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ABSTRACT

This paper attends to demonstrate the usefulness of Design of Experiments (DOE) method in magnetic refrigeration (MR) understanding and optimization. A numerical DOE is applied to a simple 1D finite difference model describing an Active Magnetic Regenerative Refrigeration (AMRR) system. The heat transfer fluid is water, the regenerator consists of stacked gadolinium plates and the model is based on the assumption of an equivalent single plate. A two-level 2^{7-3} fractional DOE based on Box methodology is used to evaluate the effect of seven parameters on the temperature span, namely material and fluid thicknesses, length, equivalent width, mass flow rate, cycle frequency and magnetocaloric effect (MCE).

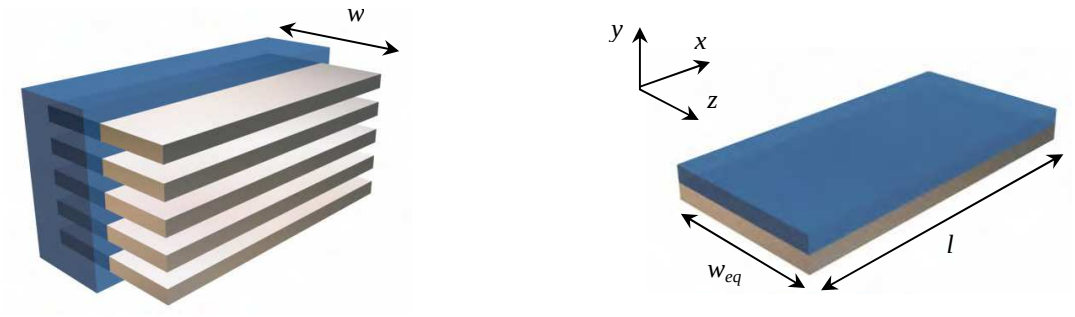
1. INTRODUCTION

Room temperature magnetic refrigeration has been shown to be an efficient cooling technology and a good alternative to the conventional vapour compression system. It could avoid using greenhouse gases and lower energy consumption by 20-30% over conventional technology (Geschneidner, 2008). MR is based on the magnetocaloric effect, an intrinsic property of some magnetic materials which heat and cool when a magnetic field is respectively applied and removed. But the temperature span is limited to a couple of degrees, e.g. near 2K/T for gadolinium. Steyert has solved this limitation by proposing a special MR cycle, namely Active Magnetic Regenerative Refrigeration, which leads to greater temperature span (Barclay and Steyert, 1982).
However a better design of AMRR systems is needed to make them efficient enough to be commercialized.
Many models intend to describe AMRR systems (Allab *et al.*, 2005), (Bouchekara *et al*., 2008), (Petersen *et al*., 2008) (Engelbrecht *et al*., 2005), etc. but few focus on optimizing AMRR systems efficiency. It is partly because there are numerous parameters in such systems (induction value, regenerator thickness, cycle period, etc.), and testing every configuration can be very long and tedious. In this context DOE appears to be a suitable method to optimize AMRR systems. DOE is a powerful tool for system optimization. It gives information on the effect of parameters on a system response. DOE allows focusing on the most important parameters and avoiding useless tests. It can be used either with an experimental approach or a numerical one (numerical DOE). Significant reduction of numerical cost could be made by applying numerical DOE to electromagnetic, thermal or coupled model.
AMRR systems and their numerical modelling are presented on the second part of this paper. The third part deals with numerical DOE and its application to our AMRR model. Finally conclusions are given on the last part of this paper.

2. AMRR SYSTEMS DESCRIPTION AND NUMERICAL MODEL

2.1 Description of AMRR systems

The simplest system consists of stacked gadolinium plates (Figure 1 a)), between which a heat transfer fluid flows (in our case water). This geometry is a simple one but is a good reference case to validate DOE method. We assume that such a system is equivalent to a single plate and fluid layer of width w_{eq} (Figure 1 b)).



a) Stacked plates bed b) Equivalent single plate bed

Figure 1: Schemes of the AMRR system

The system undergoes an AMRR cycle, which consists in four steps:
1 – **Magnetization**: magnetocaloric material is adiabatically magnetized
2 – **Hot blow**: water flows from the cold end to the hot end
3 – **Demagnetization**: magnetocaloric material is adiabatically demagnetized
4 – **Cold blow**: water flows from the hot end to the cold end

2.2. Governing equations for heat transfer

The time-dependent model used for the present paper is based on the one developed by Allab *et al.* and more details can be found in (Allab *et al.*, 2005). Main assumptions and simplifications are as follows:
- Pumps, heat loads and field source are not modelled
- The variation of temperature is supposed to be negligible along y and z directions
- Material and fluid properties are assumed constant
- Magnetic field is applied and removed instantaneously
- MCE is constant against T_r and x
- Flow is supposed laminar
- During hot and cold blow periods, fluid flows at constant velocity v_x
- Friction losses are neglected
- Conduction in material and fluid is neglected

The last assumption is restrictive and can lead to over-estimation of the temperature span. Yet DOE method rests valid and technical solutions can be used to limit axial conduction. Under these assumptions, thermal exchanges during hot and cold blow are described by the governing equations (1) and (2) respectively for the fluid and the magnetocaloric material.

$$m_f c_f \left(\frac{\partial T_f(x,t)}{\partial t} + v_x \frac{\partial T_f(x,t)}{\partial x} \right) = hA\left(T_r(x,t) - T_f(x,t)\right) \quad (1)$$

$$m_r c_r \left(\frac{\partial T_r(x,t)}{\partial t} \right) = hA\left(T_f(x,t) - T_r(x,t)\right) \quad (2)$$

Heat transfer coefficient is calculated using equation (3) with a, m and n empirical coefficients depending on system geometry and flow characteristic.

$$h = \frac{\lambda_f a \mathrm{Re}^m \mathrm{Pr}^n}{D_h} \tag{3}$$

Initial condition for the numerical simulation is that temperature is constant in material and fluid and equal to ambient temperature, corresponding to equation (4).

$$T_f(x,0) = T_r(x,0) = T_0 \tag{4}$$

Boundary conditions are Neumann conditions (null heat flux) everywhere except at the fluid entrance and exit where they are Dirichlet conditions. During hot blow fluid enters at cold reservoir temperature and during cold blow fluid enters at hot reservoir temperature. Since the model does not take heat loads into account, temperatures of hot and cold reservoirs are equal to the average of fluid exit temperature from previous cycle respectively during hot blow and cold blow.

2.3. Numerical considerations

Equations system has been implemented in MATLAB® with an explicit finite-difference scheme. To ensure stability the Courant-Friedrich-Levy (CFL) condition must be verified. We use a centred scheme in which case the CFL condition is given by equation (5).

$$Co = v_x \frac{\Delta t}{\Delta x} < 1 \tag{5}$$

Calculations have been carried out until the system reached a steady state, with respect to criteria given in equations (6) and (7). For all the calculations, the value of eps has been set to 10^{-2}.

$$T_{cold}(Nc) - T_{cold}(Nc-1) < eps \tag{6}$$

$$T_{hot}(Nc) - T_{hot}(Nc-1) < eps \tag{7}$$

3. NUMERICAL DOE

3.1. Concepts and methodology

DOE concepts relevant to the present work are given in this section but more details may be found elsewhere (Demonsant, 1996). The aim of DOE is to maximize the information gained from a given number of experiments or simulations, leading to time and money savings. The instinctive strategy consists in changing one factor at a time, but such a strategy can lead to incorrect conclusions and excessive cost and time delay. Indeed every factor effect is calculated for a specific combination of other factors and it is uncertain that the effect would be the same for another combination. Moreover trials are performed disorderly and every new trial can contradict conclusions drawn with previous ones leading to trials proliferation.
Conversely DOE is a structured experiment strategy which involves simultaneous changing of more than one factor at a time, and reduction of experiments number is obtained through factorial design. DOE strategy is usually composed of two steps. First a screening phase is achieved to identify most significant factors. This step is followed by an optimization, which focuses on critical factors to determine the levels to set them to get the best response. The work reported in the present paper is a numerical DOE, aims to determine critical factors on our AMRR modeling.

3.2. Application to AMRR model

A couple of numerical DOE have been performed on topics as various as mechanics (Shena *et al*., 2008), heat exchange (Rouquette *et al*., 2007), magnetism (Lee *et al*., 2006) or biochemistry (Kalil *et al*., 2000), but never, to the author's knowledge on MR. A numerical DOE has therefore been performed to study the

influence of factors on the efficiency of AMRR systems, through the use of a two-level fractional design. Some of the inputs of the numerical model, namely magnetocaloric material and heat transfer fluid properties, have been set to a nominal value. Others inputs have been used as factors for the numerical DOE. Each factor can take 2 values (Table 1), namely level - and level + corresponding to a variation of -/+50% around its nominal value (level 0). For instance factor 1 corresponding to regenerator thickness is set either to 0.5 mm or 1.5 mm.

Table 1: Factors level

Factor label	1	2	3	4	5	6	7
Factor	b_r	b_f	l	w_{eq}	q	$\frac{1}{f}$	*MCE*
Level 0	1.0 (mm)	0.12 (mm)	50 (mm)	514 (mm)	3 (mL/s)	250 (ms)	1.5 (°C)
Level -/+	-/+50 %	-/+50 %	-/+50 %	-/+50 %	-/+50 %	-/+50 %	-/+50 %

There are different methods to generate a fractional design, among which Taguchi and Box. The latter has been used because interactions between factors could hardly be anticipated. As seven factors are studied, a full design would lead to 2^7=128 simulations, allowing calculation of factors effects and interactions from second-order to seventh-order effects. To reduce the number of simulations, a fraction of the full design has been used. Such a reduction involves confusions (called aliases) between some factors and interactions effects and also between some interactions effects. The choice of the fractional design is a compromise between a small number of simulations and few confusions. Most of time third-order interactions are supposed negligible, so the fractional design has been chosen so that factors are aliased with third-order interactions, namely a 2^{7-3} fractional design given in Table 2.

Box methodology consists in using a full factorial design completed using Box generators (Box *et al.*, 1978). In the present case, the 2^{7-3} fractional design has been built using a 2^4 full design corresponding to columns 1 to 4 of Table 2. The three remaining columns (5 to 7) are constructed using generators proposed by Box *et al*. For example column 5 is the product of columns 1, 2 and 3. To study the efficiency of the different AMRR systems, the temperature span at steady state has been chosen as DOE response. Every simulation has been run using the AMRR numerical model presented in section 2 to determine the value of the response and according to the chosen DOE, e.g. for run 1 factors 1 to 6 have been set to their level - and factor 7 to its level +.

Table 2: 2^{7-3} fractional design

Run	1	2	3	4	5=1.2.3	6=2.3.4	7=-1.3.4	ΔT (K)
1	-	-	-	-	-	-	+	27.4
2	-	-	-	+	-	+	-	15.6
3	-	-	+	-	+	+	-	9.9
4	-	-	+	+	+	-	+	75.4
5	-	+	-	-	+	+	+	4.3
6	-	+	-	+	+	-	-	3.3
7	-	+	+	-	-	-	-	6.6
8	-	+	+	+	-	+	+	58.2
9	+	-	-	-	+	-	-	4.3
10	+	-	-	+	+	+	+	24.2
11	+	-	+	-	-	+	+	69.8
12	+	-	+	+	-	-	-	31.4
13	+	+	-	-	-	+	-	3.2
14	+	+	-	+	-	-	+	9.2
15	+	+	+	-	+	-	+	13.7
16	+	+	+	+	+	+	-	8.4

Effects of factors and second-order interactions have been calculated from the results of the fractional design. The effect of a given factor X_j is calculated according to equation (8), where the operator l_{ij} takes

the value of the *j*-th factor for the *i*-th run and Y_i is the *i*-th response value. It corresponds to the variation of the response relative to the average response when factor X_j goes from level - to level +. Similarly the effect of an interaction between factors X_j and X_k is calculated according equation (9).

$$e_{X_j} = \frac{2}{n}\sum_{i=1}^{n} l_{ij}\left(Y_i - \overline{Y}\right) \tag{8}$$

$$e_{X_j X_k} = \frac{2}{n}\sum_{i=1}^{n} l_{ij} l_{ik}\left(Y_i - \overline{Y}\right) \tag{9}$$

Effects give information on the impact of factors and interactions on the response, both in magnitude and direction but it is easier to compare factors and interactions between them in calculating their contribution according to equation (10), with SS_{X_j} the sum of squares of factor X_j and TSS the total sum of squares. In the specific case of a two-level design, equation (10) simplifies to equation (11). Contributions are in a way relative effect.

$$CTR_{X_j} = \frac{SS_{X_j}}{TSS} \tag{10}$$

$$CTR_{X_j} = \frac{n\left(e_{X_j}/2\right)^2}{\sum_{i=1}^{n}\left(Y_i - \overline{Y}\right)^2} \tag{11}$$

The alias structure of this design is easily determined considering Box generators used for it. Factors are aliased to third-order interactions (or more) which are supposed negligible so effects are attributed to factors. Factors effects calculated from the results of the fractional design are given in Table 3. But second-order interactions are aliased three to three so every calculated effect actually corresponds to the sum of three interactions, called contrast. Effects of these contrasts calculated from the results of the fractional design are given in Table 4.

Table 3: Factors effects and contributions

Factor	1	2	3	4	5	6	7
Effect (K)	-4.6	-18.9	22.7	10.8	-9.8	2.8	24.9
Contribution (%)	1.0	**16.3**	**23.7**	**5.3**	**4.4**	0.4	**28.5**

Table 4: Contrasts effects and contributions

Contrast	1.2 + 3.5 - 6.7	1.3 + 2.5 - 4.7	1.4 - 3.7 + 5.6	1.5 + 2.3 + 4.6	1.6 - 2.7 + 4.5	-1.7 + 2.6 + 3.4	2.4 + 3.6 - 5.7
Effect (K)	-4.9	-2.2	-15.3	-6.0	9.0	-7.5	2.0
Contribution (%)	1.1	0.2	**10.7**	1.6	**3.7**	2.6	0.2

3.3 Results analysis

Both magnitude and direction of effects have to be examined. Table 3 clearly shows that factors 1 and 6, namely material thickness and cycle frequency, are negligible for their contribution is less than 1%. Material thickness influences neither convective coefficient nor heat exchange area so has a low effect on temperature span. It is reminded that in opposition with classical methods which often impose factors to be varied at constant regenerator volume, DOE method lets it change so that a variation of material thickness does not affect hydraulic diameter. Frequency impacts on heat exchange period of each cycle so on the dynamic of heat transfers (and power) but as the temperature span is calculated when steady state is reached, frequency does not impact it. It leads to assume every interaction composed with factor 1 or 6 negligible so it helps

analysing contrasts in clearing up some confusions. As expected the most significant factor is MCE, whose contribution almost reaches 29%. The effect on the temperature span is positive, meaning that the higher the MCE is, the higher the temperature span is. The second critical factor is regenerator length with a contribution near 24%. It has a positive effect on response meaning that a longer regenerator gives higher temperature span. This result is in accordance with experimental results. The third critical factor is fluid channel thickness (16%), with a negative effect on the response. It can be analysed as following: when this factor increases, at constant mass flow rate the fluid velocity decreases leading to a smaller convective coefficient and less exchanges between fluid and material. The equivalent width also has a significant (5%) positive effect on temperature span. If width is increased, heat exchange surface is increased and involves more heat exchange between material and fluid. The last significant factor is mass flow rate (4%). A higher mass flow rate gives higher fluid velocity (at constant cross-flow section) and so higher convective coefficient.

When focussing on interactions, the most significant one is length*MCE. This effect is not easy to analyse and requires further investigations. Others interactions have non-negligible effects but are also difficult to explain. Even if confusions in some contrasts are not ended, this 2^{7-3} fractional design allows factor ranking because factors are not aliased with significant interactions.

4. CONCLUSION

A numerical DOE based on a 1D finite difference model has been used to study seven factors and rank their effects on the performance of an AMRR system. The seven factors studied are material and fluid thicknesses, length, equivalent width, mass flow rate, cycle frequency and MCE. The studied response is the temperature span at steady state. The use of DOE allowed quantification and ranking of the importance of the factors relative to one another. It is shown that the most critical factor is MCE, justifying the current focus on novel giant MCE materials. There are two others critical factors, namely length and fluid thickness that can be respectively increased and decreased to improve temperature span and compensate for a low MCE. Secondly equivalent width and mass flow rate have a significant impact but really lower than the three latter. Finally material thickness and cycle frequency do not seem to have a significant effect on temperature span relative to other factors. Results are valid in the simulation domain used for this DOE i.e. for factor values between level - and level + and under assumptions presented in section 2 and some results may not fit to experimental results but this paper attends to demonstrate the usefulness of DOE method in MR understanding and optimization. Information on seven factors and their second-order interactions have been drawn running only 16 simulations leading to important numerical cost savings.

Future work will focus on implementing a more refine model and perform a numerical DOE with multiple responses (refrigeration capacity, COP, etc.) because temperature span may not be sufficient to describe AMRR efficiency. Then non significant factors will be set to an interesting value (in term of cost, compactedness, etc.) and critical factors used to optimize AMRR system using response surface method or other methods of optimization.

NOMENCLATURE

Symbols

m	mass	(kg)
c	specific heat capacity	(J/kg/K)
v_x	fluid velocity	(m/s)
T	temperature	(K)
t	time coordinate	(s)
x	spatial coordinate	(m)
h	convective coefficient	(W /m²/K)
A	area of heat exchange	(m^2)
λ	thermal conductivity	(W/m/K)
Re	Reynolds number	(-)
Pr	Prandtl number	(-)
Co	Courant number	(-)
Δt	time step size	(s)
Δx	spatial step size	(m)
Nc	number of cycles	(-)
eps	steady state criterion	(K)
b	thickness	(mm)
l	length	(mm)
w	width	(mm)
w_{eq}	equivalent width	(mm)
q	volume flow rate	(mL/s)
f	cycle frequency	(Hz)
MCE	magnetocaloric effect	(K)
ΔT	temperature span	(K)
e	effect	(K)
n	number of runs	(-)
X_j	j-th factor	
Y_i	response of i-th run	(K)
$\overline{Y}$	average response	(K)
CTR	contribution	(%)
SS	sum of squares	
TSS	total sum of squares	

Subscripts

f	fluid
r	regenerator
$cold$	regenerator cold end
hot	regenerator hot end

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