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Enhancing Positioning Accuracy Through RSS Based Ranging And Weighted Least Square Approximation

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Abstract— In this paper, localization based on Received Signal Strength (RSS) is investigated assuming a path loss log normal shadowing model. RSS-based estimation schemes of ranges are investigated; three different schemes are studied: Mean, median and mode. Estimation of position is performed using weighted least square approximation. We show that the positioning accuracy depends on the used estimator of ranges from RSS observables. We suggest that typical median estimator must be replaced by maximum likelihood estimator (mode) to enhance the positioning accuracy. Monte Carlo simulations show that the estimation scheme based on the mode estimator performs better than those based on the median or the mean estimator; and that the use of Weighted Least square approximation enhances the accuracy comparing to typical unweighted least square approximation.

Index Terms—Localization, RSS, Location Estimation, Weighted least square, ranging, Path Loss, Log Normal Shadowing.

I. INTRODUCTION

Nowadays, Location Based Services (LBSs) are more and more required by people and industries. Security is the main motivation for civilian mobile position location whose implementation is nowadays mandatory for the emergency calls. Besides security, the second leading application for wireless localization is intelligent transportation systems (ITSs). Personal tracking, navigation assistance and position-dependent billing are also new LBSs in expansion [1]. Furthermore, the location information is not only valuable for itself to provide new services but also to improve cellular communication systems at various levels. This is the scope of the FP7 WHERE project [2].

Location methods based on Received Signal Strength (RSS) have an important advantage compared with others methods since RSS is usually available whatever is the Radio Access Network (RAN) [3]. Nevertheless, the precision and accuracy of RSS is different from one RAN to another. The challenge here is to merge hybrid RSSs characterized by different accuracies and coming from different systems in order to enhance the position accuracy. In the following, hybrid RSS fusion relates to an algorithm which make use of RSS observables coming from different RANs (Cellular, WLAN, UWB, etc). This is the typical case in 4G networks where nodes with different technological platforms are integrated

and in which the MS may be connected conjointly to cellular Base Station (BS) and wireless Access Point (AP) [4].

AN	Anchor Node
LS	Least Square
ML	Maximum Likelihood
MS	Mobile Station
RSS	Received Signal Strength
d	Distance between transmitter and receiver (m)
d_0	Reference distance generally equal to 1 meter
L	Pathloss at distance d (dB)
L_0	Pathloss at distance d_0 (dB)
n_p	Pathloss exponent
λ	Wavelength (m)
σ_{sh}	Standard Deviation of shadowing (dB)
$\mathbf{x} = (x, y)$	Coordinates of the MS
$\mathbf{x}_k = (x_k, y_k)$	Coordinates of the k^{th} AN
l	Length of the simulated area
N_{Trial}	Number of Trials in Monte Carlo simulations

TABLE I: List of different used abbreviations and symbols.

Historically, RSS can be used in either fingerprinting or trilateration. Fingerprinting with RSS refers to the type of algorithms that first collect RSS fingerprints of a scene and then estimate the location of the MS by matching on-line measurements with the closest location fingerprints [5]. RSS lateration consists in estimating the ranges from collected RSSs assuming a path loss model and then computing position using these different estimated ranges. Generally, to estimate range from RSS the median estimator is used [6], [7], [8], [9]. This estimator do not require the knowledge of shadowing which affects the RSS measurements, and it is useful when no information about shadowing is available. Nevertheless, in the case of a non Gaussian distribution, this estimator performs worse than the Maximum Likelihood estimator (ML) given by the mode of the distribution.

In the present study, we investigate different schemes of positioning based on RSS. Assuming a log normal shadowing model for path loss, three different estimators of ranges from RSS observables are investigated: The mean, the median and the mode estimators. Then, Weighted Least Square approximation is applied on estimated ranges in order to estimate position. These different estimators are evaluated by Monte Carlo simulations and suggest that mode estimator is the best estimator and that the Weighted Least Square

approximation may enhance positioning accuracy.

The rest of the paper is organized as follows. Section II investigates the log normal shadowing model and presents the different radio propagation parameters which may affect the positioning accuracy. Section III presents the three estimation schemes of ranges based on RSS observables. Then, section IV presents the mathematical formulation of the Weighted Least Square approximation on the ranges. In section V, the performances of each estimation scheme are evaluated and discussed using Monte Carlo simulations. Finally, our concluding remarks are given in section VI.

In order to simplify the lecture of this paper, a list of abbreviations and symbols that are used in the paper is given in Table I.

II. LOG NORMAL SHADOWING PATH LOSS MODEL

The simple analysis often used in coexistence studies limits the propagation characteristics to the large scale of the signal at given distances (pathloss). In mathematical terms, the mean received power (around which there will still be shadowing and multipath) will vary with distance with an exponential law. The total pathloss at a distance, d , will then be L , often modeled as [10]:

$$L = L_0 + 10n_p \log\left(\frac{d}{d_0}\right) \quad (1)$$

d_0 , d , n_p and L_0 are defined in Table I. L_0 is given by:

$$L_0 = 20 \log\left(\frac{4\pi d_0}{\lambda}\right) \quad (2)$$

In fact this expression of L represent only the mean loss of the power. The measured loss varies about this mean according to a zero-mean Gaussian random variable, $X_{\sigma_{sh}}$, with standard deviation σ_{sh} . Shadowing is caused by obstacles between the transmitter and receiver that attenuate signal power through absorption, reflection, scattering, and diffraction. The complete path loss equation expressed in dB is then given by:

$$L = L_0 + 10n_p \log\left(\frac{d}{d_0}\right) + X_{\sigma_{sh}} \quad (3)$$

This model can be used for both indoor and outdoor environments. For each environment or/and radio link, a characteristic value of each parameter, n_p and σ_{sh} , is used. These values can be determined by calibration via measurement companions. Furthermore, the frequency and the bandwidth affect these parameters. The most common values of n_p are shown by Table II for different types of environments.

Type of environment	Path loss exponent n_p
Free Space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building LOS	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factory	2 to 3

TABLE II: Path Loss Exponent for different environments [4].

The log normal shadowing model is very interesting for localization because it defines a linear relation between RSS and the logarithm of the distance between MS and AN. Nevertheless, the precision of estimated distance decreases as the separation between MS and AN increases. As a rule of thumb, if $n_p = 2$ then RSS drops by 6 dB every time distance doubles. This sub-linear attenuation rate means that the difference in RSS between 1 m and 2 m is similar to the difference between 10 m and 20 m : exactly 6 dB (Fig. 1). Taking this into account, a constant level of noise can result in ever increasing error when RSS is used to estimate distance; if RSS noise is sufficient that we cannot tell the difference between 1 and 1.5 m , we also cannot tell the difference between 10 m and 15 m . As shown in Fig. 1, changes in RSS due to distance become small relative to noise, even if the level of noise remains the same over distance [11].

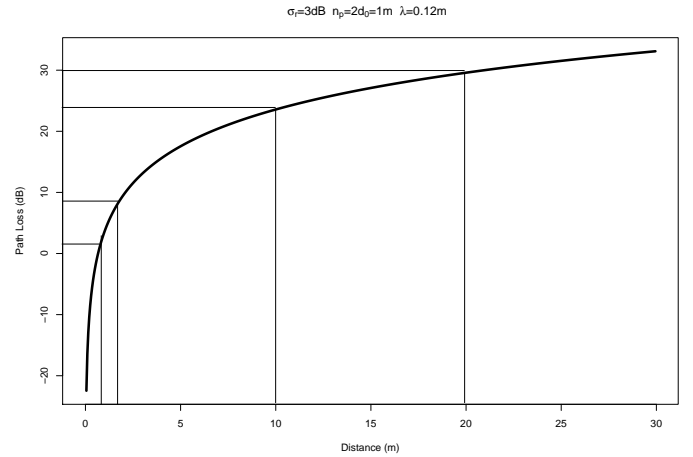


Fig. 1: Variation of path loss with respect to distance using Log Normal Shadowing model: Error increases over distance depending on both noise and attenuation rate. As the path loss flattens out, differences in RSS become small relative to noise level.

III. RSS-BASED RANGING

In this section, we investigate the RSS-based ranging schemes which consist in the estimation of ranges from RSS observables. Let's consider the log normal shadowing described by the equation (3) as the used path loss model where we assume that the shadowing term $X_{\sigma_{sh}}$ is zero-mean Gaussian :

$$X_{\sigma_{sh}} \sim \mathcal{N}(0, \sigma_{sh}^2) \quad (4)$$

From (3) and (4) we derive the fact that the distance d follows a Log-Normal distribution :

$$p_d(d, L) = \frac{1}{\sqrt{2\pi}dS} e^{-\frac{(\ln d - M)^2}{2S^2}} \quad (5)$$

where

$$S = \frac{\sigma_{sh} \ln 10}{10n_p} \quad (6)$$

$$M = \frac{(L - L_0) \ln 10}{10n_p} + \ln d_0 \quad (7)$$

As d follows a Log-Normal distribution, the mean, median and mode of estimated distance \hat{d} are given respectively by [12]:

$$\hat{d}_{LS} = e^{M + \frac{S^2}{2}} \quad (8)$$

$$\hat{d}_{median} = e^M \quad (9)$$

$$\hat{d}_{ML} = e^{M - S^2} \quad (10)$$

From equations (8) to (10), one can notice that the only estimator that does not consider the knowledge of shadowing, given by the term S , is the median. Thus, this estimator may be practical when no information about shadowing is available. Once the MS get this knowledge, the best estimator will be the mode which is the ML estimator. The mean estimator is not a good choice as it over estimates the distance, and it is very inaccurate especially for strong values of S .

To better evaluate the performances of these different estimators, we derived for each estimator its variance. we obtained the estimated variances of mean, median, and mode estimators of distance are, respectively, given by:

$$\hat{\sigma}_{LS}^2 = \hat{d}_{LS}^2 e^{2S^2} (e^{S^2} - 1) = e^{2M+3S^2} (e^{S^2} - 1) \quad (11)$$

$$\hat{\sigma}_{median}^2 = \hat{d}_{median}^2 e^{S^2} (e^{S^2} - 1) = e^{2M+S^2} (e^{S^2} - 1) \quad (12)$$

$$\hat{\sigma}_{ML}^2 = \hat{d}_{ML}^2 (1 - e^{-S^2}) = e^{2M-2S^2} (1 - e^{-S^2}) \quad (13)$$

IV. RSS-BASED POSITIONING

Once the MS gets the necessary amount of RSS observables (3 at least in 2D scenario), it can perform the first step by estimating the different ranges $(\hat{d}_k)_{k=1,\dots,K}$ with respect to the K discovered AN in the scene. These ranges can be estimated using one of the three estimators given by (8), (9) or (10). Thus, we obtain the system :

$$\begin{aligned} (x - x_1)^2 + (y - y_1)^2 &= \hat{d}_1^2 \\ &\dots \\ (x - x_K)^2 + (y - y_K)^2 &= \hat{d}_K^2 \end{aligned} \quad (14)$$

Subtracting the first one ($k = 1$) from others equations of (14) results in

$$2 \begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ \dots & \dots \\ x_K - x_1 & y_K - y_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} h'_2 + \hat{d}_1^2 - \hat{d}_2^2 \\ \dots \\ h'_K + \hat{d}_1^2 - \hat{d}_K^2 \end{bmatrix} \quad (15)$$

where $h'_k = x_k^2 - x_1^2 + y_k^2 - y_1^2$ for k in $(2, \dots, K)$.

The least square solution is then given by [6]:

$$\mathbf{x} = \frac{1}{2} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{h} \quad (16)$$

where

$$\mathbf{A} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ \dots & \dots \\ x_K - x_1 & y_K - y_1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} \quad (17)$$

$$\mathbf{h} = \begin{bmatrix} h'_2 + \hat{d}_1^2 - \hat{d}_2^2 \\ \dots \\ h'_K + \hat{d}_1^2 - \hat{d}_K^2 \end{bmatrix} \quad (18)$$

In order to enhance the performances of LS regression, we introduce the matrix of covariance of estimated ranges. Three covariance matrices are then defined depending on used ranges estimator. For the mean, median and mode estimator, respectively, this covariance matrix is given by:

$$\mathbf{R}_{LS} = \text{diag}((\hat{\sigma}_{LS,k}^2)_{k=2..K}) \quad (19)$$

$$\mathbf{R}_{median} = \text{diag}((\hat{\sigma}_{median,k}^2)_{k=2..K}) \quad (20)$$

$$\mathbf{R}_{ML} = \text{diag}((\hat{\sigma}_{ML,k}^2)_{k=2..K}) \quad (21)$$

The weighted least square solution is then given by [6]:

$$\mathbf{x} = \frac{1}{2} (\mathbf{A}^T \mathbf{R}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{R}^{-1} \mathbf{h} \quad (22)$$

where R can be R_{LS} , R_{median} , or R_{ML} .

V. SIMULATIONS RESULTS AND DISCUSSIONS

In this section, we evaluate the performances of the set of studied estimation schemes described in section III and IV through Monte Carlo simulations. The different steps of the simulation are the following:

- 1) K random ANs and one targeted MS are uniformly drawn in an area of $l \times l m^2$.
- 2) Different path losses ($L - L_0$) are computed for each link k between the MS and the k^{th} AN. For each link, log normal shadowing model is applied with appropriate n_p , λ and σ_{sh} . Table III shows the used parameters for indoor and outdoor scenarios respectively.
- 3) The different estimation schemes are then evaluated for three different scenarios:

- Indoor.
- Outdoor.
- Indoor/Outdoor.

Parameters	Indoor	Outdoor
n_p	1.6 to 1.8	2 to 4.0
$\lambda(m)$	0.12	0.333
$\sigma_{sh} (dB)$	2 to 5	2 to 5
$l(m)$	15	1000

TABLE III: List of radio parameters used in simulations for both indoor and outdoor scenarios.

All simulations have been done with a number of trials equal to $N_{Trial} = 300$. For each studied scenario, the correspondent figure (Fig. 2 to Fig. 4 respectively) compares the cumulative density functions of the studied estimation schemes with respect to the positioning error in order to suggest the best estimation scheme. For each estimator of ranges (mode, median and mean), we carried simulations using respectively typical unweighted Least Square and

weighted Least Square.

The Fig. 2 and Fig. 3 are obtained respectively for indoor and outdoor scenarios with the parameters described in Table III. These figures show that the estimation schemes based on the mode estimator for ranges performs better than those usually used based on median and mean estimators. Moreover, these figures show obviously that the Weighted Least Square approximation performs better than typical unweighted Least Square approximation. Nevertheless, the Weighted Least Square approximation is more complex and consumes more resources since it performs the estimation of different variances before estimating the position of the MS.

In order to compare the performances of these different estimation schemes in the case of hybrid RSS fusion, we carried simulations in a typical 4G scenario where the MS can be connected conjointly to cellular BSs and wireless APs (IEEE 802.15.4 or IEEE 802.11 for example). The Fig. 4 shows the performances of different estimation schemes for this scenario with $l = 1000m$. This figure is obtained by reproducing the same simulations conditions assumed in Fig. 3 but with adding two indoor links into a square of $l = 15m$. The position of MS is chosen randomly in the square $15 \times 15m^2$. This is done by respecting the different assumed parameters (n_p , λ and σ_{sh}) for indoor scene given by Table III for each additional link. This figure emphasizes the expected conclusions and shows that the proposed estimation scheme based on the mode estimator of ranges and the Weighted Least Square approximation enhances the performances of hybrid RSS-based localization compared with Typical Least Square approximation and typical used median estimator for ranges.

Comparison between Fig. 3 and Fig. 4 shows that the enhancement performed by the use of Weighted Least Square approximation, after adding indoor links, is major than the enhancements shown in Fig. 2 and Fig. 3. These observations believe that the proposed estimation scheme based on the mode estimator of ranges is more reliable when hybrid RANs are used. Furthermore in this type of scenarios, estimators may experience short and long range at the same time. In this case, the precisions of estimated distances from RSS observables can be very different as explained in Fig. 1. We believe that the use of Weighted Least Square approximation merges more smartly these RSS observables because it takes in account their variances.

Nevertheless, the performances of RSS based localization can be enhanced when adding heterogeneous observables like the Time of Arrival (ToA). The approach consists then to fuse RSS with ToA based ranges to estimate position and this fusion enhances the localization accuracy especially when the ToA observables are precisely estimated (in UWB standard for example) [13]. Moreover, without dealing with ranges, we can estimate position from RSS directly when assuming Log Normal Shadowing Model for path loss and this approach may enhance the RSS based localization performances [14].

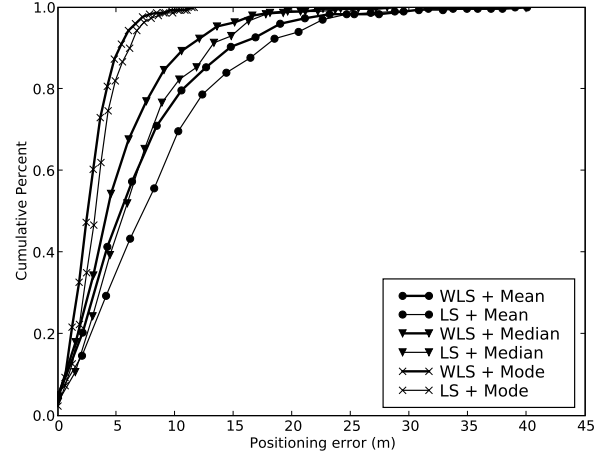


Fig. 2: Compared cumulative density function of four studied estimators for indoor scenario - $l = 15m$, $K = 5$.

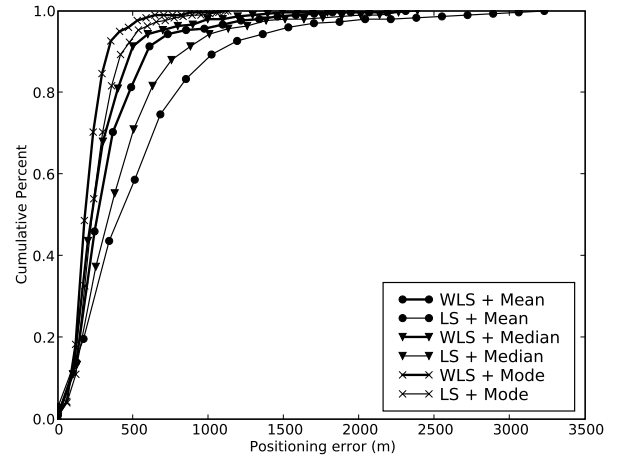


Fig. 3: Compared cumulative density function of four studied estimators for outdoor scenario - $l = 1000m$, $K = 5$.

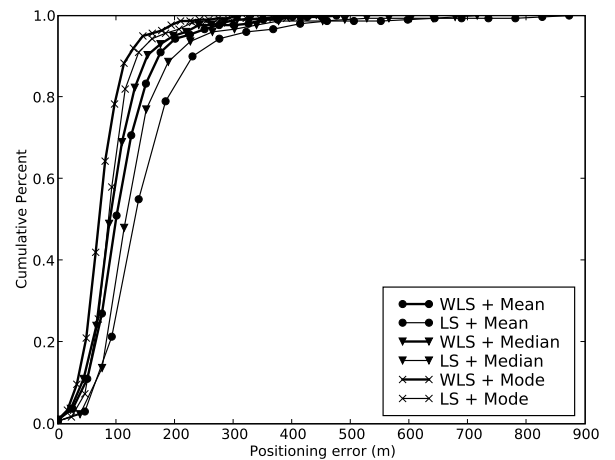


Fig. 4: Compared cumulative density function of four studied estimators for hybrid scenario - $l = 1000m$, $K = 5BS + 2AP$.

VI. CONCLUSION

In this paper, we studied hybrid RSS-based localization estimators assuming a path loss log normal shadowing model. We distinguished three estimation schemes of ranges from RSS observables based respectively on the mode, median and mean estimators. The studied positioning schemes consist then in two steps: estimation of ranges from RSS using mean, median or mode estimators; and estimation of location using weighted least square approximation on previously estimated ranges. We showed that estimation of ranges from RSS and consequently positioning accuracy can be enhanced using mode estimator rather than median or mean estimators usually used in past studies. Furthermore, the use of Weighted Least Square approximation instead of typical Least Square approximation may enhance positioning accuracy using hybrid RSS observables coming from different radio access networks. Next step will be to evaluate performance in more realistic scenarios and especially by using more realistic path loss model with adequate parameters.

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