



HAL
open science

A-contrario ringing detection and Shannon-compliant images

Gwendoline Blanchet, Lionel Moisan, Bernard Rougé

► **To cite this version:**

Gwendoline Blanchet, Lionel Moisan, Bernard Rougé. A-contrario ringing detection and Shannon-compliant images. 2009. hal-00388032v1

HAL Id: hal-00388032

<https://hal.science/hal-00388032v1>

Preprint submitted on 26 May 2009 (v1), last revised 7 Jan 2010 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A-CONTRARIO RINGING DETECTION AND SHANNON-COMPLIANT IMAGES

Gwendoline Blanchet¹, Lionel Moisan², Bernard Rougé³

¹ Centre National d'Etudes Spatiales, Toulouse, France

² Université Paris Descartes, MAP5, CNRS UMR 8145, Paris, France

³ École Normale Supérieure de Cachan, CMLA, CNRS UMR 8536, Cachan, France

ABSTRACT

According to Shannon Sampling Theory, cardinal sine (sinc) interpolation is the optimal way to reach subpixel accuracy from a properly sampled digital image. However, for most images sinc interpolation tends to produce an artifact called ringing, that consists in undesirable oscillations near objects contours. In this work, we propose a way to detect this ringing artifact in an a-contrario framework. Using Euler zigzag numbers, we compute the probability that neighboring gray-levels form an alternating sequence by chance, and characterize this way ringing blocks as structures that would be very unlikely in a random image. We then show two applications where the associated algorithm is used to test or enforce the compliance of an image with sinc interpolation.

Index Terms— Shannon sampling Theory, ringing, sinc interpolation, Euler zigzag numbers, a-contrario detection.

1. INTRODUCTION

Inadequate sampling or processing of a digital image may cause various artifacts, including blur, aliasing, blocking, ringing, etc. Among them, the ringing artifact can be characterized by the presence of undesirable oscillations of the image intensities, generally located near objects contours. These oscillations are generally caused by the use of frequency domain techniques (Fourier Transform, wavelets) or by non-monotone image filters (deconvolution, non-positive interpolation kernels, etc.).

In this paper, we propose a new method to detect image zones containing high-frequency ringing, that is, oscillations at Nyquist frequency (period of 2 pixels). This high-frequency ringing may appear in particular when an image is interpolated in Fourier domain (the so-called “sinc interpolation”). Image shrinking for example, can be performed optimally (in the L^2 sense) by bandlimiting an image with a hard cutoff in the frequency domain prior to downsampling. However, this process generally causes ringing artifacts in the shrunk image, since the multiplication with a window function in the frequency domain is equivalent in the image domain to the convolution with an oscillating sinc kernel. High-frequency ringing also appears when an aliased image (that

is, an image that has not been sampled above Nyquist rate) is interpolated in Fourier domain, and, as we shall see later, this phenomenon can be used on purpose to detect aliasing. High-frequency ringing is also typical of periodization artifacts caused by the use of the Discrete Fourier Transform on images. More generally, as soon as subpixel interpolation is required on an image, ringing detection is a very interesting tool not only to decide whether the optimal sinc interpolation can be used or not, but also to choose an appropriate image preprocessing filter or to select an alternative interpolation kernel.

Among ringing metrics proposed in the literature, very few of them are no-reference metrics. Marziliano et al. [1] propose to detect vertical edges, then to measure the oscillations heights on a given support fixed in advance. Oguz [2] defines the Visible Ringing Measure after an edge map obtained from morphological processing. It has been recently used by Yang et al. [3] to measure the efficiency of a ringing removal algorithm for JPEG2000 images. Other papers (e.g. [4, 5]) essentially investigate ringing as a compression artifact. In all cases, no criterion is given to ensure that no ringing would be found in noise, so that an arbitrary threshold has to be set.

In Section 2, we propose an a-contrario probabilistic model to detect suspiciously large oscillating patterns in an image. We obtain a criterion that permits to detect ringing artifacts in a contrast-invariant way, while offering the warranty that no detection occurs in a white noise image (whatever the noise distribution is). The resulting algorithm is then applied in Section 3 to test the compliance of an image with sinc interpolation, and later in Section 4 to perform optimal ringing-free image reduction.

2. A-CONTRARIO RINGING DETECTION

As we can see on Fig. 2 (right image), high-frequency ringing results in alternated gray levels on a given line. If g_1, g_2, \dots, g_n are the successive gray levels encountered on such a line, we shall say that they form an *alternating sequence* if

$$\forall k \in \{1, \dots, n-2\}, \quad (g_{k+2} - g_{k+1}) \cdot (g_{k+1} - g_k) < 0.$$

In other terms, the signs of $g_2 - g_1, g_3 - g_2, \dots, g_n - g_{n-1}$ are alternating like $+, -, +, \dots$ or $-, +, -, \dots$. To decide whether an alternating sequence should be considered as normal or suspicious in an image, we propose to use the a-contrario framework [6] and compute the probability to obtain, by chance, an alternating sequence with length at least n . If the gray-levels (g_k) are obtained from distinct pixels of a white noise image — that is, a random image U such that all $U(x, y)$ are independent and identically distributed (i.i.d.) —, then each sign $g_{k+1} - g_k$ has a probability $1/2$ to be positive, and a probability $1/2$ to be negative. However, the successive signs are not independent, as shown by the following theorem.

Theorem 1 *Let U be a discrete white noise image, that is, i.i.d. gray values drawn according to a given probability measure admitting a density with respect to Lebesgue measure. Then, for any sequence of $n \geq 3$ distinct pixels, the probability that the corresponding n gray levels form an alternating pattern is*

$$p_n = \frac{2A_n}{n!},$$

where A_n is the n -th Euler zigzag number, that can be obtained from the exponential generating function

$$\tan\left(\frac{x}{2} + \frac{\pi}{4}\right) = \tan x + \sec x = \sum_{n \geq 0} A_n \frac{x^n}{n!}.$$

From classical expansions of A_n and $n!$, it is not difficult to show that

$$p_n \underset{n \rightarrow \infty}{\sim} \tilde{p}_n, \quad \text{with} \quad \tilde{p}_n = 4 \cdot \left(\frac{2}{\pi}\right)^{n+1},$$

and the approximation is very good even for small n (see Table 2). If the successive signs were independent, the probability to obtain an alternating pattern would be $q_n = 2^{2-n}$, which asymptotically underestimates the actual probability p_n by a factor $4/\pi = 1.273\dots$ per point.

Now we come to the definition of an horizontal ringing block. In the following, u is a gray-level image, that is a real-valued function defined on $\Omega = \{0, \dots, M-1\} \times \{0, \dots, N-1\}$.

n	A_n	$n!$	p_n	$\left \frac{\tilde{p}_n}{p_n} - 1 \right $
3	2	6	0.667	$1.45 \cdot 10^{-2}$
4	5	24	0.417	$3.86 \cdot 10^{-3}$
5	16	120	0.267	$1.44 \cdot 10^{-3}$
10	50521	3628800	0.0278	$5.62 \cdot 10^{-6}$
20	$3.7 \cdot 10^{14}$	$2.4 \cdot 10^{18}$	$3.04 \cdot 10^{-4}$	$9.56 \cdot 10^{-11}$

Table 1. Euler zigzag numbers (A_n) and $n!$ permit to compute the probability p_n that a sequence of gray values alternates in a white noise image (see Theorem 1). Note on the right column that the approximation of p_n by its asymptotics \tilde{p}_n becomes rapidly excellent as n increases.

A rectangle of Ω is a subset $R = I \times J$ of Ω , where I and J are intervals of \mathbb{Z} (that is, sets made of consecutive integers). We shall say that R has size $l \times w$ (length l , width w) if $|I| = l$ and $|J| = w$.

Definition 1 *Let $u : \Omega \rightarrow \mathbb{R}$ be a gray-level image. A discrete rectangle $R = I \times J$ of Ω is a horizontal ringing block (HRB) if $(u(x, y))_{x \in I}$ is an alternating sequence for all $y \in J$.*

Thanks to Theorem 1, the probability to observe a $l \times w$ HRB in a random image is $(p_l)^w$. Let us consider the domain $D_\infty = \{3, \dots, +\infty\} \times \{1, \dots, +\infty\}$ and for any $\alpha > 0$, the subdomain

$$D_\alpha = \{(l, w) \in D_\infty, (p_l)^w \leq \alpha\}.$$

Let us call $n(\alpha)$ the minimum number of discrete regions (quarter-planes) of the kind $\{l_0, \dots, +\infty\} \times \{w_0, \dots, +\infty\}$ (l_0 and w_0 being arbitrary integers) needed to cover exactly D_α . Considering the shape of D_α (see Fig. 1), it is easy to show that $n(\alpha)$ is the number of horizontal (or vertical) lines required to draw the boundary of D_α . The function $\alpha \mapsto n(\alpha)$ is nonincreasing, and since $p_3 = 2/3$, $n(\alpha)$ is bounded from above by $\lceil \frac{\log \alpha}{\log(2/3)} \rceil$ (upper integer part). Typical values of $n(\alpha)$ are $n(10^{-6}) = 10$, $n(10^{-8}) = 11$, $n(10^{-10}) = 13$, $n(10^{-23}) = 20$.

Definition 2 *Let $u : \Omega \rightarrow \mathbb{R}$ be a $M \times N$ gray-level image. If R is a $l \times w$ HRB of u , we write*

$$\alpha(R) = MN \cdot (p_l)^w,$$

and say that R is ε -meaningful if $\varphi(\alpha(R)) \leq \varepsilon$, where

$$\varphi(\alpha) = \alpha \cdot n\left(\frac{\alpha}{MN}\right). \quad (1)$$

We say that R is maximal ε -meaningful if there is no ε -meaningful HRB of u that contains strictly R .

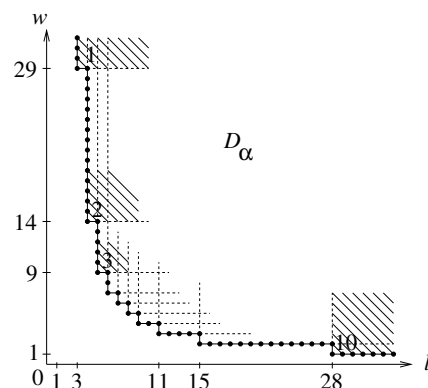


Fig. 1. The D_α domain for $\alpha = 10^{-5}$ can be covered by 10 quarter-planes, that is, $n(\alpha) = 10$.

The previous definition finds its justification in the following theorem, that guarantees that on a white noise image, at most ε maximal ε -meaningful HRBs (on average) will be found by chance (a-contrario property, see [6]).

Theorem 2 *For any $\varepsilon > 0$ and any $M \times N$ image u , let us call $m_\varepsilon(u)$ the number of maximal ε -meaningful HRBs of u . Then for any white noise image U , one has*

$$\forall \varepsilon > 0, \quad \mathbb{E} \left(m_\varepsilon(U) \right) \leq \varepsilon.$$

Proof — Any maximal ε -meaningful HRB R of U satisfy $\alpha(R) \leq \bar{\alpha}$, where $\bar{\alpha}$ is the maximum value of α such that $\varphi(\alpha) \leq \varepsilon$. By definition of $n(\bar{\alpha})$, there exists a finite subset $C_{\bar{\alpha}}$ of $D_{\bar{\alpha}}$ such that

$$D_{\bar{\alpha}} = \bigcup_{(l_0, w_0) \in C_{\bar{\alpha}}} \{l_0, \dots + \infty\} \times \{w_0, \dots + \infty\}.$$

Let us call *minimal* ε -meaningful HRB of U any $l \times w$ HRB of U such that $(l, w) \in C_{\bar{\alpha}}$. By definition of $C_{\bar{\alpha}}$, any ε -meaningful HRB of U (maximal or not) can be written as the union of minimal ε -meaningful HRBs of u . Hence, it is possible to build the set of maximal ε -meaningful HRBs of U from the set of minimal ε -meaningful HRBs of U , by applying a recursive fusion process, and the number of maximal HRBs obtained this way will be smaller than the number of original minimal HRBs. Now, since the total number of possible minimal ε -meaningful HRBs is $MN \cdot n\left(\frac{\bar{\alpha}}{MN}\right)$ and the probability of one of these rectangles to be a HRB is less than $\frac{\bar{\alpha}}{MN}$, the expected number of minimal ε -meaningful HRBs is less than

$$MN \cdot n\left(\frac{\bar{\alpha}}{MN}\right) \cdot \frac{\bar{\alpha}}{MN} = \varphi(\bar{\alpha}) \leq \varepsilon,$$

which concludes the proof. \square

3. ALGORITHM

Thanks to the previous results, we can tell if ringing is present in a $M \times N$ image using the following procedure :

1. Choose a bound ε on the expected number of false alarms, typically $\varepsilon = 1$ (at most one false alarm per image);
2. Find $\bar{\alpha}$ such that $\varphi(\bar{\alpha}) \leq \varepsilon$ (if $M = N = 1000$ and $\varepsilon = 1$, then $\bar{\alpha} = 1/11$ is convenient);
3. Find all horizontal extrema of u , that is, all pixels (x, y) such that $u(x-1, y), u(x, y), u(x+1, y)$ form an alternating sequence;
4. Among these points, find all maximal rectangles, and dilate them by one pixel horizontally;
5. Among these rectangles, select maximal ε -meaningful HRBs by checking the condition $\alpha(R) \leq \bar{\alpha}$.

Note that this algorithm is contrast-invariant (that is, the result is unchanged if u is changed into $g \circ u$, where $g :$

$\mathbb{R} \rightarrow \mathbb{R}$ is an increasing contrast change), and that it only depends on one parameter (the choice of ε , that controls the expected number of false alarms). We checked that the number of maximal ε -meaningful HRBs on white noise images remained small (generally 0 or 1 for $\varepsilon = 1$, almost always 0 for $\varepsilon = 0.1$), as guaranteed by Theorem 2.

4. APPLICATION TO SINC INTERPOLATION

As we mentioned before, the so-called “sinc interpolation” (also known as Fourier Interpolation) is the optimal way to reach subpixel accuracy from a properly-sampled image. However, most real-world images are undersampled (hence aliased), because undersampling yields a better appearance, making images look crisper despite the fact that they are less precise as regards subpixel accuracy. A good way to check if an image is “Shannon-compliant” (which means that it is well interpolated by the sinc kernel) is to resample it on a translated grid (typically by half of a pixel) and to see if ringing appears on the result. Since we focused on horizontal ringing, this amounts to the following procedure :

1. Compute the periodic component p of u (see [7]);
2. Translate p by $(1/2, 0)$ in the Fourier domain, that is, compute the image q whose Discrete Fourier Transform is $\hat{q}(a, b) = \hat{p}(a, b) \cdot \exp\left(\frac{-i\pi a}{M}\right)$;
3. Find all maximal 1-meaningful HRBs on q .

The step 1 permits to avoid ringing artifacts due to the fact that the Discrete Fourier Transform implicitly assumes that images are periodic, causing strong (non Shannon-compliant) discontinuities on image borders. It can be discarded but in that case, meaningful HRBs found near the image border have to be discarded too after step 3.

We applied this procedure to the “caps” image taken from the Live database [8] (see Fig. 2). As expected, no ringing was found on the original image, whose quality is good. However, several ringing blocks were found on the translated image, which shows that this image is not Shannon-compliant, as the sinc interpolation cannot be applied without creating ringing artifacts.

5. SHANNON-COMPLIANT IMAGE REDUCTION

Thanks to Shannon Sampling Theorem, we know that if we want to reduce an image to a given size without creating aliasing, the “optimal way” (in the L^2 sense) consists in changing its Fourier spectrum into a smaller one by cutting high-frequency components. Unfortunately, this hard frequency cut-off procedure has another drawback: it produces ringing (see Fig. 3, top row), due to the strong discontinuity it creates in the Fourier domain. Since the algorithm we proposed is able to detect ringing, we can try to perform Shannon-compliant image reduction by removing these ringing artifacts. A simple way to do that is to attenuate the



Fig. 2. The “caps” image from the Live database is not compliant with sinc interpolation: ringing is detected (black zones on the left image) after an horizontal translation of 0.5 pixel, especially near the topright corner of the cap. The image on the right zooms on this part, with contrast magnification and no detection marked.

high-frequency components of the shrunk image up to the point where the ringing artifacts disappear (but not further in order to introduce as little blur as possible). In practice, we considered a family of parametric filters and used the ringing detection to select the optimal value of the parameter, that is the first one yielding no ringing (see Fig. 3, bottom row). Compared to systematic lowpass filtering (see [9] in particular), the interest of this method is that it does not introduce a systematic amount of blur, but only the minimal necessary amount, so that if the initial image is already blurry, the obtained shrunk image will be much sharper and precise than with generic lowpass filtering. Of course, another interest is that the algorithm guarantees that the resulting image is Shannon-compliant, which opens interesting perspectives concerning subpixel accuracy.

6. CONCLUSION

We proposed a new method to detect suspicious oscillating blocks in an image while keeping control of the number of false alarms in white noise. The resulting algorithm, thanks to its ability to detect high-frequency ringing, can be used to test the adequacy of an image with sinc interpolation, or to reduce an image to the sharpest possible Shannon-compliant one. Other applications involving high-frequency ringing artifacts could be investigated in a similar way, and the possible extension of the method to more general oscillating structures (lower frequencies, oblique directions, etc.) could lead to interesting new applications (e.g., blind deconvolution with ringing control).

7. REFERENCES

[1] P. Marziliano, F. Dufaux, S. Winkler, and T. Ebrahimi, “Perceptual blur and ringing metrics: Application to JPEG2000,” *Elsevier Signal Processing: Image Communication*, vol. 19, no. 2, pp. 163–172, 2004.

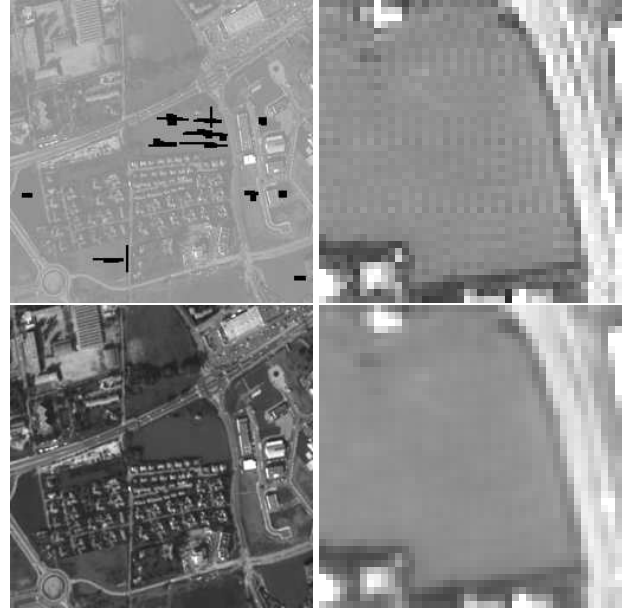


Fig. 3. The “nimes” image (©CNES) is shrunk using a hard frequency cutoff (top left), which causes ringing artifacts (marked in black), visible on the magnified part (top right). The ringing detector is then used to select an appropriate attenuation filter in the Fourier domain (cos filter), which result in a better-looking and ringing-free image (bottom row) that complies well with sinc interpolation.

[2] S.H. Oguz, *Morphological post-filtering of ringing and lost data concealment in generalized lapped orthogonal transform based image and video coding*, Ph.D. dissertation, Univ. of Wisconsin, Madison, 1999.

[3] S. Yang, Y. Hu, T. Nguyen, and D. Tull, “Maximum-likelihood parameter estimation for image ringing-artifact removal,” in *IEEE Trans. on Circuits and Systems for Video Tech.*, 2001, vol. 11, pp. 963–973.

[4] H.R. Sheikh, A.C. Bovik, and L. Cormack, “No-reference quality assessment using natural scene statistics: Jpeg2000,” *IEEE Trans. Image Processing*, vol. 14, no. 11, pp. 1918 – 1927, 2005.

[5] H. Liu, N. Klomp, and I. Heynderickx, “Perceptually relevant ringing region detection method,” in *proc. EUSIPCO*, 2008.

[6] A. Desolneux, L. Moisan, and J.-M. Morel, *From Gestalt Theory to Image Analysis - A Probabilistic Approach*, Springer, 2008.

[7] L. Moisan, “Periodic plus smooth image decomposition,” preprint, MAP5, Université Paris Descartes, 2008.

[8] H.R. Sheikh, Z. Wang, L. Cormack, and A.C. Bovik, *LIVE Image Quality Assessment Database Release 2*, <http://live.ece.utexas.edu/research/quality>, 2005.

[9] G. Blanchet, L. Moisan, and B. Rougé, “A linear prefilter for image sampling with ringing artifact control,” in *International Conference on Image Processing*, 2005, vol. 3, pp. 577–580.