(\frac{2}{3}\right)^\alpha \right\} \quad (20)$$

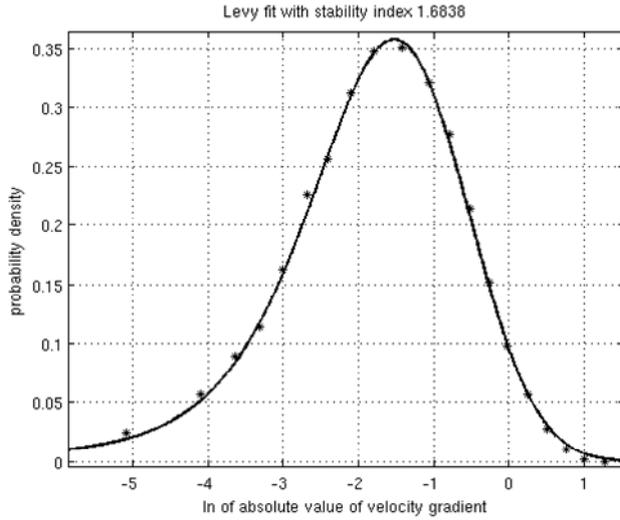


Figure 6: Fitting of (Stewart et al., 1970) pdf with a stable law; Stability parameter is found to be 1.684

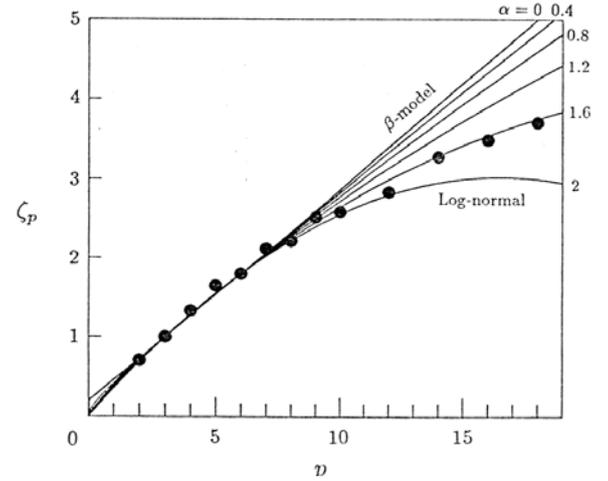


Figure 7: fitting by Kida, 1991 of Anselmet et al., 1984 data obtained in Modane wind tunnel.

Unfortunately, error bars around values of these moments increases with the order of the moments considered since poorly resolved tails of distribution are more and more solicited. Moreover knowledge of the pdf is only equivalent to the knowledge of all the moments of the distribution (i.e. an infinite number of moments!). Lastly a distribution having a discrete support (like a Poisson distribution) or a continuous support (like a stable law) may yield similar set of moments (up to a given order) whereas they are both physically and fundamentally different (due to the topological nature of their support). These arguments point out that moment of a pdf may not be the most appropriate way to describe accurately a pdf and that a direct fitting of the pdf, whenever available, shall be preferred. Yet some scaling properties of these moments with Reynolds number will be used in the forthcoming section and it will shed a new light on the cascade of stretching events described in previous sections.

### Scaling of hyper flatness with Reynolds Number and Tennekes' simple model for the small scale structure of turbulence

Before going on, let us discuss a bit further the way structure functions can be related to vortex distribution. Actually most values of (Rimbert and Séro-Guillaume, 2003), compatible with the present description have been taken in the near-dissipation range ( $\eta < r < 10\eta$ ) where according to (Chevillard et al., 2005) turbulence intermittencies are known to be “rapidly increasing”. This may be related to the fact that it is actually the range of scale where the velocity difference is a proper picture of Richardson turbulent cascade. Let us have a look at how, in this modeling, structure function can be computed:

$$\left\langle |u(x+r) - u(x)|^p \right\rangle = \int |u(x+r, \omega) - u(x, \omega)|^p d\mathbf{P}(\omega) \quad (24)$$

Where  $\omega$  is a random events (i.e. a realization of the vortex tangle) whose probability measure is  $\mathbf{P}(\omega)$ . Now let us assume that point  $x$  is in the neighbourhood of a stretched vortex of radius  $R$  (actually  $R(\omega)$ ). If  $r < R$  then both measurements point in the structure function can be located inside the vortex (let us call this case 1) whereas when  $r > R$ , at most one point can be located inside the vortex (case 2: one point outside and case 3: two points outside). In the near dissipation range, it will be assumed that viscosity is still non existent so that velocity can be assumed to be that of a solid rotation at an angular velocity of  $\Omega$  inside the vortex. Outside it is the result of the application of Biot-savart law, the vortices and their intensity superseding

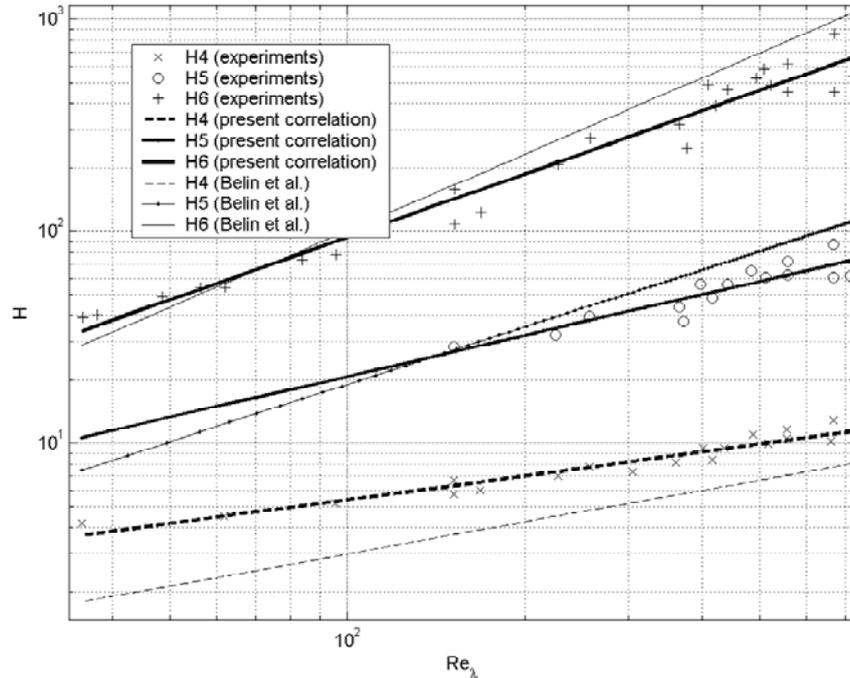
the electric wires and their current (Lamb, chapter VII, 1932). Since the geometry of the current is random and somewhat unknown (it is a tangle of random self-avoiding vortices), the velocity outside a vortex can be assumed to be a random variable of mean, the average velocity of the fluid. Anyway, in case 2 or 3,  $u(x+r)$  and  $u(x)$  are not much correlated whereas in case 1, one gets (approximatively):

$$u(x+r) - u(x) = r\Omega^1 \quad (25)$$

So that

$$\left\langle |u(x+r) - u(x)|^p \right\rangle = \int r^p \Omega(\omega)^p d\mathbf{P}(\omega) = r^p \int \Omega^p d\mathbf{P}(\Omega) \quad (26)$$

And structure function can be related to moments of the distribution of vorticity. Actually this can only happen in case 1 which is more frequent in the near-dissipation range. This may explain the so-called increase of turbulent intermittencies in this range.



**Figure 8: scaling of hyper flatness with Reynolds number up to a Taylor scale based Reynolds number of 750**

Combining the fitting of the scaling of hyper flatness with Reynolds number of the kind  $H_p = a(p) \cdot Re_\lambda^{b(p)}$  (Cf. Figure 8, result are summed up in Table 2) and using equations (17-20), this has led to the following scaling relation (cf. Rimbert and Séro-Guillaume, 2003 for details)

$$\sigma^\alpha \sim \left[ (1,06 \pm 0,08) \cdot \ln(\sqrt{Re_\lambda}) \right] + 0,00 \pm 0,20 \quad (27)$$

In this work a thorough attention to incertitude determination was made (using standard techniques, see Beck and Arnold 1977 for instance). Moreover both numerical simulations (Kerr, 1985) and experimental results (Belin et al., 1997) have been used to obtain that correlation. Note that changing the value of the stability index from 1.65 to 1.70 does not change the overall analysis but the results are now on the extreme side of the 95% confidence interval.

<sup>1</sup> Actually the hypothesis that the fluid is in solid rotating motion is not necessary. Since  $r$  is kept fixed, the only thing that matter is that velocity increment at fixed  $r$  and  $\Omega$  shall be proportional so that it still works if the vortex is, for instance, a Burgers vortex (Burgers, 1948)

**Table 2: scaling of hyper flatness with Reynolds number (Rimbert and Séro-Guillaume, 2003)**

| Order | Hyper-flatness                  | Moment scaling parameter               |
|-------|---------------------------------|--|
| 4     | $H_4 = 0,95.Re_\lambda^{0,376}$ | $\xi_4 = 0.38 \ln(Re_\lambda) - 0.045$ |
| 5     | $H_5 = 1,07.Re_\lambda^{0,642}$ | $\xi_5 = 0.64 \ln(Re_\lambda) + 0.069$ |
| 6     | $H_6 = 0,99.Re_\lambda^{0,989}$ | $\xi_6 = 0.99 \ln(Re_\lambda) - 0.013$ |

Now, let us recall that the ratio between Taylor scale  $\lambda$  and Kolmogorov scale  $\eta$  is equal to (Tennekes and Lumley, 1972):

$$\frac{\lambda}{\eta} = 15^{1/4} . Re_\lambda^{1/2} \quad (28)$$

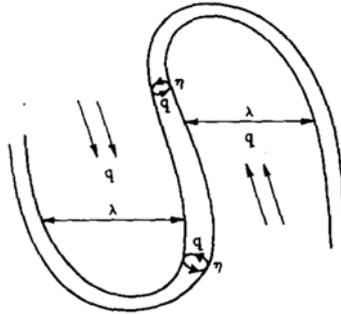
So that (21) leads to

$$\sigma^\alpha = \ln\left(\frac{\lambda}{\eta}\right) \quad (29)$$

Where  $\lambda$  stands for the Taylor length scale i.e. the mean size of the dissipative eddies. Moreover following section 2.3 this equation also reads

$$H = \ln\left(\frac{\lambda}{\eta}\right) \quad (30)$$

This means that the Hencky strain i.e. the number of stretching and bending events needed to describe the self-avoiding vortex is related to the ratio between the Taylor micro scale and the Kolmogorov scale.

**Figure 9: Tennekes' simple model for the small scale structure of turbulence**

This seems quite natural and the picture that is suggested can be related to Tennekes (1968) simple model for the small scale structure of turbulence (cf .Figure 9): a population of vortices of the size of the Taylor microscale is stretching a population of vortex tubes whose radii is of the order of the Kolmogorov scale. As Tennekes pointed out by making a budget of turbulent energy dissipation per unit volume: the following relationship is recovered:

$$\varepsilon = \nu \frac{q^2}{\eta^2} \frac{\eta^2}{\lambda^2} = \nu \frac{q^2}{\lambda^2} \quad (31)$$

Where  $q$  stands for the scale of velocity fluctuation (cf .Figure 9).

## Conclusion

In this paper a bridge between vortex stretching and linear polymer growth has been drawn leading to log stable law of intermittency parameter inversely proportional to the Flory exponent. The number of stretching and folding stage needed to map a vortex onto a three dimensional self-avoiding walk has been proved to be the logarithm of the ratio between Taylor micro scale (i.e. the size of the mean dissipative eddies) and Kolmogorov scale (i.e. the

size of the smallest dissipative eddies). For the time being, this modeling has been verified on four independent sets of experimental and numerical results (Stewart et al. 1970; Anselmet et al.; 1984, Kerr; 1985, Belin et al., 1997) giving it some generality<sup>2</sup>.

How interesting this modeling may be (since it does not contain any free parameter, eventually, values of Taylor and Kolmogorov scale depend on the experiment taken into account), let us emphasize that it can only be considered as a kinematic description: it shows a possible way for the vortices to evolve. The dynamics of vortex evolution is another matter that should be taken into account (this shall be rather simple, in the inertial range, but boundary conditions on the integral scale and Kolmogorov scale are another, tougher, matter). The goal of this work would be, for instance, to explain the relaxation mechanism of homogeneous and isotropic turbulence (cf. Comte-Bellot and Corrsin, 1965). Other topics of interest would be the introduction of transition in the behaviour vortices (for instance vortex breakdown or reconnections cf. Tabeling and Willaime, 2002) which may lead to some change in the modeling (maybe in the value of the stability parameter). Lastly reconnection with the pioneering work of Kolmogorov on pulverization (cf. Rimbart and Séro-Guillaume, 2004) which has been the very incentive of this work will be considered in some forthcoming papers.

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<sup>2</sup> The question whether turbulence is universal or not ...or shall be universal for instance at small scale...etc. However interesting it is, will not be tackled with in this paper.