



HAL
open science

Clockless Components and Relativity

Philippe Matherat

► **To cite this version:**

| Philippe Matherat. Clockless Components and Relativity. 2009. hal-00389739v1

HAL Id: hal-00389739

<https://hal.science/hal-00389739v1>

Preprint submitted on 29 May 2009 (v1), last revised 19 Dec 2011 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Clockless Components and Relativity

Philippe Matherat
CNRS - LTCI (UMR 5141)
Institut Télécom - Télécom-ParisTech - Comelec
<http://matherat.net/>

Abstract

It is natural to think that the conceptual frame of electronic asynchronous DI-systems (Delay-Insensitive) is well adapted to the description of distributed systems constituted with *mobile components* because when components move, distances change and then also delays of propagation. But a closer look at this question shows that it would be interesting to place this study in the frame of relativity theory, even for non-moving components. The main reason is: *time-like intervals* of this theory have an *objective character* and introduce a partial time-order which also is objective. It means that this time-order relation can be measured the same by all observers, whatever the propagation speed of signals and whatever the movements of the events emitters and receivers.

Moreover these intervals are well adapted to the discussion of the meaning of Udding's rules in term of causality, these rules being one way to define DI-systems.

We propose a new formalism based on trace theory in order to deal with this partial order for intervals issued from relativity.

Keywords: Asynchronous circuits and systems, Delay-Insensitive, Relativity, Mobile components, Trace theory, Partial time-order

1 Introduction

The notion of DI-systems (*Delay-Insensitive*) [1–7], in which delays are lapses of time during events propagations, leads us to consider that a partial order can be sufficient to specify a computation.

This way of thinking emerged after a research initiated by the will to remove the clock from digital electronic systems (*clockless systems*). In this frame, we have to

understand that electronic components of a distributed system can synchronize their actions, not by referring to a common unique time coordinate, but by the own nature of their logical functions and their communications.

It is remarkable that these considerations are very similar to those which lead A. Einstein to formulate his special theory of relativity [8,9], which one can see as the search for a method to synchronize local times of distant physical systems. In this theory, propagation *distances* and *durations* are measured differently by different observers. But it's not the case for *intervals* which are their generalization to relativistic space-time. As they are measured the same by all observers, we can say they have an *objective character*, they are *physical facts*. Among these intervals, *time-like intervals* of propagations define an objective partial time-order for events [10,11].

A DI computation, as defined by a partial order for propagations, is seen the same by all observers distributed in space-time, whatever their movements. In this frame, a DI computation also has an *objective character*. This is not the case for *simultaneity in different places* needed for *isochronic forks* of SI computations (Speed-Independent), for the implementation of which we have to add constraints on distances and speeds.

Then, it can seem natural to place the study of clockless systems implementation from the beginning in the frame of relativistic space-time, even if the components of the distributed system have no relative movements. There is no additional complexity in doing so, and it has the advantage of an easy extension to moving components, which is required for *mobile embedded systems*.

A way to define DI-systems is to use *Udding's rules* [2,3], which are written in the formalism of *trace theory*. But they can seem not adapted to relativistic frame because these traces describe, not a partial order, but a total time-order. In fact, a closer look at these rules shows that they make clear the cases in which events have to be non-ordered, although this non-ordering is written in a language which admit only total ordering.

We have to distinguish the case where a and b are non-ordered, from the case where the two orders ab and ba are possible. For doing this, we propose to define a new syntax for traces, adapted to the partial order of relativity, named R-traces. And we propose a new writing for relativistic Udding's rules.

Before this, we propose a reminder of important results of special relativity theory, selected for their interest in introducing the objective partial time-order of *time-like intervals*.

2 Relativity and objectivity

We represent the propagating movement of an edge of a digital Boolean signal by a trajectory in a cartesian coordinates system (O, x, t) . In order to simplify, we consider only uniform rectilinear movements and then only one dimension for space (x variable). The t coordinate represents the instant in the time dimension. x and t are real numbers.

Position and instant are relative to a coordinates system which is a material body constituted with measuring apparatus (rulers and clocks). We will name it “observer”.

Let us suppose that we have two coordinates systems: K associated to (O, x, t) , and K' associated to (O', x', t') . K' is moving, referred to K , in such a way that its origin O' has the speed v referred to K . At the origin of times ($t = t' = 0$), O and O' are in the same place. Then the absciss of O' referred to K is vt .

In the classical (newtonian) mechanics, the formulas for changing the coordinates system are:

$$x' = x - vt \tag{1}$$

$$t' = t \tag{2}$$

The formula (2) express that the time is the same for all coordinates systems, in other words for all observers, whatever their movements. Time is unique and independent of space. We name it the *global time* of the classical mechanics. It allows us to define a *simultaneity in different places* which has an objective character, because there is a consensus between all observers about this simultaneity.

In the theory of special relativity [8, 10], the formulas for changing coordinates system are (if c is the velocity of light in vacuum):

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

These formulas are known under the name *Lorentz transformation*. (We can extend them to 3 dimensions in space by adding $y' = y$ and $z' = z$.)

It's easy to verify that if we inverse these formulas in order to obtain x and t as functions of x' and t' , we find formulas with the same form as (3) and (4) except that v is replaced by $-v$. Then, neither K nor K' is privileged (this is named *reciprocity*).

It can be showed that, at the difference of what is well known in classical mechanics, lengths and durations are not independent of the movement of the observer:

- A body which is measured of length l when measured by an observer with no relative movement, is measured of length l/a when it is measured by a moving observer, with $1/a = \sqrt{1 - \frac{v^2}{c^2}} < 1$. One say that there is *contraction of length*. This contraction is reciprocal, *i.e.* it is indifferently a contraction whenever we measure from K to K' or from K' to K .
- A lapse of time which is measured as a duration d when measured by an observer with no relative movement, is measured as a duration da when measured by a moving observer. It is *dilatation of time* and is also reciprocal.

Lengths and durations no more have an *objective character*, because they cannot be the *object of a consensus* among all observers.

Let us consider two events A and B in two distinct places, which are simultaneous in K , for example with these coordinates:

$$A \begin{cases} x = 0 \\ t = 0 \end{cases} \quad B \begin{cases} x = 1 \\ t = 0 \end{cases}$$

Let us compute their coordinates in K' , we obtain:

$$A \begin{cases} x' = 0 \\ t' = 0 \end{cases} \quad B \begin{cases} x' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ t' = \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \end{cases}$$

This shows that these two events are not simultaneous in K' . But K has nothing special, it is in nothing privileged, it is not *at rest*. As a consequence, there is no special interest in the fact that two distant events are simultaneous in one coordinates system. This *simultaneity in different places* has *no objective character*, is *not a physical fact*.

A sentence like “the distance separating the two points A and B at time $t = 0$ ” has no meaning, because it supposes that all observers can measure the same distance. The difficulty doesn’t come from the *contraction of length*, but from the impossibility to attribute a meaning to the words “*the same instant*” in this case.

If we define “object” and “objectivity” by the possibility of a consensus among the observers, then we have to look for things which are conserved in a Lorentz transformation. An *object* is something which is “*Lorentz-invariant*”.

3 Objectivity of *intervals*

In classical physics, space is independent of time and is euclidean with 3 dimensions. The words “the distance separating the two points A and B at time $t = 0$ ” have a meaning and the value of this distance is the same for all observers. This is proved in considering the two points:

$$A \begin{cases} x_A \\ y_A \\ z_A \end{cases} \quad B \begin{cases} x_B \\ y_B \\ z_B \end{cases}$$

We can compute the distance D with the aid of the theorem of Pythagoras:

$$D^2 = (x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2$$

We can easily verify that this distance is the same in K and in K' if we use formulas (1) et (2).

We saw that it's not the case in relativistic physics. Would it be possible to find another expression that could look like such a distance, but which is Lorentz-invariant?

Let us consider, not two points in space, but two events in space-time:

$$A \begin{cases} x_A \\ y_A \\ z_A \\ t_A \end{cases} \quad B \begin{cases} x_B \\ y_B \\ z_B \\ t_B \end{cases}$$

and let us define the *interval* I in this way:

$$I^2 = (x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2 - c^2 (t_A - t_B)^2$$

If we write the expression for I^2 with coordinates in K' , and if we replace them by their value from (3) and (4), we find that the expression obtained for I^2 is the same than the one written with coordinates in K . As a consequence, the interval is Lorentz-invariant, and has an objective character.

Then the *sign* of I^2 has an objective character. Let us consider 3 types of intervals (which we will discuss in the case of only one spatial dimension):

- Interval zero:

$$I^2 = 0 \quad \Rightarrow \quad (x_A - x_B) = \pm c (t_A - t_B)$$

This is the case of the propagation of light from A to B or from B to A , at a velocity c or $-c$. The interval is zero whatever the distance from A to B .

- Time-like interval:

$$I^2 < 0 \quad \Rightarrow \quad (x_A - x_B)^2 < c^2 (t_A - t_B)^2$$

A body can go from A to B (or the converse) at a speed lower than c . There is an objective time-order for the pair (A, B) , compatible with the propagation of the body between A and B in the space-time.

- Space-like interval:

$$I^2 > 0 \quad \Rightarrow \quad (x_A - x_B)^2 > c^2 (t_A - t_B)^2$$

No body, not even light, can travel from A to B (nor the converse). In this case, it's easy to verify that we can always find an observer (*i.e.* a speed v) for which A and B are simultaneous. We can also find a second observer which sees A before B and a third which sees B before A . These 3 observers can always be found for each pair of events linked by such a space-like interval. There is no objective time-order for these pairs of events. Nevertheless, in this case there is an objective spatial-order.

These 3 types of *intervals* are defining on space-time an *objective partial order* for events, a time-order for time-like intervals, and a spatial order for space-like intervals.

4 Space-time diagrams

It is still possible to graphically represent the movements of bodies or of edges of signals on a cartesian map (O, x, t) associated to the coordinates system K , or also on another one associated to K' . But this is less interesting than in classical mechanics, because lengths and durations are not objects.

But, we just saw that *intervals* are “objects”, and that they define a partial order on space-time. Then, it's interesting to graphically represent it. The corresponding representation is named *space-time diagram* (Figure 1).

In order to show that it's not a cartesian map, but only the representation of a partial order with 2 dimensions, we draw two axes labeled x and t which are not crossing each-other. The two slanted lines which are crossing each-other on the A point are representing two light rays, travelling at the velocity c in the two opposite directions of the space. They delimitate two regions named *past of A* and *future of A*. These regions are said *inside the light cones of A*.

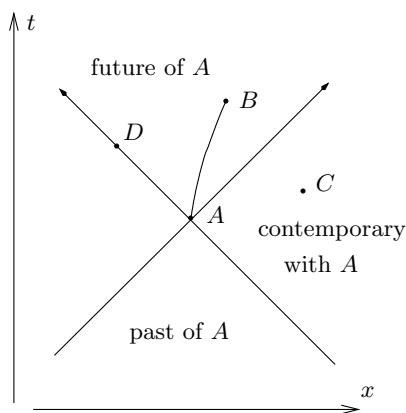


Figure 1: A space-time diagram

A point like D is at an interval zero to A because a light ray can travel from A to D . It is said that D is *on the cone*. A point like B is inside the cone of future of A because a body can travel from A to B with a speed lower than c . B is said *after* A . A point like C is said “contemporary with A ”. The interval (A, C) is space-like. C is *at right* of A (spatial order) but there is no time-order for (A, C) .

5 Computers, causality and space-time

When we define the specifications for a communication protocol between components in a distributed system in order to define a computation, we have to write logical causality relations of two types: 1) actions of changing the state of some component, 2) actions of communicating from some component to another.

When we make an electronic implementation of this computation and of these actions, we have to deal with time-orders. It is essential that logical causal relations can be implemented by *physical facts, i.e. by objective relations in space-time*. Time-like intervals, defining an objective time-order, can implement logical causality.

Then, time-like intervals will intervene in two ways for implementing causality in a distributed system:

- Successive states of one component are linked each-other by time-like intervals because the component travels at a speed lower than the light speed,
- Communications are implemented by propagations of signals which are time-like intervals.

In Figure 1, B is after A in an objective way, and this can be linked to the causality. But the duration of propagation from A to B is not measured the same by all observers. This duration is similar to what is named *delay* in the context of electronic circuits. Causality can be assured even if delays are not known, either because they are not objective or because components are moving. What is needed for implementing DI-circuits is very similar to what is possible in the frame of relativity with time-like intervals.

On the other hand, the notion of *isochronic fork* has no equivalent in the relativistic space-time. It's a *simultaneity in different places*, and it cannot be linked with something objective related to time. Nevertheless, it can be possible to implement it, but only if we limit relative distances and speeds between components, in order to limit the max value for delays. This is a very different context from the present one where we are considering only partial orders and not values of distances or durations.

6 Time-order and causal-order relations

In Figure 2, which is a space-time diagram, we represented two *propagation events* a and b , which are for example edges of signals implementing changes of Boolean variables. Here, we are using the word “event” in the same way as Bertrand Russell [11,12], *i.e.* for denoting an *interval* and not a *point* in space-time. O_a and O_b are the origins of events a and b , E_a and E_b are their extremities.

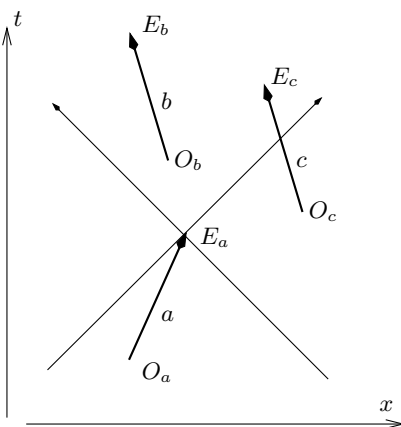


Figure 2: Time-order for propagation events

Definition: b is entirely inside the cone of future of E_a . All points of b are *after*

all points of a . We say that it defines the time order for a and b and we write this relation with the three symbols:

$$a \triangleleft b$$

This relation is *irreflexive*, *anti-symmetric* and *transitive*. We say that a is *before* b or b is *after* a .

Définition: Let us consider now the situation between events a and c when we have neither $a \triangleleft c$, nor $c \triangleleft a$. There are points of c which are with some points of a in a space-like interval. We say that a is *contemporary with* c and we write:

$$a \bowtie c$$

This relation is *symmetric*. Let us note that it is *non-transitive* for we can have $a \bowtie c$ and $c \bowtie b$ but not $a \bowtie b$ (it's the case in our Figure).

The relations \triangleleft and \bowtie are *physical* and *objective*. For this reason, we can represent them on space-time diagrams.

In order to deal with causal partial order, which differs from time-order, let us consider the Figure 3, in which E and R are components each with no spatial width, and a , b and c are communication events between E and R . Let us note that it can be possible to have $b \triangleleft c$ even if b is not the cause of c .

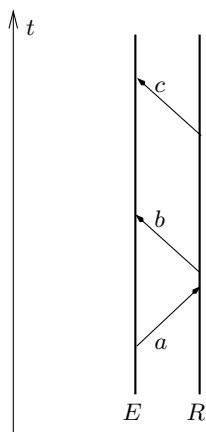


Figure 3: Partial causal order

With this Figure, we tried (in vain) to represent the case where a is the cause of b and of c , but with no relation of causality between b and c . The causal order is a logical order, distinct of the time-order. We have to choose other denotations for the causal order.

Definition: In order to express “ a is the cause of b ”, we write:

$$a \rightarrow b$$

Définition: In order to express that we have neither $a \rightarrow b$ nor $b \rightarrow a$, we write:

$$a \leftrightarrow b$$

Figure 3 is a try to represent the case where $(b \triangleleft c) \wedge (b \leftrightarrow c)$, but the causal relation cannot be drawn on a Figure.

We will consider that:

$$(a \rightarrow b) \Rightarrow (a \triangleleft b)$$

which express that if a is the cause of b , then the a event has to be arrived at its destination (has to be received by R) before the b event begins its propagation (be emitted by R). In fact, this is postulated in order to describe the link which we wish to have between *causality* and *time* in implementing logical machines. It is linked with another postulate:

$$(a \bowtie b) \Rightarrow (a \leftrightarrow b)$$

These two implications have no converse, but we have:

$$\begin{aligned} (a \triangleleft b) &\Rightarrow (a \rightarrow b) \vee (a \leftrightarrow b) \\ (a \leftrightarrow b) &\Rightarrow (a \triangleleft b) \vee (b \triangleleft a) \vee (a \bowtie b) \end{aligned}$$

On Figure 3, if the environment E is placed more at left, it would be possible to have $a \triangleleft (b \bowtie c)$, even with the same timing for R . Because when R is emitting c , it cannot know if E already received b or not (if there is no acknowledge).

Delay-Insensitivity is a way to express that several timing configurations are compatible with the causal order of the computation. A change in delays can cause a change in time-orders. We have here an example where the wanted causal order is $a \rightarrow (b \leftrightarrow c)$, but where possible time-orders are: $(a \triangleleft b \triangleleft c)$, $(a \triangleleft c \triangleleft b)$ or $(a \triangleleft (b \bowtie c))$.

7 Structures of relativistic traces

In this section, we propose a formalism for relativistic traces (which we name R-traces) for electronic circuits, imitated from [3–6].

Lower-case letters of the beginning of the alphabet (a, b, c, \dots) denote *propagation events*, i.e. *not localized points* in space-time but *propagation of signals*. Then, a letter (or symbol) as a denotes a space-time interval.

We replace the concatenation operator “;” (concatenation of symbols or of strings of symbols) by new operators for partial time-order. We choose “ \triangleleft ” and “ \bowtie ” symbols, and we add parentheses “(” and “)”.

Let us note that R-traces *don't* represent the partial order of *causality* (which we will denote with “ \rightarrow ” and “ \leftrightarrow ”). The partial causal order will be found later, for *structures* of R-traces.

R-trace structures are defined as triples $S = \langle \mathbf{i}S, \mathbf{o}S, \mathbf{t}S \rangle$, where:

- $\mathbf{i}S$ is a finite set of symbols which denotes propagations from the environment to the component (*inputs*),
- $\mathbf{o}S$ is a finite set of symbols which denotes propagations from the component to the environment (*outputs*),
- $\mathbf{t}S$ denotes a set (finite or not) of finite-length sequences (named *R-traces*) written with symbols in the set $\mathbf{a}S \cup \{\triangleleft, \bowtie, (,)\}$, where $\mathbf{a}S$ (*alphabet* of S) is $\mathbf{i}S \cup \mathbf{o}S$, and with the following syntax.

R-trace *structures* are denoted by the capitals (R, S, T, \dots) and *R-traces* are denoted by the lower-case letters (s, t, u, \dots) of the end of the alphabet.

- The empty R-trace, denoted “ ε ”, is a R-trace of the structure S : $\varepsilon \in \mathbf{t}S$.
- A symbol alone can be a R-trace, for example: $a \in \mathbf{t}S$.
- A R-trace s can be extended with a new symbol a in this way: $s \triangleleft a \in \mathbf{t}S$, where the triangle denotes that s is entirely in the cone of past of the origin of a .
- A symbol-event in a R-trace can be replaced by two symbols with no time-order between them. We will write this: $(a \bowtie b)$.
- When such a parenthesis is preceded by an order-triangle “ \triangleleft ”, each symbol in the parenthesis denotes an event which is inside the cone of future of the extremity of the preceding sequence. For example, $s \triangleleft (a \bowtie b)$ denotes that, both and independently, s is before a and before b .
- When such a parenthesis is followed by an order-triangle “ \triangleleft ”, the symbols in the parenthesis all denote events preceding in time the symbol following the parenthesis. For example, $(a \bowtie b) \triangleleft t$ denotes that t is both after a and after b .

- In a R-trace, a symbol-event can be replaced by a R-trace, put inside parentheses. For example,
 $s \triangleleft ((a \triangleleft b) \bowtie c) \triangleleft t$ is a R-trace. This R-trace denotes that s is before a and before c , and that t is after b and after c .
- As a consequence, we have the associativity:
 $(a \bowtie (b \bowtie c)) \equiv ((a \bowtie b) \bowtie c)$, even if the relation “ \bowtie ” is non-transitive, because this parenthesis with 3 symbols denotes that each of the 3 symbols is in a time-order relation only with what is outside the largest parenthesis.
- If a R-trace t is in conformity with the preceding rules, we will write $t \propto \mathbf{a}S$, and we will say: “ t is a valid R-trace formed on the alphabet $\mathbf{a}S$ ”. It’s a string written with symbols in $\mathbf{a}S \cup \{\triangleleft, \bowtie, (,)\}$.

Example of a R-trace: the following R-trace can be a part of the history of a Muller C-element:

$$(a \bowtie b) \triangleleft c \triangleleft a \triangleleft b \triangleleft c \triangleleft b \triangleleft a \triangleleft c \triangleleft (a \bowtie b) \triangleleft c \quad (5)$$

7.1 Operations on R-traces

We define the *concatenation* of two R-traces t and u as $t \triangleleft u$. This operation will be extended to concatenation of sets of R-traces. For example, $\mathbf{t}R \triangleleft \mathbf{t}S$ denotes the set of R-traces formed by concatenating one R-trace from $\mathbf{t}R$ and one R-trace from $\mathbf{t}S$.

We define the operation *star* of the R-trace t , which is written $[t]^*$, as the set constituted with traces obtained after a finite number of concatenations of t :

$$[t]^* = \{\varepsilon, t, t \triangleleft t, t \triangleleft t \triangleleft t, t \triangleleft t \triangleleft t \triangleleft t, \dots\}$$

Applied to a set of R-traces, the *star* operator will produce the set of all traces obtained after a finite number of concatenations of R-traces of the set.

We define the *prefix* operator on a R-trace: let t be a R-trace such that we have $t \propto \mathbf{a}S$, the string u is a prefix of t if $u \propto \mathbf{a}S$ and if there exists $v \propto \mathbf{a}S$ such that $t = u \triangleleft v$. We will also consider that ε and t are prefixes of t .

The **pref** operator applied to t will produce the set of all prefixes of t . Applied to a set of traces, it will produce the set of the prefixes of all the R-traces of the set.

Let t be a R-trace, and A a set of symbols. We will define $t \downarrow A$, the operation of *projection* of t on A like this:

- If $t = \varepsilon$, then $t \downarrow A = \varepsilon$

- If $t = a$ and $a \in A$, then $t \downarrow A = a$
- If $t = a$ and $a \notin A$, then $t \downarrow A = \varepsilon$
- If $t = (u)$ and $u \downarrow A \neq \varepsilon$, then $t \downarrow A = (u \downarrow A)$
- If $t = (u)$ and $u \downarrow A = \varepsilon$, then $t \downarrow A = \varepsilon$
- If $t = u \triangleleft v$ and $u \downarrow A \neq \varepsilon$ and $v \downarrow A \neq \varepsilon$, then $t \downarrow A = u \downarrow A \triangleleft v \downarrow A$
- If $t = u \triangleleft v$ and $u \downarrow A \neq \varepsilon$ and $v \downarrow A = \varepsilon$, then $t \downarrow A = u \downarrow A$
- If $t = u \triangleleft v$ and $u \downarrow A = \varepsilon$ and $v \downarrow A \neq \varepsilon$, then $t \downarrow A = v \downarrow A$
- If $t = u \triangleleft v$ and $u \downarrow A = \varepsilon$ and $v \downarrow A = \varepsilon$, then $t \downarrow A = \varepsilon$
- If $t = u \bowtie v$ and $u \downarrow A \neq \varepsilon$ and $v \downarrow A \neq \varepsilon$, then $t \downarrow A = u \downarrow A \bowtie v \downarrow A$
- If $t = u \bowtie v$ and $u \downarrow A \neq \varepsilon$ and $v \downarrow A = \varepsilon$, then $t \downarrow A = u \downarrow A$
- If $t = u \bowtie v$ and $u \downarrow A = \varepsilon$ and $v \downarrow A \neq \varepsilon$, then $t \downarrow A = v \downarrow A$
- If $t = u \bowtie v$ and $u \downarrow A = \varepsilon$ and $v \downarrow A = \varepsilon$, then $t \downarrow A = \varepsilon$

Example of such a projection: the R-trace (5), projected on $\{a, c\}$ will produce the R-trace:

$$a \triangleleft c \triangleleft a \triangleleft c \triangleleft a \triangleleft c \triangleleft a \triangleleft c$$

which is a repeat of $a \triangleleft c$, and we can also write:

$$[a \triangleleft c]^4$$

7.2 Operations on R-trace structures

Concatenation of R-trace structures:

$$R; S = \langle \mathbf{i}R \cup \mathbf{i}S, \mathbf{o}R \cup \mathbf{o}S, \mathbf{t}R \triangleleft \mathbf{t}S \rangle$$

Union of R-trace structures:

$$R|S = \langle \mathbf{i}R \cup \mathbf{i}S, \mathbf{o}R \cup \mathbf{o}S, \mathbf{t}R \cup \mathbf{t}S \rangle$$

Star of a R-trace structure:

$$*[R] = \langle \mathbf{i}R, \mathbf{o}R, [\mathbf{t}R]^* \rangle$$

Prefix of a R-trace structure:

$$\mathbf{pref}R = \langle \mathbf{i}R, \mathbf{o}R, \mathbf{pref}[\mathbf{t}R] \rangle$$

(If $\mathbf{pref}R = R$, we will say that R is “prefix-closed”.)

Projection of a R-trace structure (on the set A):

$$R \downarrow A = \langle \mathbf{i}R \cap A, \mathbf{o}R \cap A, \{t \downarrow A \mid t \in \mathbf{t}R\} \rangle$$

Weave of R-trace structures:

$$R||S = \langle \mathbf{i}R \cup \mathbf{i}S, \mathbf{o}R \cup \mathbf{o}S, \{t \bowtie (\mathbf{a}R \cup \mathbf{a}S) \mid t \downarrow \mathbf{a}R \in \mathbf{t}R \wedge t \downarrow \mathbf{a}S \in \mathbf{t}S\} \rangle$$

Example: if $R = \langle \{a\}, \{c\}, \{a \triangleleft c\} \rangle$, and $S = \langle \{b\}, \{c\}, \{b \triangleleft c\} \rangle$, then:

$$R||S = \langle \{a, b\}, \{c\}, \{(a \bowtie b) \triangleleft c, a \triangleleft b \triangleleft c, b \triangleleft a \triangleleft c\} \rangle$$

7.3 Component and environment

A R-trace structure S is said to be *non-empty* if $\mathbf{t}S \neq \emptyset$.

The dialog between a *component* and its *environment* is specified by a R-trace structure S (named *command*), which is non-empty, and such that $\mathbf{i}S \cap \mathbf{o}S = \emptyset$. Symbols in $\mathbf{i}S$ denote signals emitted by the environment and received by the component. Symbols in $\mathbf{o}S$ denote signals emitted by the component and received by the environment.

We make no distinction between the specification of the component on one hand and the specification of the environment on the other hand. We only specify the *pair* component-environment, defined by a dialog, described by only one R-trace structure.

Example: the dialog between a Muller C-element and its environment is defined by:

$$\begin{aligned} \mathbf{pref} * & \quad < \{a, b\}, \{c\}, \\ & \quad \{(a \bowtie b) \triangleleft c, a \triangleleft b \triangleleft c, b \triangleleft a \triangleleft c\} > \end{aligned} \quad (6)$$

If we define the following *atomic commands* as particular R-trace structures:

$$\begin{aligned} a? & = < \{a\}, \emptyset, \{a\} > \\ b? & = < \{b\}, \emptyset, \{b\} > \\ c! & = < \emptyset, \{c\}, \{c\} > \end{aligned}$$

Then, we can rewrite (6):

$$\mathbf{pref} * [(a? || b?); c!]$$

Note that, as a consequence of the definition of the *projection* of R-traces, the operation $a? || b?$ denotes the 3 possible time relations $a \bowtie b$, $a \triangleleft b$, and $b \triangleleft a$, and then is the same as the relation $a? \leftrightarrow b?$. The writing of R-trace *structures* with “||” and “;” corresponds to the *partial causal order* that we previously wrote with “ \leftrightarrow ” and “ \rightarrow ”. We can also write, for a Muller C-element:

$$\mathbf{pref} * [(a? \leftrightarrow b?) \rightarrow c!]$$

The notation with “||” and “;”, for the DI-component, has the advantage to be compatible with the classical notation (as for example in [6]), but we have to note that *it now has a different meaning*: R-trace structures are a description of a *partial order*, the order relation of *causality for events which are propagations* in relativistic space-time.

Remark: *propagation delays are already included in this description*. It was not the case in the classical description.

8 Relativistic Udding’s rules

The “Udding’s rules” [3] are a formalization of the intuitive idea of DI-systems, which was previously introduced by the metaphor of FRW (Foam Rubber Wrapper) [1].

The FRW metaphor shows how the change of propagation delays can change the time-order of signals, and suggest how component specifications can be invariant under these changes in time-ordering.

Udding translated this intuitive notion of FRW and write it as logical constraints imposed on the syntax of trace-structures which specify components, so that these specifications be invariant under the changes of time-ordering caused by the changes of propagation delays.

In the classical frame of traces [4], each trace denote a total time-order. When there is a change of time-order due to the change of propagation delays, it is said that: “the order at the environment frontier is not the same as the order at the component frontier”.

But this way of thinking is no more possible in the relativistic frame, because there is no “simultaneity in different places”, and this prevents us to find a meaning to the words “the time-order at the frontier”. We cannot define a time-order at the frontier if this frontier has a width, in other words if the frontier is not a single point in space. And it is not possible to continue to use traces if traces denote a total order. For this reason, we introduced R-traces which denote a partial order. Another important semantic change introduced with R-traces is that they denote, not an order in a single place in space, but an order for *propagations*. This semantic change is linked with the change of meaning of the word *event* which is now an *interval* (time-like).

If we adapt Udding’s rules to R-traces, they still continue to be constraints on the syntax of the specifications of components. But now R-traces are for the specification of a *single* object, that is a *pair* component-environment. And R-traces are the expression of the dialog inside this pair, more precisely the causality relations for propagations. Because these causality relations will be compatible with the time-order in relativistic space-time, this will make DI-systems compatible, not only with a change in place&route on a chip at the design stage, but also with *movement of components during a computation*.

In this section, we propose a new writing for Udding’s rules and we use space-time diagrams to illustrate the discussion. In these diagrams, the component and the environment will be each a point in space represented by a near vertical line. Propagation events will be represented by more slanted arrows because their speed is higher (as in Figure 3).

We choose E and R for the names of respectively the environment and the component. R will also be the name of the R-trace structure which describes the specification of the dialog in the pair. We will discuss R_0 rule after R_1 rule because we consider R_0 as a special case of R_1 . (We refer to the names of the rules in [3].)

8.1 R_1 rule

This rule is for 2 successive symbols of the same type (2 inputs or 2 outputs). Classically, in a context of total ordering, the 2 relative orders have to be present and with the same consequences:

$$\forall s, t \in \mathbf{tR}, (a, b \in \mathbf{iR}) \vee (a, b \in \mathbf{oR}) ; \\ \text{sabt} \in \mathbf{tR} \Leftrightarrow \text{sbat} \in \mathbf{tR}$$

Let us try to discuss this rule on Figure 4. On the left, we draw the case of an a signal which has a lower speed than b signal. On the right, we tried to draw the case of a b signal which is so late that it is inside the cone of future of a (time order $a \triangleleft b$). In the classical writing, at left the order is not the same for E and R , at right the order is the same. In the relativistic writing, there is no causal order between a and b .

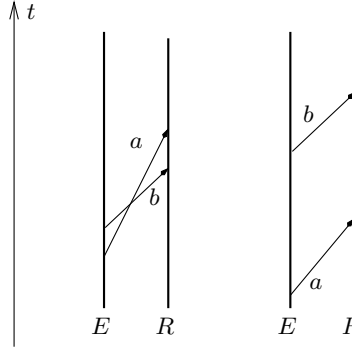


Figure 4: About R_1 rule (two inputs case)

For the new writing of R_1 rule, we propose:

$$\forall s, t \in \mathbf{tR}, (a, b \in \mathbf{iR}) \vee (a, b \in \mathbf{oR}) ; \\ (\quad s \triangleleft a \triangleleft b \triangleleft t \in \mathbf{tR} \quad \vee \quad s \triangleleft (a \bowtie b) \triangleleft t \in \mathbf{tR} \quad) \\ \Rightarrow \quad s \triangleleft (a \leftrightarrow b) \triangleleft t \in \mathbf{tR}$$

where $(a \leftrightarrow b)$ is a condensed writing for denoting the presence of all the 3 time-orders $a \triangleleft b$, $(a \bowtie b)$ and $b \triangleleft a$.

It is the case we discussed in section 6. If there is no signal in order to inform E that a is arrived before the departure of b (acknowledge), then there is no possibility for a consensus between E and R on the order of a and b . But it's acceptable if the order has no consequence on the subsequent computation.

8.2 R_0 rule

It's not possible to transmit on a same channel (same symbol) 2 events with no signal between them (on another channel, of the same type or of the opposite type). Because there could be a confusion on the number of events or the possibility of loosing events. Classically:

$$\forall s \in \mathbf{tR}, a \in \mathbf{aR} ; \quad saa \notin \mathbf{tR}$$

R_1 rule forces us to consider saa as $s \triangleleft (a \leftrightarrow a)$. What is the meaning of this, in terms of causality?

If the two a are with no causal relation between them, then they are independently caused by the preceding s . May be repeating a can have a meaning, but in this case the second a has to come after an acknowledge of the first, if there is a possibility of physical confusion.

For the new writing of R_0 rule, we propose:

$$\forall s \in \mathbf{tR}, a \in \mathbf{aR} ; \quad s \triangleleft (a \leftrightarrow a) \notin \mathbf{tR}$$

(None of the 3 time-orders can be present.)

8.3 R_2 rule

This rule is for 2 symbols of opposite type (one input and one output). The two events can be causally ordered, and generally the order has a meaning. There is no ambiguity. In our writing, it's the case of $a \rightarrow b$ implemented by $a \triangleleft b$.

But if the two events are not ordered, or if they are ordered but with the possibility of the two opposite orders, then the two orders have to cause the same consequences, because the order can be changed by the propagations. Classically:

$$\begin{aligned} & \forall s, t \in \mathbf{tR}, (a \in \mathbf{iR} \wedge b \in \mathbf{oR}) \vee (a \in \mathbf{oR} \wedge b \in \mathbf{iR}) ; \\ & (sab \in \mathbf{tR} \wedge sba \in \mathbf{tR}) \Rightarrow (sabt \in \mathbf{tR} \Leftrightarrow sbat \in \mathbf{tR}) \end{aligned}$$

We will discuss this rule with the help of Figure 5. On the right, a and b signals are clearly not ordered (we have $a \bowtie b$), but on the left part of the Figure which represents $a \triangleleft b$ we cannot see if there is a causal order ($a \rightarrow b$) or if it is a special case of the non-ordered case ($a \leftrightarrow b$).

The goal of this R_2 rule is to force us, if we want to implement ($a \leftrightarrow b$), to make the case $a \triangleleft b$ have the same consequences than $b \triangleleft a$ or $a \bowtie b$. For the new writing of R_2 rule, we propose:

$$\forall s, t \in \mathbf{tR}, (a \in \mathbf{iR} \wedge b \in \mathbf{oR}) \vee (a \in \mathbf{oR} \wedge b \in \mathbf{iR}) ;$$

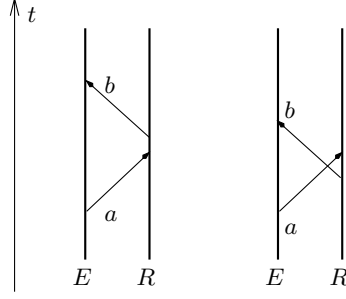


Figure 5: About R_2 rule

$$\begin{aligned}
 & \left[(s \triangleleft a \triangleleft b \in \mathbf{t}R \quad \wedge \quad s \triangleleft b \triangleleft a \in \mathbf{t}R) \right. \\
 & \quad \left. \vee \quad s \triangleleft (a \bowtie b) \in \mathbf{t}R \right] \\
 & \quad \Rightarrow \\
 & \left[(s \triangleleft a \triangleleft b \triangleleft t \in \mathbf{t}R \quad \vee \quad s \triangleleft (a \bowtie b) \triangleleft t \in \mathbf{t}R) \right. \\
 & \quad \left. \Rightarrow \quad s \triangleleft (a \leftrightarrow b) \triangleleft t \in \mathbf{t}R \right]
 \end{aligned}$$

8.4 R'_2 rule

In some cases, R_2 rule is too constraining. In order to explain this, we go back to Figure 5 (which is about R_2). Let us suppose that the two drawings in this Figure represent the same non-ordered case $s \triangleleft (a \leftrightarrow b)$. In the left, we can see that R has more information than E about the time-ordering of a and b , and also R knows that E sees the same order than R . But E cannot make such an inference for presuming the time-order seen by R . For example, E can believe that it is $(a \bowtie b)$ because it cannot know the propagation delays.

In this case, we can authorize only R to take a specific decision. For example (Figure 6) R can emit d in the case where a and b are not causally ordered but where we have $a \triangleleft b$.

In the case of c which is of the same type of a , we have to impose c exists whatever the order seen by R , because c is emitted by E which has less information than R . This is done by R'_2 rule, classically:

$$\begin{aligned}
 & \forall s, t \in \mathbf{t}R, (a, c \in \mathbf{i}R \wedge b \in \mathbf{o}R) \\
 & \quad \vee \quad (a, c \in \mathbf{o}R \wedge b \in \mathbf{i}R); \\
 & (sabt c \in \mathbf{t}R \wedge sbat \in \mathbf{t}R) \Rightarrow sbatc \in \mathbf{t}R
 \end{aligned}$$

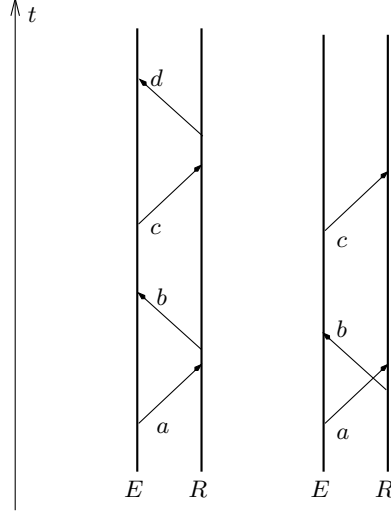


Figure 6: About R'_2 rule

For a new writing, we propose:

$$\begin{aligned}
& \forall s, t \in \mathbf{tR}, (a, c \in \mathbf{iR} \wedge b \in \mathbf{oR}) \\
& \quad \vee (a, c \in \mathbf{oR} \wedge b \in \mathbf{iR}); \\
& \left[(s \triangleleft a \triangleleft b \triangleleft t \triangleleft c \in \mathbf{tR} \wedge s \triangleleft b \triangleleft a \triangleleft t \in \mathbf{tR}) \right. \\
& \quad \left. \vee s \triangleleft (a \bowtie b) \triangleleft t \triangleleft c \in \mathbf{tR} \right] \\
& \Rightarrow s \triangleleft (a \leftrightarrow b) \triangleleft t \triangleleft c \in \mathbf{tR}
\end{aligned}$$

8.5 Group of R_3 rules

If a is an input and b an output, and if there is no causal order between them ($a \leftrightarrow b$), is it possible for the arrival of a to prevent the departure of b in the case ($a \triangleleft b$)? If we don't want this possibility to exist, we have to exclude it by a rule. Classically, this rule is:

$$sa \in \mathbf{tR} \wedge sb \in \mathbf{tR} \Rightarrow sab \in \mathbf{tR} \quad (7)$$

If we extend this to the combinations with symbols of the same type, these rules can serve to a classification of components: 1) Two input symbols can be mutually exclusive if the environment has to make a choice (case of inputs of memory components), 2) Two output symbols can be mutually exclusive if the component has to make a choice (case of arbitration devices).

We have to write a rule of the R_3 group if we want to forbid a choice. For example, if we want to exclude memories, we can write: $\forall s \in \mathbf{tR}, a \in \mathbf{iR} \wedge b \in \mathbf{iR}$; then (7).

In the relativistic frame, we have to replace the second part of (7) by: $s \triangleleft (a \leftrightarrow b) \in \mathbf{tR}$. For a new writing of (7), we propose:

$$\left[s \triangleleft (a \bowtie b) \in \mathbf{tR} \quad \vee \quad (s \triangleleft a \in \mathbf{tR} \wedge s \triangleleft b \in \mathbf{tR}) \right] \\ \Rightarrow s \triangleleft (a \leftrightarrow b) \in \mathbf{tR} \quad (8)$$

The 3 following rules differ only by the combination of types of symbols, after which we add (8).

R'_3 rule. This first rule defines the more constrained class of components. Neither the environment, nor the component can make a choice (Muller-C and Fork are in this class).

$$\forall s \in \mathbf{tR}, \quad a \neq b \in \mathbf{aR}; \quad \text{then (8)}.$$

R''_3 rule. This second rule accepts memories. a and b are not two inputs.

$$\forall s \in \mathbf{tR}, \quad a \neq b \in \mathbf{aR} \quad a \notin \mathbf{iR} \vee b \notin \mathbf{iR}; \quad \text{then (8)}.$$

R'''_3 rule. Not only 2 inputs can be mutually exclusive but also 2 outputs (arbiters).

$$\forall s \in \mathbf{tR}, (a \in \mathbf{iR} \wedge b \in \mathbf{oR}) \vee (a \in \mathbf{oR} \wedge b \in \mathbf{iR}); \text{ then (8)}.$$

Definition of relativistic DI-components. Imitating Udding, we can now define the class of relativistic DI-components as the specifications which obey to R_0 , R_1 , R'_2 and R'''_3 .

9 Generalization of classical rules

We have to show how the new symbols a , b , \triangleleft , \bowtie can be transformed when we restrict them to the classical space-time. We need a type of “projection” which transforms R-traces in traces. A R-trace denotes a *partial* time-order for *propagations*, but a classical trace denotes a *total* time-order for *points* of space-time (input or output port of a component). The same symbol a is not denoting the same object in a R-trace as in a trace. However, we can define such a “projection”:

- $a \triangleleft b \in \mathbf{tR}$ will be projected to: $ab \in \mathbf{tR}$.
- $(a \bowtie b) \in \mathbf{tR}$ will be projected to:
 $ab \in \mathbf{tR} \wedge ba \in \mathbf{tR}$.

- $a \leftrightarrow b \in \mathbf{tR}$ will be projected to: $ab \in \mathbf{tR} \wedge ba \in \mathbf{tR}$.

In making this, we project the absence of order and replace it by the presence of the two opposite orders, but it's imposed by the syntax of classical traces. Graphically, on space-time diagrams, we transform one R-trace in two classical traces by intersecting slanted arrows separately with the two vertical lines (one intersection at the component and one at the environment). We obtain two traces which have to be present together in the specification of the component.

We can note that all the relativistic Udding's rules have at their end something of the form:

$$\Rightarrow (a \leftrightarrow b) \in \mathbf{tR}$$

which is projected to something of the form:

$$\Rightarrow ab \in \mathbf{tR} \wedge ba \in \mathbf{tR}$$

In other words, all these rules impose the absence of causal order, and their restrictions to classical space-time impose the presence of the two opposite orders.

We can easily verify that the new rules are projected to classical rules.

In fact, we can note that it's precisely because the Udding rules have the property to impose the presence of the two opposite orders that their restriction from relativistic case to classical case is possible. And this is the reason why the notion of DI-systems can be generalized to relativistic space-time with its partial time-order.

10 Conclusion

We proposed a new formalism for traces (named R-traces) allowing to describe the dialog between a DI component and its environment in the relativistic space-time, and also of course in its restriction to the classical space and time. There is no additional complexity in doing so, and it is also adapted to moving components. A R-trace structure is a writing of the specification of a component-environment pair as the list of partial time-orders allowed for the communication events in this dialog.

In the frame of this formalism, we proposed a new writing for Udding's rules, which is not complex and which can make clearer the meaning of these rules in terms of causality.

With the example of the specification of the Muller-C element, which is (with the Fork) the sole primitive component of [5, 6] implicated in a concurrency, we showed that this new formalism can be written with the same form as the classical one for the specifications of DI components. But in this frame, the previous writing has

a new meaning: R-trace structures are the order relations of causality for events, including propagation delays.

In this frame, a distributed system is no more a composition of components which communicate, but it becomes a composition of component-environment pairs, no one of the two members of a pair being describable for its own. It's a composition of *interfaces*, if we name *interface* the dialog between the two members of a pair. This leads to exciting new questions about the way to deal with such systems...

References

- [1] Molnar, C.E., Fang, T.P., Rosenberger, F.U.: Synthesis of Delay-insensitive Modules. In: 1985 Chapel Hill Conference on VLSI, pp. 67–86. (1985).
- [2] Udding, J.T.: Classification and Composition of Delay-Insensitive Circuits. PhD Thesis, Eindhoven University of Technology, Netherlands, (Sept 1984).
- [3] Udding, J.T.: A Formal Model for Defining and Classifying Delay-Insensitive Circuits and Systems. Distributed Computing 1, 197–204, Springer-Verlag (1986).
- [4] Van de Snepscheut, J.L.A.: Trace Theory and VLSI Design. LNCS 200, Springer-Verlag (1985).
- [5] Ebergen, J.C.: Translating Programs into Delay-Insensitive Circuits. PhD thesis, Technische Universiteit Eindhoven, (Oct 1987).
- [6] Ebergen, J.C.: A Formal Approach to Designing Delay-Insensitive Circuits. Distributed Computing 5, 107–119, Springer-Verlag (1991).
- [7] Verhoeff, T.: A Theory of Delay-Insensitive Systems. Thesis, Eindhoven University of Technology (1994).
- [8] Einstein, A.: Zur Elektrodynamik bewegter Körper. Annalen der Physik 17, 891–921, (1905).
- [9] Einstein, A.: The collected papers of Albert Einstein, Vol. 2. English translation, Princeton University Press (1989).
- [10] Russell, B.: ABC of Relativity. Routledge (1997). First Ed. George Allen & Unwin (1925).

[11] Russell, B.: Philosophical Consequences of Relativity. Britannica 13th Ed. (1926).

[12] Russell, B.: The Analysis of Matter. Routledge (1992). First Ed. (1927).