Adaptive regulation - Rejection of unknown multiple narrow band disturbances

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Ioan Doré Landau[1], Aurelian Constantinescu[2] and Marouane Alma[1]
[1]GIPSA-LAB, Dept. of Automatic Control, ENSIEG BP 46, 38402 Saint-Martin d'Hères, France
ioan-dore.landau@gipsa-lab.grenoble-inp.fr
[2]Electrical Engineering Dept,Ecole de Technologie Supérieure, 1100 rue Notre-Dame Ouest, Montréal (Québec) H3C 1K3, Canada

***Abstract*— The paper presents a methodology for feedback adaptive control of active vibration systems in the presence of time varying unknown multiple narrow band disturbances. A direct adaptive control scheme based on the internal model principle and the use of the Youla-Kucera parametrization is proposed. This approach is comparatively evaluated with respect to an indirect adaptive control scheme based on the estimation of the disturbance model. The evaluation of the methodology is done in real time on an active suspension system and on an active vibration control system using an inertial actuator.**

***Index Terms*— direct adaptive control, internal model principle, Youla-Kucera parametrization, adaptive disturbance rejection, multiple narrow band disturbances**

I. INTRODUCTION

One of the basic problems in control is the attenuation (rejection) of unknown disturbances without measuring them. The common framework is the assumption that the disturbance is the result of a white noise or a Dirac impulse passed through the "model of the disturbance". While in general one can assume a certain structure for such "model of disturbance", its parameters are unknown and may be time varying. This will require to use an adaptive approach. To be more specific, the disturbances considered can be defined as "finite band disturbances". This includes single or multiple narrow band disturbances or sinusoidal disturbances. Furthermore for robustness reasons the disturbances should be located in the frequency domain within the regions where the plant has enough gain (see explanation in section III).

Solutions for this problem, provided that an "image" of the disturbance can be obtained by using an additional transducer, have been proposed by the signal processing community and a number of applications are reported ([12], [13], [6], [17]). However, these solutions (inspired by Widrow's technique for adaptive noise cancellation ([32])) ignore the possibilities offered by feedback control systems and require an additional transducer. The principle of this *signal processing solution* for adaptive rejection of unknown disturbances is that a transducer can provide a measurement, highly correlated with the unknown disturbance. This information is applied to the control input of the plant through an adaptive filter (in general a Finite Impulse Response - FIR) whose parameters are adapted such that the effect of the disturbance upon the output is minimized. The disadvantages of this approach are:

- It requires the use of an additional transducer.
- Difficult choice for the location of this transducer (it is probably the main disadvantage).
- It requires the adaptation of many parameters.

To achieve the rejection of the disturbance (at least asymptotically) without measuring it, a *feedback solution* can be considered. As mentioned earlier, the common framework is the assumption that the disturbance is the result of a white noise or a Dirac impulse passed through the "model of the disturbance" [1]. Several problems have been considered within this framework leading to adaptive feedback control solutions:

1) Unknown plant and disturbance models ([14]).
2) Unknown plant model and known disturbance ([29], [33]).
3) Known plant and unknown disturbance model ([8], [2], [3], [31], [28], [11], [18], [19], [22]).

The present paper will focus on the last case, since this is the situation encountered in many applications. Among the various approaches considered for solving this problem, the following ones may be mentioned:

1) Use of the internal model principle ([16], [20], [5], [30], [31], [2], [3], [18], [19], [22]).
2) Use of an observer for the disturbance ([28], [11]).
3) Use of the "phase-locked" loop structure considered in communication systems ([8], [7]).

Of course, since the parameters of the disturbance model are unknown, all these approaches lead to an adaptive implementation which can be of *direct* or *indirect* type.

From the user point of view and taking into account the type of operation of existing adaptive disturbance compensation systems one has to consider two modes of operation of the adaptive schemes:

- *Self-tuning* operation (the adaptation procedure starts either on demand or when the performance is unsatisfactory and the current controller is frozen during the estimation/computation of the new controller parameters).

[1]Throughout the paper it is assumed that the order of the disturbance model is known but the parameters of the model are unknown (the order can be estimated from data if necessary).

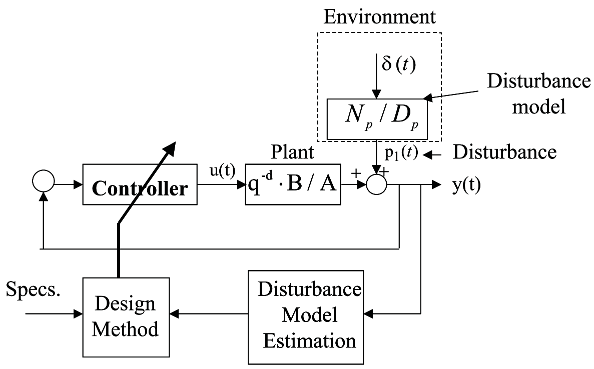

Fig. 1. Indirect adaptive control scheme for rejection of unknown disturbances

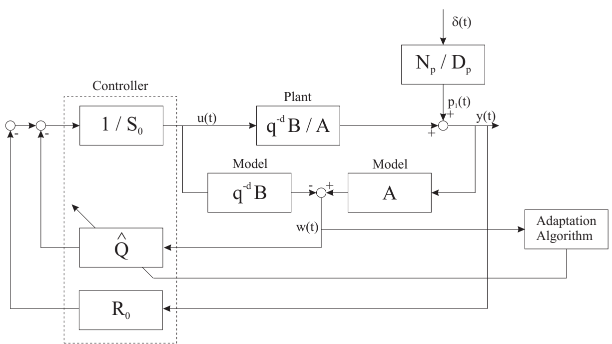

Fig. 2. Direct adaptive control scheme for rejection of unknown disturbances

- *Adaptive* operation (the adaptation is performed continuously and the controller is updated at each sampling).

Using the internal model principle, the controller should incorporate the model of the disturbance ([16], [20], [5], [30]). Therefore the rejection of unknown disturbances raises the problem of adapting the internal model of the controller and its re-design in real-time.

One way for solving this problem is to try to estimate in real time the model of the disturbance and re-compute the controller, which will incorporate the estimated model of the disturbance (as a pre-specified element of the controller). While the disturbance is unknown and its model needs to be estimated, one assumes that the model of the plant is known (obtained for example by identification). The estimation of the disturbance model can be done by using standard parameter estimation algorithms (see for example [25], [27]). This will lead to an indirect adaptive control scheme. The principle of such a scheme is illustrated in figure 1. The time consuming part of this approach is the redesign of the controller at each sampling time. The reason is that in many applications the plant model can be of very high dimension and despite that this model is constant, one has to re-compute the controller because a new internal model should be considered. This approach has been investigated in [8], [18], [19].

However, by considering the Youla-Kucera parametrization of the controller (known also as the Q-parametrization), it is possible to insert and adjust the internal model in the controller by adjusting the parameters of the Q polynomial (see figure 2). It comes out that in the presence of unknown disturbances it is possible to build a direct adaptive control scheme where the parameters of the Q polynomial are directly adapted in order to have the desired internal model without recomputing the controller (polynomials R_0 and S_0 in figure 2 remain unchanged). The number of the controller parameters to be directly adapted is roughly equal to the number of parameters of the denominator of the disturbance model. In other words, the size of the adaptation algorithm will depend upon the complexity of the disturbance model.

This paper focuses on the direct feedback adaptive control for the case of unknown and time-varying frequency narrow band disturbances. The direct adaptive control scheme to be presented([22]) takes advantage of the Youla-Kucera parametrization for the computation of the controller. For evaluation purposes (complexity and performance) an indirect adaptive control scheme based on the Internal Model Principle will be also presented.

The paper is organized as follows. Section II is dedicated to a brief review of the plant, disturbance and controller representation as well as of the Internal Model Principle. Some robustness issues are addressed in section III. The direct and the indirect adaptive control schemes for disturbance rejection are presented in sections IV and V, respectively. The application to an active suspension system, including the real-time results, is presented in VI. The application to the active vibration control system using an inertial actuator, including real time results is presented in section VII . Some concluding remarks are given in section VIII.

II. Plant representation and controller structure

The structure of a linear time invariant discrete time model of the plant (used for controller design) is:

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d-1}B^{*}(z^{-1})}{A(z^{-1})} \ , \qquad (1)$$

with:

$$d = \text{the plant pure time delay in number of sampling periods}$$

$$\begin{aligned} A &= 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A} \ ; \\ B &= b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} = q^{-1} B^{*} \ ; \\ B^{*} &= b_1 + \cdots + b_{n_B} z^{-n_B+1} \ , \end{aligned}$$

where $A(z^{-1})$, $B(z^{-1})$, $B^{*}(z^{-1})$ are polynomials in the complex variable z^{-1} and n_A, n_B and $n_B - 1$ represent their orders[2]. The model of the plant may be obtained by system identification. Details on system identification of the models considered in this paper can be found in [26], [9], [23], [21], [1], [10].

[2]The complex variable z^{-1} will be used for characterizing the system's behavior in the frequency domain and the delay operator q^{-1} will be used for describing the system's behavior in the time domain.

Since in this paper we are focused on regulation, the controller to be designed is a RS-type polynomial controller ([24], [26]) - see also figure 5.

The output of the plant $y(t)$ and the input $u(t)$ may be written as:

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + p_1(t) \; ; \quad (2)$$

$$S(q^{-1}) \cdot u(t) = -R(q^{-1}) \cdot y(t) \; , \quad (3)$$

where q^{-1} is the delay (shift) operator ($x(t) = q^{-1}x(t+1)$) and $p_1(t)$ is the resulting additive disturbance on the output of the system. $R(z^{-1})$ and $S(z^{-1})$ are polynomials in z^{-1} having the orders n_R and n_S, respectively, with the following expressions:

$$R(z^{-1}) = r_0 + r_1 z^{-1} + \ldots + r_{n_R} z^{-n_R} = R'(z^{-1}) \cdot H_R(z^{-1}) \; ; \quad (4)$$

$$S(z^{-1}) = 1 + s_1 z^{-1} + \ldots + s_{n_S} z^{-n_S} = S'(z^{-1}) \cdot H_S(z^{-1}) \; , \quad (5)$$

where H_R and H_S are pre-specified parts of the controller (used for example to incorporate the internal model of a disturbance or to open the loop at certain frequencies).

We define the following sensitivity functions:

- Output sensitivity function (the transfer function between the disturbance $p_1(t)$ and the output of the system $y(t)$):

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{P(z^{-1})} \; ; \quad (6)$$

- Input sensitivity function (the transfer function between the disturbance $p_1(t)$ and the input of the system $u(t)$):

$$S_{up}(z^{-1}) = -\frac{A(z^{-1})R(z^{-1})}{P(z^{-1})} \; , \quad (7)$$

where

$$\begin{aligned} P(z^{-1}) &= A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \\ &= A(z^{-1})S'(z^{-1}) \cdot H_S(z^{-1}) + z^{-d}B(z^{-1})R'(z^{-1}) \cdot H_R(z^{-1}) \quad (8) \end{aligned}$$

defines the poles of the closed loop (roots of $P(z^{-1})$). In pole placement design, $P(z^{-1})$ is the polynomial specifying the desired closed loop poles and the controller polynomials $R(z^{-1})$ and $S(z^{-1})$ are minimal degree solutions of (8) where the degrees of P, R and S are given by: $n_P \leq n_A + n_B + d - 1$, $n_S = n_B + d - 1$ and $n_R = n_A - 1$. Using the equations (2) and (3), one can write the output of the system as:

$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p_1(t) = S_{yp}(q^{-1}) \cdot p_1(t) \; . \quad (9)$$

For more details on RS-type controllers and sensitivity functions see [26].

Suppose that $p_1(t)$ is a deterministic disturbance, so it can be written as

$$p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \; , \quad (10)$$

where $\delta(t)$ is a Dirac impulse and $N_p(z^{-1})$, $D_p(z^{-1})$ are coprime polynomials in z^{-1}, of degrees n_{N_p} and n_{D_p}, respectively. In the case of stationary disturbances the roots of $D_p(z^{-1})$ are on the unit circle. The energy of the disturbance is essentially represented by D_p. The contribution of the terms of N_p is weak compared to the effect of D_p, so one can neglect the effect of N_p.

Internal Model Principle: *The effect of the disturbance given in (10) upon the output:*

$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \; , \quad (11)$$

where $D_p(z^{-1})$ is a polynomial with roots on the unit circle and $P(z^{-1})$ is an asymptotically stable polynomial, converges asymptotically towards zero if and only if the polynomial $S(z^{-1})$ in the RS controller has the form:

$$S(z^{-1}) = D_p(z^{-1})S'(z^{-1}) \; . \quad (12)$$

In other terms, the pre-specified part of $S(z^{-1})$ should be chosen as $H_S(z^{-1}) = D_p(z^{-1})$ and the controller is computed using (8), where P, D_p, A, B, H_R and d are given[3].

Using the Youla-Kucera parametrization (Q-parametrization) of all stable controllers ([4], [30]), the controller polynomials $R(z^{-1})$ and $S(z^{-1})$ get the form:

$$R(z^{-1}) = R_0(z^{-1}) + A(z^{-1})Q(z^{-1}) \; ; \quad (13)$$

$$S(z^{-1}) = S_0(z^{-1}) - z^{-d}B(z^{-1})Q(z^{-1}) \; . \quad (14)$$

The (central) controller (R_0, S_0) can be computed by poles placement (but any other design technique can be used). Given the plant model (A, B, d) and the desired closed-loop poles specified by the roots of P one has to solve:

$$P(z^{-1}) = A(z^{-1})S_0(z^{-1}) + z^{-d}B(z^{-1})R_0(z^{-1}) \; . \quad (15)$$

Equations (13) and (14) characterize the set of all stabilizable controllers assigning the closed loop poles as defined by $P(z^{-1})$ (it can be verified by simple calculations that the poles of the closed loop remain unchanged). For the purpose of this paper $Q(z^{-1})$ is considered to be a polynomial of the form:

$$Q(z^{-1}) = q_0 + q_1 z^{-1} + \ldots + q_{n_Q} z^{-n_Q} \; . \quad (16)$$

To compute $Q(z^{-1})$ in order that the controller incorporates the internal model of the disturbance one has to solve the diophantine equation:

$$S'(z^{-1})D_p(z^{-1}) + z^{-d}B(z^{-1})Q(z^{-1}) = S_0(z^{-1}) \; , \quad (17)$$

where $D_p(z^{-1})$, d, $B(z^{-1})$ and $S_0(z^{-1})$ are known and $S'(z^{-1})$ and $Q(z^{-1})$ are unknown. Equation (17) has a unique solution for $S'(z^{-1})$ et $Q(z^{-1})$ with: $n_{S_0} \leq n_{D_p} + n_B + d - 1$, $n_{S'} = n_B + d - 1$, $n_Q = n_{D_p} - 1$. One sees that the order n_Q of the polynomial Q depends upon the structure of the disturbance model.

III. Robustness considerations

As it is well known, the introduction of the internal model for the perfect rejection of the disturbance (asymptotically) will have as effect to raise the maximum value of the modulus of the output sensitivity function S_{yp}. This may lead to unacceptable values for the modulus and the delay margins if the controller design is not appropriately done

[3]Of course it is assumed that D_p and B do not have common factors.

(see [26]). As a consequence, a robust control design should be considered assuming that the model of the disturbance is known, in order to be sure that for all situations an acceptable modulus margin and delay margin are obtained.

On the other hand at the frequencies where perfect rejection of the disturbance is achieved one has $S_{yp}(e^{-j\omega}) = 0$ and

$$\left|S_{up}(e^{-j\omega})\right| = \left|\frac{A(e^{-j\omega})}{B(e^{-j\omega})}\right| \ . \tag{18}$$

Equation **(**18**)** corresponds to the inverse of the gain of the system to be controlled. The implication of equation **(**18**)** is that cancellation (or in general an important attenuation) of disturbances on the output should be done only in frequency regions where the system gain is large enough. If the gain of the controlled system is too low, $|S_{up}|$ will be large at these frequencies. Therefore, the robustness vs additive plant model uncertainties will be reduced and the stress on the actuator will become important. Equation (18) also implies that serious problems will occur if $B(z^{-1})$ has complex zeros close to the unit circle (stable or unstable zeros) at frequencies where an important attenuation of disturbances is required. It is mandatory to avoid attenuation of disturbances at these frequencies.

Since on one hand we would not like to react to very high frequency disturbances and on the other hand we would like to have a good robustness it is often wise to open the loop at $0.5f_s$ (f_s is the sampling frequency) by introducing a fixed part in the controller $H_R(q^{-1}) = 1 + q^{-1}$ (for details see [26] and section II).

IV. Direct adaptive control for disturbance attenuation

The objective is to find an estimation algorithm which will directly estimate the parameters of the internal model in the controller in the presence of an unknown disturbance (but of known structure) without modifying the closed loop poles. Clearly, the Q-parametrization is a potential option since modifications of the Q polynomial will not affect the closed loop poles. In order to build an estimation algorithm it is necessary to define an *error equation* which will reflect the difference between the optimal Q polynomial and its current value.

In [30], such an error equation is provided and it can be used for developing a direct adaptive control scheme. This idea has been used in [31], [2], [3], [22]. Using the Q-parametrization, the output of the system in the presence of a disturbance can be expressed as:

$$\begin{aligned} y(t) &= \frac{A(q^{-1})[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})]}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \\ &= \frac{S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})}{P(q^{-1})} \cdot w(t) \ , \end{aligned} \tag{19}$$

where w(t) is given by (see also figure 2):

$$\begin{aligned} w(t) &= \frac{A(q^{-1})N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \\ &= A(q^{-1}) \cdot y(t) - q^{-d} \cdot B(q^{-1}) \cdot u(t) \ . \end{aligned} \tag{20}$$

In the time domain, the internal model principle can be interpreted as finding Q such that asymptotically $y(t)$ becomes zero. Assume that one has an estimation of $Q(q^{-1})$ at instant t, denoted $\hat{Q}(t,q^{-1})$. Define $\varepsilon^0(t+1)$ as the value of $y(t+1)$ obtained with $\hat{Q}(t,q^{-1})$. Using (19) one gets:

$$\varepsilon^0(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\hat{Q}(t,q^{-1}) \cdot w(t) \ . \tag{21}$$

One can define now the *a posteriori* error (using $\hat{Q}(t+1,q^{-1})$) as:

$$\varepsilon(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\hat{Q}(t+1,q^{-1}) \cdot w(t) \ . \tag{22}$$

Replacing $S_0(q^{-1})$ from the last equation by (17) one obtains

$$\varepsilon(t+1) = [Q(q^{-1}) - \hat{Q}(t+1,q^{-1})] \cdot \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + v(t+1) \ , \tag{23}$$

where

$$v(t) = \frac{S'(q^{-1})D_p(q^{-1})}{P(q^{-1})} \cdot w(t) = \frac{S'(q^{-1})A(q^{-1})N_p(q^{-1})}{P(q^{-1})} \cdot \delta(t)$$

is a signal which tends asymptotically towards zero.

Define the estimated polynomial $\hat{Q}(t,q^{-1})$ as: $\hat{Q}(t,q^{-1}) = \hat{q}_0(t) + \hat{q}_1(t)q^{-1} + \ldots + \hat{q}_{n_Q}(t)q^{-n_Q}$ and the associated estimated parameter vector : $\hat{\theta}(t) = [\hat{q}_0(t)\, \hat{q}_1(t) \ldots \hat{q}_{n_Q}(t)]^T$. Define the fixed parameter vector corresponding to the optimal value of the polynomial Q as: $\theta = [q_0\, q_1 \ldots q_{n_Q}]^T$. Denote:

$$w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) \tag{24}$$

and define the following observation vector:

$$\phi^T(t) = [w_2(t) \quad w_2(t-1) \ldots w_2(t-n_Q)] \ . \tag{25}$$

Equation (23) becomes

$$\varepsilon(t+1) = [\theta^T - \hat{\theta}^T(t+1)] \cdot \phi(t) + v(t+1) \ . \tag{26}$$

One can remark that $\varepsilon(t)$ corresponds to an adaptation error ([24]).

From equation (21) one obtains the *a priori* adaptation error:

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t) \ ,$$

with

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1) \ ; \tag{27}$$

$$w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) \ ; \tag{28}$$

$$w(t+1) = A(q^{-1}) \cdot y(t+1) - q^{-d}B^*(q^{-1}) \cdot u(t) \tag{29}$$

where $B(q^{-1})u(t+1) = B^*(q^{-1})u(t)$.

The *a posteriori* adaptation error is obtained from (22):

$$\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t) \ .$$

For the estimation of the parameters of $\hat{Q}(t,q^{-1})$ the following parameter adaptation algorithm is used ([24]):

$$
\begin{aligned}
\hat{\theta}(t+1) &= \hat{\theta}(t)+F(t)\phi(t)\varepsilon(t+1)\ ; && (30)\\
\varepsilon(t+1) &= \frac{\varepsilon^0(t+1)}{1+\phi^T(t)F(t)\phi(t)}\ ; && (31)\\
\varepsilon^0(t+1) &= w_1(t+1)-\hat{\theta}^T(t)\phi(t)\ ; && (32)\\
F(t+1) &= \frac{1}{\lambda_1(t)}\left[F(t)-\frac{F(t)\phi(t)\phi^T(t)F(t)}{\frac{\lambda_1(t)}{C)}+\phi^T(t)F(t)\phi(t)}\right] && (33)\\
1 &\geq \lambda_1(t)>0; 0\leq\lambda_2(t)<2 && (34)
\end{aligned}
$$

where $\lambda_1(t),\lambda_2(t)$ allow to obtain various profiles for the evolution of the adaption gain $F(t)$ (for details see [24], [26] and section VI).

In order to implement this methodology for disturbance rejection (see figure 2), it is supposed that the plant model $\frac{z^{-d}B(z^{-1})}{A(z^{-1})}$ is known (identified) and that it exists a controller $[R_0(z^{-1}),S_0(z^{-1})]$ which verifies the desired specifications in the absence of the disturbance. One also supposes that the degree n_Q of the polynomial $Q(z^{-1})$ is fixed, $n_Q=n_{D_p}-1$, i.e. the structure of the disturbance is known.

The following procedure is applied at each sampling time for *adaptive* operation:

1) Get the measured output $y(t+1)$ and the applied control $u(t)$ to compute $w(t+1)$ using (29).
2) Compute $w_1(t+1)$ and $w_2(t)$ using (27) and (28) with P given by (15).
3) Estimate the Q-polynomial using the parametric adaptation algorithm (30) - (33).
4) Compute and apply the control (see figure 2):

$$S_0(q^{-1})\cdot u(t+1)=-R_0(q^{-1})\cdot y(t+1)-\hat{Q}(t,q^{-1})\cdot w(t+1)\ . \quad (35)$$

For the *self tuning* operation of the adaptive scheme, the estimation of the Q- polynomial starts once the level of the output is over a defined threshold. A parameter adaptation algorithm (30)-(33) with *decreasing adaption gain* is used and the estimation is stopped when the adaption gain is below a pre-specified level[4]. During estimation of the new parameters, the controller is kept constant. The controller is updated once the estimation phase is finished. For a stability analysis of this scheme see [22].

V. Indirect adaptive control for disturbance attenuation

Indirect adaptive control for the attenuation of unknown disturbances consists in two steps: (1) Identification of the disturbance model; (2) Computation of a digital controller using the identified disturbance model as internal model. Details on this approach can be found in [22].

[4]The magnitude of the adaptation gain gives an indication upon the variance of the parameter estimation error - see for example [24].

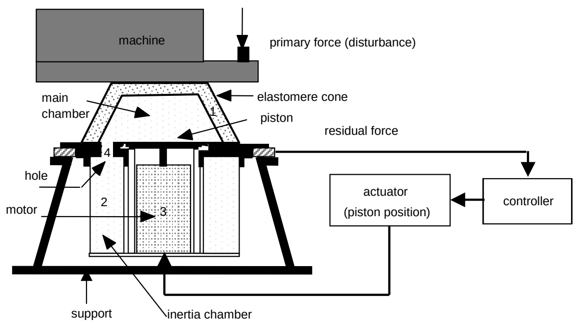

Fig. 3. Active suspension system (scheme)

VI. Application 1 - Adaptive rejection of narrow band disturbances on an active suspension

A. The active suspension

The structure of the system (the active suspension) used in this paper is presented in figure 3. Two photos of the system are presented in figure 4 (Courtesy of Hutchinson Research Center and Laboratoire d'Automatique de Grenoble). It consists of the active suspension, a load, a shaker and the components of the control scheme. The mechanical construction of the load is such that the vibrations produced by the shaker, fixed to the ground, are transmitted to the upper side of the active suspension. The active suspension is based on a hydraulic system allowing to reduce the over-pressure at the frequencies of the vibration modes of the suspension.

The controller will act upon the piston (through a power amplifier) in order to reduce the residual force. The sampling frequency is $800Hz$. The equivalent control scheme is shown in figure 5. The system input, $u(t)$ is the position of the piston (see figures 3, 5), the output $y(t)$ is the residual force measured by a force sensor. The transfer function $(q^{-d_1}\frac{C}{D})$, between the disturbance force, u_p, and the residual force $y(t)$ is called *primary path*. In our case (for testing purposes), the primary force is generated by a shaker controlled by a signal given by the computer. The plant transfer function $(q^{-d}\frac{B}{A})$ between the input of the system, $u(t)$, and the residual force is called *secondary path*. The input of the system being a position and the output a force, the secondary path transfer function has a double differentiator behavior.

The control objective is to reject the effect of unknown narrow band disturbances on the output of the system (the residual force). The system has to be considered as a "black box".

B. Results obtained on the active suspension

For the active suspension the disturbance will be a time-varying frequency sinusoid, so we shall consider $n_{D_p}=2$ and $n_Q=n_{D_p}-1=1$.

The identification procedure in open and closed-loop operation for the active suspension is discussed in detail in [23], [21], [1]. The frequency characteristic of the identified

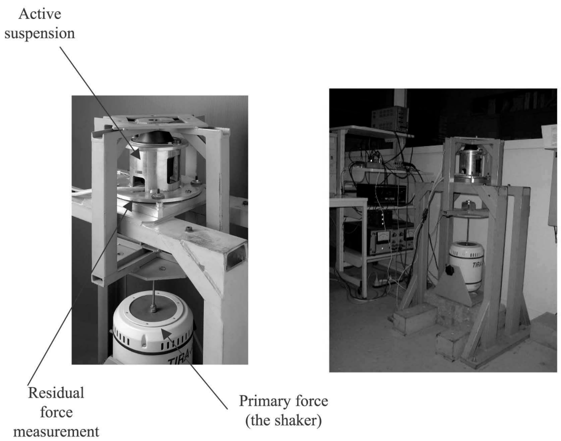

Fig. 4. Active suspension system (photo)

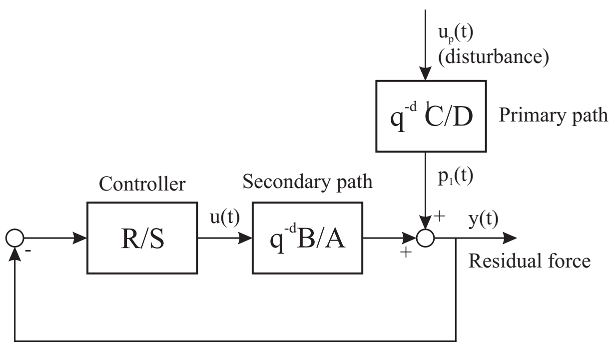

Fig. 5. Block diagram of active vibration suppression systems

primary path model (open-loop identification), between the signal u_p sent to the shaker in order to generate the disturbance and the residual force $y(t)$, is presented in figure 6. The first vibration mode of the primary path model is near $32Hz$. The frequency characteristic of the identified secondary path model (closed-loop identification), is presented also in figure 6. This model has the following complexity: $n_B = 14$, $n_A = 16$, $d = 0$. The identification has been done using as excitation of the piston a PRBS (Pseudo Random Binary Sequence) with frequency divider $p = 4$ (for details on the PRBS signals see [26]). There exist several very low damped vibration modes on the secondary path, the first one being at $31.8Hz$ with a damping factor 0.07. The identified model of the secondary path has been used for the design and implementation of the controller.

The central controller (without the internal model of the disturbance) has been designed using the pole placement method and the secondary path identified model. The resulting nominal controller has the following complexity: $n_R = 14$, $n_S = 16$ and it satisfies the imposed robustness constraints on the sensitivity functions(for the design methodology see[26])[5].

In order to evaluate the performances of direct and indirect methods in real time, time-varying frequency sinusoidal disturbances between 25 and $47Hz$ have been used (the first vibration mode of the primary path is near $32Hz$).

[5]Any design method allowing to satisfy these constraints can be used.

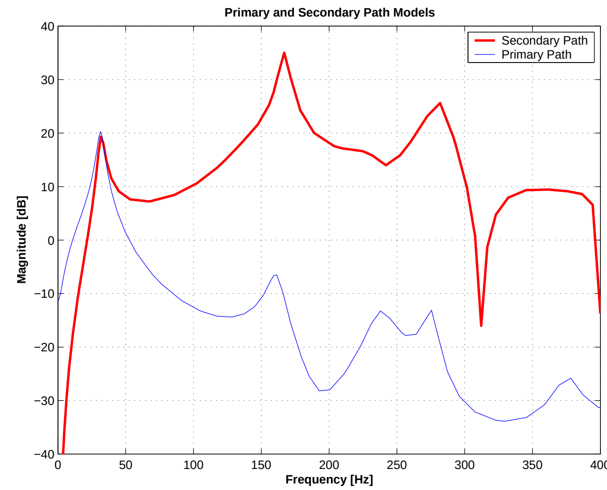

Fig. 6. Frequency characteristics of the primary and secondary paths

For both direct and indirect adaptive control methods, two protocols have been defined.

- *Protocol 1 : Self-tuning operation*
  The system operates in closed loop with a frozen controller. As soon as a change of the disturbance is detected (by measuring the variance of the residual output), the estimation algorithm is started with the last frozen controller in operation. When the algorithm converges (a criterion has to be defined - see below), a new controller is computed and applied to the system. The adaptation algorithm is stopped and one waits for a change of frequency.
- *Protocol 2 : Adaptive operation*
  The estimation algorithm works permanently (once the loop is closed) and the controller is recomputed at each sampling. The adaptation gain in this case does not tend asymptotically to zero.
- *Start up*: For comparison purpose the system is started in open-loop for both protocols. After 5 seconds (4000 samples) a sinusoidal disturbance of $32Hz$ is applied on the shaker. The model of the disturbance is estimated and an initial controller is computed (same initial controller for both direct and indirect adaptive control). In the case of the self-tuning operation the adaptation algorithm is stopped while in the case of the adaptive operation the adaptation algorithm continues to be active.

After the *start up* ends, every 15 seconds (8000 samples) sinusoidal disturbances of different frequency are applied (32Hz, 25Hz,32Hz,47Hz,32Hz).

a) Protocol 1 : Self-tuning operation. Real time experimental results: The measured residual force obtained in self-tuning operation with the direct adaptation method is presented in figure 7 and with the indirect adaptation method in figure 8 . We note in general a faster convergence speed of the direct adaptive control scheme compared to the indirect

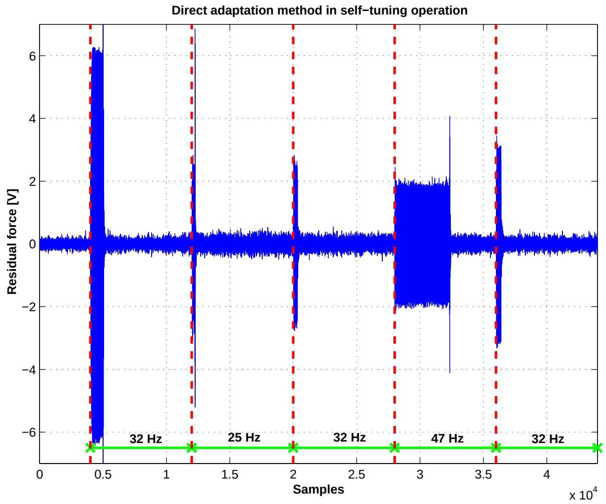

Fig. 7. Time domain results with the direct adaptation method in self-tuning operation

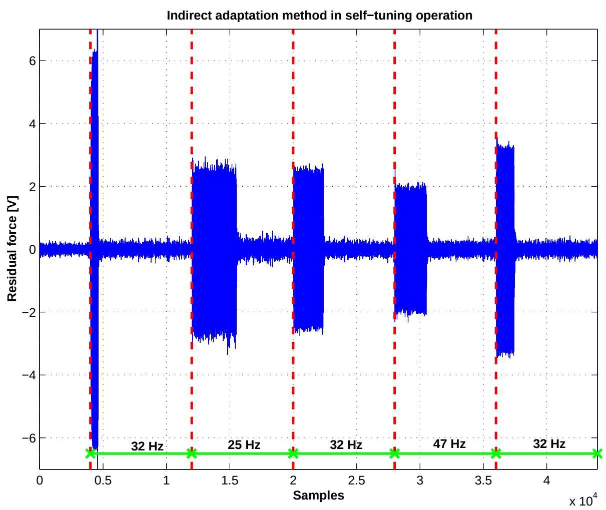

Fig. 8. Time domain results with the indirect adaptation method in self-tuning operation

one (except for 47Hz).

For the self-tuning protocol, the spectral densities of the residual force obtained in open and in closed loop, respectively, using the direct adaptation scheme (after the algorithm converges) are presented in figure 9. The results are given for the three frequencies used: 25, 32 and 47 Hz. We remark that the attenuations are larger than 49 dB for all the frequencies. Similar results are obtained with the indirect adaptation algorithm. For details see [9].

In *self-tuning* operation, one uses an adaptation gain $F(t)$ with *variable forgetting factor*, with $\lambda_0 = 0.97$ and the initial forgetting factor $\lambda_1(0) = 0.97$ (the forgetting factor is given by $\lambda_1(t) = \lambda_0\lambda_1(t-1) + 1 - \lambda_0$, with $0 < \lambda_0 < 1$). For the *variable forgetting factor* the adaptation gain tends asymptotically towards zero. The convergence criterion has been fixed as a threshold on the trace value of the adaptation gain matrix. For details see [9].

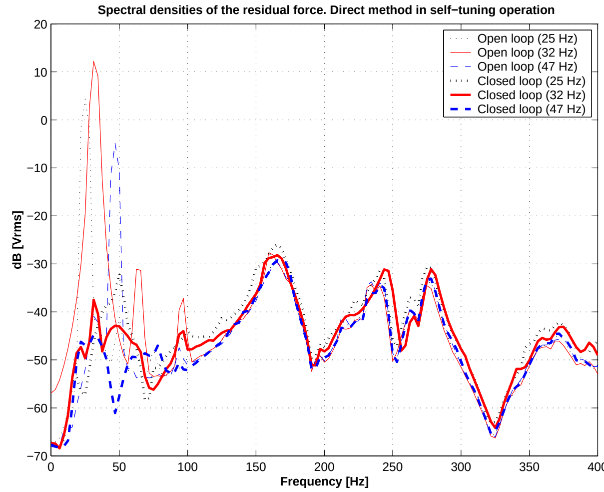

Fig. 9. Spectral densities of the residual force in open and in closed loop, with the direct adaptation method in self-tuning operation

The detection of a change of frequency is done using the variance of the measured residual force computed on a sliding window of 50 samples.

b) Protocol 2 : Adaptive operation. Real time experimental results: The measured residual force obtained in adaptive operation is presented in figure 10 for the direct adaptation method and in figure 11 for the indirect adaptation method. An adaptation gain with *variable forgetting factor combined with a constant trace* ([24], [26]) has been used in order to be able to track automatically the changes of disturbance characteristics. The low level threshold of the trace has been fixed at $3 \cdot 10^{-9}$ for the direct algorithm and at $5 \cdot 10^{-7}$ for the indirect one (note that in the indirect adaptive scheme there are more parameters to estimate than in the direct adaptive scheme). The attenuation obtained with the indirect adaptive scheme in adaptive operation is less good than in the self tuning operation and less good than the one obtained with the direct adaptive scheme. We note that the direct adaptive control scheme in adaptive operation gives better results than in self tuning operation (compare figures 7 and 10).

The spectral densities of the residual force for the direct adaptive scheme (after the algorithm converges) are similar with those obtained in *self-tuning* operation (see [9]).

According to the real time results presented above, one can conclude that the direct adaptive control scheme gives better results than the indirect adaptive control scheme, from the point of view of the convergence speed and performance. In addition the direct adaptation scheme is much simpler than the indirect one in terms of number of operations.

c) Direct adaptive control scheme under the effect of sinusoidal disturbances with continuously time varying frequency: Consider now that the frequency of the sinusoidal disturbance varies continuously and let's use a chirp disturbance signal (linear swept-frequency signal) between 25 and 47*Hz*. The tests have been done as follows: Start up in closed loop at $t = 0$ with the central controller. Once the loop

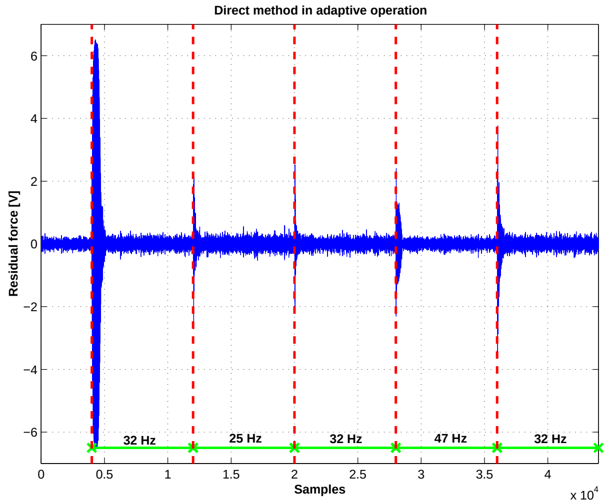

Fig. 10. Time domain results with the direct adaptation method in the adaptive case (trace $= 3 \cdot 10^{-9}$)

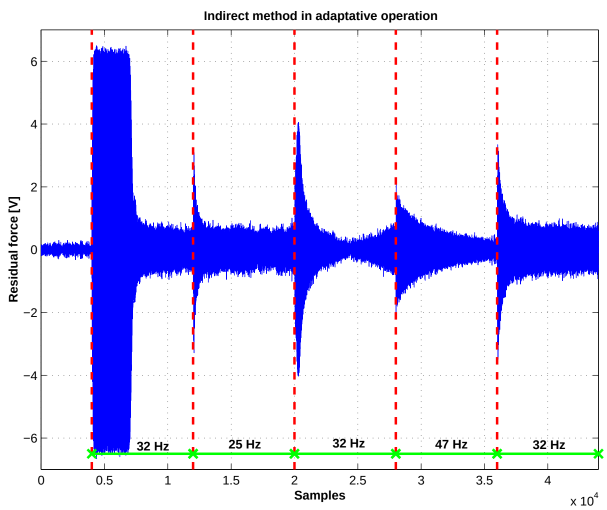

Fig. 11. Time domain results with the indirect adaptation method in the adaptive case (trace $= 5 \cdot 10^{-7}$)

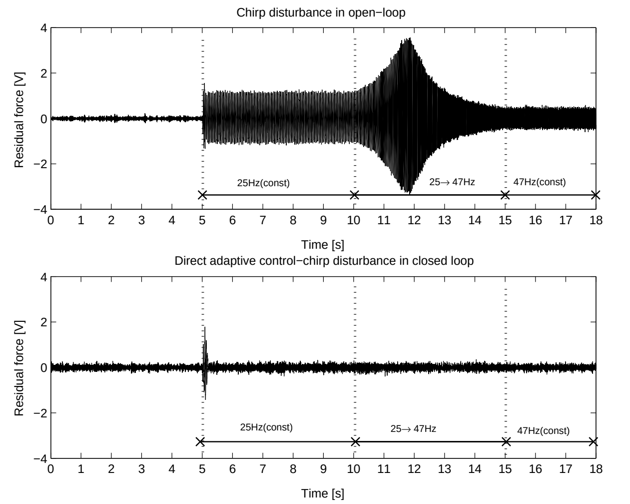

Fig. 12. Real-time results obtained with the direct adaptive method and a chirp disturbance: (a) Open loop; (b) Closed loop

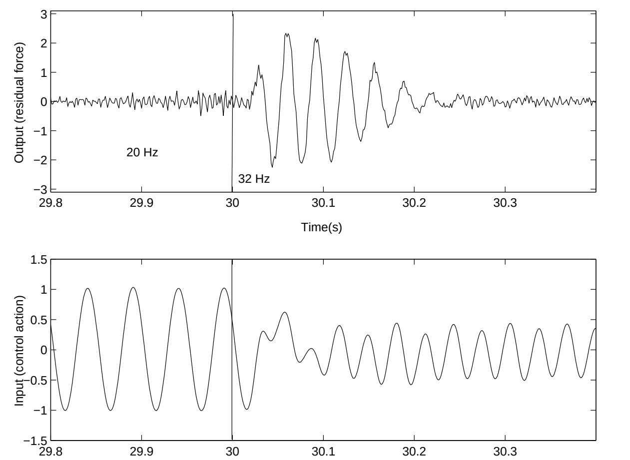

Fig. 13. Adaption transient in the direct adaptive control scheme for a step change of the disturbance frequency from 32Hz to 20Hz

is closed, the adaptation algorithm works permanently and the controller is updated (direct approach) at each sampling instant. After 5 seconds a sinusoidal disturbance of 25 Hz (constant frequency) is applied on the shaker. From 10 to 15 seconds a chirp between 25 and 47 Hz is applied. After 15 seconds a 47 Hz (constant frequency) sinusoidal disturbance is applied and the tests are stopped after 18 seconds. The time-domain results obtained in open and in closed-loop (direct adaptive control) are presented in figure 12. We can remark that the performances obtained are very good.

d) Adaptation transients for direct adaptive control: Figure 13 illustrates the adaptation transients on the input and output when a step change of the frequency of the disturbance occurs from 20Hz to 32 Hz respectively. One notes that the convergence of the output requires less than 0.25*s* This corresponds roughly to 6 periods for 32Hz. Same duration of the adaptation transient are obtained for the other frequencies step changes. These results have to be compared with the transients results given in [8], [28], [2], [3].

VII. Application 2 - Adaptive Rejection of Multiple Narrow Band Disturbances on an Active Vibration Control System Using an Inertial Actuator

A. The inertial actuator

In this application a different technological approach is used for suppressing the effect of vibrational disturbances. Instead of using an active suspension, one uses an inertial actuator which will create vibrational forces to counteract the effect of vibrational distrubances (inertial actuators use a similar principle as loudspeakers). The structure of the system is described in figure14. It consists on a standard passive damper and an inertial actuator fixed to the chassis where the vibrations should be attenuated. The testing setting is exactly the same as for the active suspension (see figure

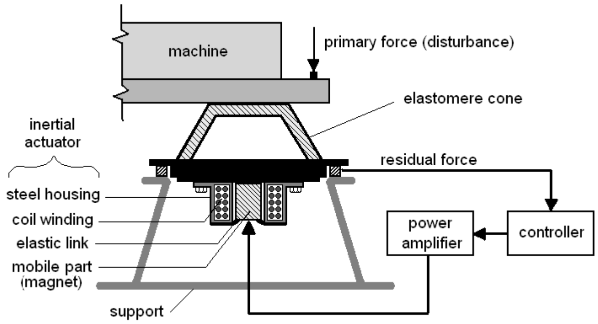

Fig. 14. Active vibration control using an inertial actuator (scheme)

4).

The controller will act on the inertial actuator (through a power amplifier) in order to reduce the residual force. The equivalent control scheme is shown in figure 5. The system input is the position of the mobile part of the actuator. Like for the active suspension, the secondary path has a double differentiator behavior. The system has to be considered as a ”black box” and the control objectives are similar to those for the active suspension; The sampling frequency is $800Hz$.

B. Results obtained with the inertial actuator

The performance of the system for rejecting multiple unknown time varying narrow band disturbances will be illustrated using the direct adaptive control scheme presented in section IV. Since two simultaneous time varying frequency sinusoids will be considered as disturbances , one should take $n_{D_p} = 4$ and $n_Q = n_{D_p} - 1 = 3$

Same procedure for system identification in open and closed loop, as for the active suspension, has been used. The frequency characteristics of the primary path (identification in open loop) and of the secondary path (identification in closed loop)are shown in Figure 15. The secondary path has the following complexity: $n_B = 12$, $n_A = 10$, $d = 0$. The identification has been done using as excitation a PRBS (with frequency divider $p = 2$ and $N = 9$). There exist several low damped vibration modes in the secondary path, the first vibration mode is at $51.58Hz$ with a damping of 0.023 and the second at $100.27Hz$ with a damping of 0.057. Only the ”adaptive ”operation regime has been considered for the subsequent results. Figure 16shows the spectral densities of the residual force obtained in open loop and in closed loop using the direct adaptation scheme (after the adaptation algorithm has converged). The results are given for the simultaneous applications of two sinusoidal disturbances ($70Hz$ and $100Hz$). One can remark a strong attenuation of the disturbances (larger than $45dB$).

Time domain results obtained with direct adaptation scheme in ”adaptive” operation regime are shown in Figure17. The disturbances are applied at $1s$ (the loop has already been closed) and step changes of their frequencies occur every $3s$.

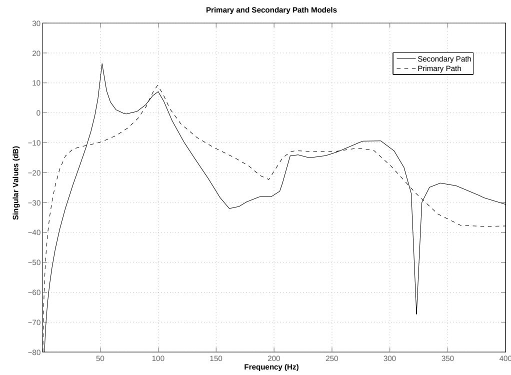

Fig. 15. Frequency characteristics of the primary and secondary paths (inertial actuator)

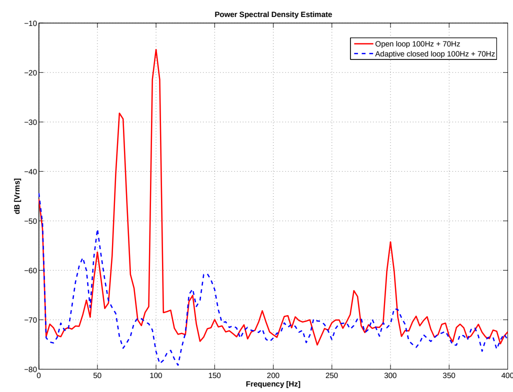

Fig. 16. Spectral densities of the residual force in open an in closed loop, with the direct adaption method in adaptive operation

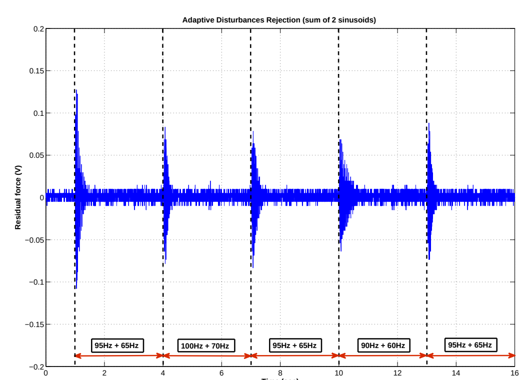

Fig. 17. Time domain results with direct adaptation method for simultaneous step changes of two sinusoidal disturbances

Figure 18 shows the corresponding evolution of the parameters of the polynomial Q. The convergence of the output requires less than $0.4s$ in the worst case.

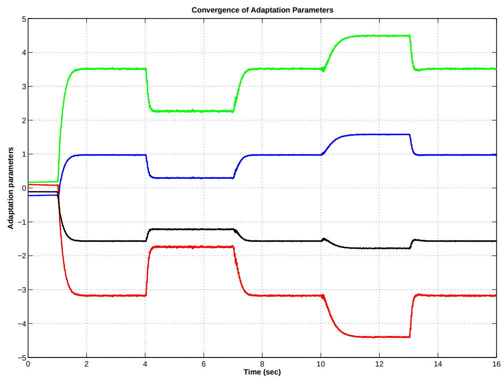

Fig. 18. Evolution of the parameters of the polynomial Q during adaptation

VIII. Conclusions

It was shown in this paper that the use of the internal model principle combined with the adaptation of the internal model implemented in a Youla - Kucera parametrized controller allows a very good rejection of the unknown time varying narrow band disturbances without requiring the use of an additional transducer. Two adaptive approaches (direct and indirect adaptation) have been presented and tested comparatively.

The results obtained in real time on active vibration control (using an active suspension or an inertial actuator) lead us to conclude that the direct adaptive control scheme provides better performance and is simpler than the indirect adaptive control scheme.

A similar approach has been used successfully on a chemical reactor and for noise cancellation in ducts. Extensions to the multivariable case have been recently done[15]

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