



**HAL**  
open science

# Extended Skyrme interaction and spin fluctuations in dense matter

Jérôme Margueron, Hiroyuki Sagawa

► **To cite this version:**

Jérôme Margueron, Hiroyuki Sagawa. Extended Skyrme interaction and spin fluctuations in dense matter. 2009. hal-00383328v1

**HAL Id: hal-00383328**

**<https://hal.science/hal-00383328v1>**

Preprint submitted on 12 May 2009 (v1), last revised 18 Jun 2009 (v2)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Extended Skyrme interaction and spin fluctuations in dense matter

J. Margueron

*Institut de Physique Nucléaire, IN2P3-CNRS and Université Paris-Sud, F-91406 Orsay Cedex, France*

H. Sagawa

*Center for Mathematical Sciences, University of Aizu, Aizu-Wakamatsu, 965-8580 Fukushima, Japan*

(Dated: May 12, 2009)

Most of the Skyrme interactions are known to predict spin or isospin instabilities beyond the saturation density of nuclear matter which contradict predictions based on realistic interactions. A modification of the standard Skyrme interaction is proposed so that the ferromagnetic instability is removed. The new terms are density dependent and modify only the spin p-h interaction in the case of spin-saturated system. Consequences for the nuclear response function and neutrino mean free path are shown. The overall effect of the RPA correlations makes dense matter more transparent for neutrino propagation by a factor of 2 to 10 depending of the density.

PACS numbers: 21.30.Fe, 21.60.Jz, 21.65.-f, 26.50.+x

Keywords: Skyrme effective interaction, nuclear matter, spin instabilities, RPA, neutrino mean free path

Skyrme interactions [1] are widely used to describe properties of nuclei such as binding energies, radii and excited states. Nevertheless, due to its very simple functional form, the standard Skyrme interaction is not suited for being extrapolated in astrophysical situation such as neutron stars. Indeed, the mean field solution obtained with such interactions become unstable for densities larger than the saturation density  $\rho_0$  and also for neutron rich matter. A recent extensive study of the symmetry energy deduced from Skyrme interactions [2] has shown that the symmetry energy becomes negative beyond the saturation density of nuclear matter for many of these interactions. This is related to the instability of the mean field with respect to the isospin density fluctuation  $\delta\rho_t$  where  $\rho_t = \rho_n - \rho_p$ . Depending on the parameters of the interaction, instabilities can occur in different channels, at various densities and at different isospins. It has been shown that there is a limited interval of densities between  $\rho_0$  and  $3\rho_0$  for which the mean field is stable in symmetric matter and also in neutron matter [3]. The stability of the ground state toward small fluctuations can be studied within the HF+RPA framework. The fluctuations around the mean field are induced by the particle-hole interaction,

$$V_{\text{ph}} = \frac{1}{N_0} \sum_{\ell} (F_{\ell} + F'_{\ell} \tau_1 \cdot \tau_2 + G_{\ell} \sigma_1 \cdot \sigma_2 + G'_{\ell} (\tau_1 \cdot \tau_2) (\sigma_1 \cdot \sigma_2)) P_{\ell}(\cos \theta), \quad (1)$$

in terms of the dimensionless Landau parameters  $F_{\ell}$ ,  $F'_{\ell}$ ,  $G_{\ell}$ ,  $G'_{\ell}$  and Legendre Polynomials  $P_{\ell}(\cos \theta)$ .  $N_0 = gm^*k_F/(2\pi^2\hbar^2)$  is the density of state around the Fermi energy and  $g$  is the degeneracy. The matter is stable unless one of these parameters of multipolarity  $\ell$  becomes lower than  $-2\ell - 1$ .

Recently, the ferromagnetic instability has regained interest [4, 5] due to the observation of extremely high magnetic fields ( $\sim 10^{15-16}$  G) in compact stars [6] and also of giant flares observed recently on 27 December

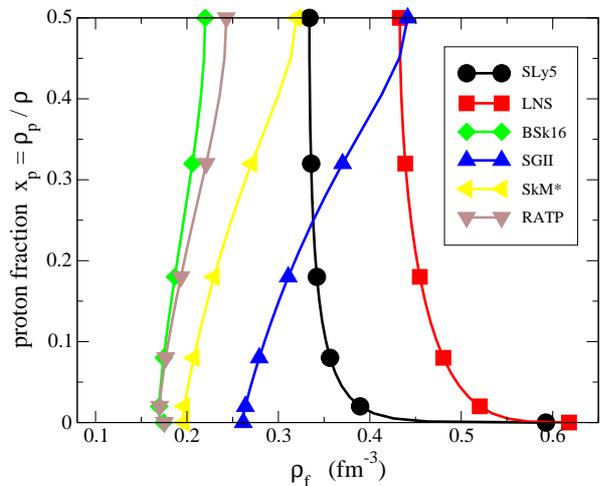


FIG. 1: (color online) Ferromagnetic phase diagram for various effective Skyrme interactions. The horizontal axis shows the critical density  $\rho_f$  at which the asymmetric matter becomes unstable, while the vertical axis shows the proton fraction  $x_p = \rho_p/\rho$ . The matter is spin symmetric for smaller density than  $\rho_f$ , while it becomes ferromagnetic for larger density than  $\rho_f$ . See the text for details.

2004 [7]. We show in Fig. 1 the critical density  $\rho_f$  at which the asymmetric matter becomes unstable with respect to the spin fluctuations. The proton fraction  $x_p = \rho_p/\rho$  is changed from symmetric matter ( $x_p = 1/2$ ) to neutron matter ( $x_p = 0$ ). We select various Skyrme interactions which are commonly used in the description of finite nuclei. Depending on the interaction, the density  $\rho_f$  can be very close to  $\rho_0$  (BSk16 [8], RATP [9]), around  $2\rho_0$  (SkM\* [10], SLy5 [11]) or nearly reach  $3\rho_0$  (SGII [12], LNS [13]) in symmetric nuclear matter ( $x_p=1/2$ ). When the proton fraction is decreasing, the density  $\rho_f$  either decrease (BSk16, RATP, SkM\*, SGII) or increase (SLy5, LNS). From Fig. 1 it could be deduced that the predic-

tion of  $\rho_f$  is varying largely among the selected Skyrme interactions.

According to microscopic calculations with realistic interactions such as diffusion Monte-Carlo [14] or Brueckner Hartree-Fock (HF) calculations [15, 16, 17], there is no ferromagnetic instability up to substantially high densities. The onset of the ferromagnetic instability beyond a few times  $\rho_0$  represents a serious limitation of the Skyrme interaction. This limitation should be cured in the calculation of nuclear matter properties such as equation of state, the response function and microscopic processes such as neutrino mean free path. It is then timely to enrich the effective Skyrme interaction in order to extend its domain of application and to take advantage of its simple form. To this end, extended Skyrme interactions shall mimic more accurately microscopic results of realistic interactions and preserve the accuracy of the original Skyrme interaction in the description of nuclei.

It is known that the spin and the spin-isospin excitations have strong impact on astronomical observables such as neutrino mean free path in neutron star,  $0\nu-$  and  $2\nu$  double beta decay processes. On the other hand, it has been recognized also that Skyrme interactions have a serious shortcoming in the spin channels. Our main purpose in this manuscript is to improve the spin dependent parts of Skyrme interactions keeping its simplicity and good properties for ground state properties. In this way, we will be able to extend the field of possible applications of Skyrme interactions for spin dependent excitations not only in finite nuclei but also infinite nuclei within the self-consistent theoretical model. It is important to make a bridge between finite and infinite systems by the self-consistent model without introducing any extra parameters. Top of realistic spin and spin-isospin interactions, we would like to introduce further tensor interactions and two-body spin-orbit interactions for constructing a global energy density functions of spin and spin-isospin channels. In this article, we first propose a simple way to get ride of the ferromagnetic instability by introducing a limited number of new terms to the standard Skyrme interaction in Sec. I. In Sec. II, we apply the new interaction to the calculation of the RPA response function in nuclear matter and then we calculate the neutrino mean free path. Conclusions are given in Sec. III.

## I. EXTENDED SKYRME INTERACTION

Some of the most recent Skyrme interactions are fitted so as to reproduce the theoretical binding energy in symmetric and neutron matter up to a density as much as few times  $\rho_0$ . As a result, the instability in the isospin channel, which occurs only if the binding energy of neutron matter becomes more attractive than that of symmetric matter, is usually either totally removed or pushed to very large densities. It is then possible to get ride of the isospin instability without modifying the standard

TABLE I: Nuclear matter properties of the Skyrme interactions SLy5, LNS, BSk16 at saturation density  $\rho_0$ ; the binding energy  $a_v$ , the incompressibility  $K_\infty$ , the symmetry energy  $a_s$ , the density  $\rho_f$  of the ferromagnetic instability.

	$\rho_0$ ( $\text{fm}^{-3}$ )	$a_v$ (MeV)	$K_\infty$ (MeV)	$a_s$ (MeV)	$\rho_f/\rho_0$
SLy5	0.16	-15.97	229.9	32	2.09
LNS	0.1746	-15.32	210.9	33.4	2.48
BSk16	0.1586	-16.05	241.6	30	1.24

Skyrme interaction. In the following, we focus on the spin instability and test the new terms added to the existing Skyrme interactions which do not show the instability in the isospin channel. We adopt three different Skyrme interactions, SLy5, LNS and BSk16 for this study. SLy5 is an interaction suited by construction to make predictions in neutron rich nuclei, LNS is an interaction which reproduces global features of G-matrix in symmetric and asymmetric nuclear matter and BSk16 reproduces known masses of nuclei with the best accuracy of rms deviation 632 keV. Some nuclear matter properties of these interactions are listed in Tab. I. The monopole Landau parameters in symmetric (SM) and neutron (NM) matter are drawn in Fig. 2 as a function of the density. As expected, these interactions has no instability in the isospin channel. Notice however that in the isospin channel, the Landau parameters  $F'_{0,\text{SM}}$  obtained from the three interaction are more attractive than the one deduced from the microscopic G-matrix calculation [18] (filled squares in Fig. 2), especially in the high density region. However, it is not necessary to correct this channel since there are no instabilities in the density range considered in Fig. 2. In the spin channels, the Landau parameters  $G_{0,\text{SM}}$ ,  $G'_{0,\text{SM}}$  and  $G_{0,\text{NM}}$  become more attractive as the density increases. The three Skyrme interactions have pathological behaviors in the spin channels  $G_0$  and  $G'_0$  for which we now propose a prescription to cure.

From the density dependence of the Landau parameters represented in Fig. 2, it is clear that the standard Skyrme interactions are not repulsive enough in the spin channel, and most probably also in the isospin channel at higher density than the saturation density. The repulsion shall come from the effect of the three body force [18]. It is then natural and compatible with density matrix expansion [19] to introduce new density dependent terms such as:

$$V^{\text{add.}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{6}t_3^s(1 + x_3^s P_\sigma)[\rho_s(\mathbf{R})]^{\gamma_s} \delta(\mathbf{r}) + \frac{1}{6}t_3^{st}(1 + x_3^{st} P_\sigma)[\rho_{st}(\mathbf{R})]^{\gamma_{st}} \delta(\mathbf{r}) \quad (2)$$

where  $P_\sigma = (1 + \sigma_1 \cdot \sigma_2)/2$  is the spin-exchanged operator,  $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$  and  $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ . In Eq. (2), we have introduced the spin density  $\rho_s = \rho_\uparrow - \rho_\downarrow$  and the spin-

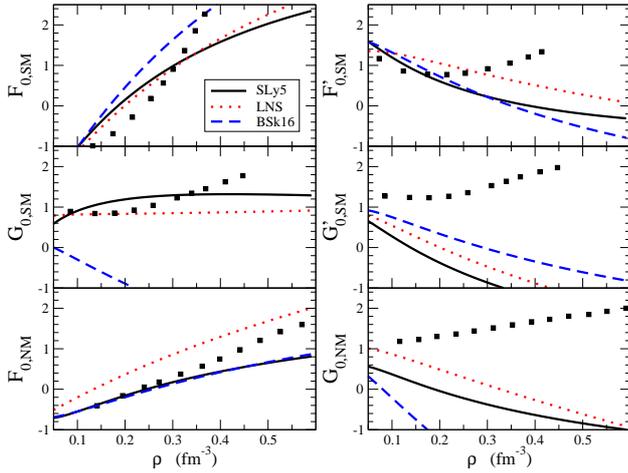


FIG. 2: (color online) Landau parameters in symmetric nuclear matter (SM) and neutron matter (NM) as a function of the density  $\rho$  obtained by the Skyrme interactions and the Brueckner HF calculations using 2BF+3BF [18]. Solid line for SLy5, dotted line for LNS, dashed line for BSk16 and filled squares for Brueckner HF calculations.

isospin density  $\rho_{st} = \rho_{n\uparrow} - \rho_{n\downarrow} - \rho_{p\uparrow} + \rho_{p\downarrow}$ . Spin symmetry is satisfied if the power of the density dependent terms  $\gamma_s$  and  $\gamma_{st}$  are even.

In the literature, it has already been proposed to add new terms to the standard Skyrme effective interaction. One of the motivation is that the HF single particle levels are usually too widely spaced at the Fermi surface. This shall be corrected by including the coupling of single particles to vibrations which increase the effective mass of the single particle states from  $m^*/m < 1$  to  $m^*/m \geq 1$  at the Fermi surface. This dynamical effect could be mimic by introducing a gradient term in the density dependent interaction (see Refs. [20, 21] and

references therein). Such a term has also been included in global fits of mass formula to make nucleosynthesis calculations as accurate as possible [22]. However, the removal of the spin instability makes symmetric matter unstable at high density [23]. Up to now, there is no satisfactory additional term which prevent the matter to fall into spin instabilities.

In the following, we adopt the notations of Ref. [11] where the density functional  $\mathcal{H}$  is expressed as a sum of the kinetic term  $\mathcal{K}$ , a zero range term  $\mathcal{H}_0$ , a density dependent term  $\mathcal{H}_3$ , an effective-mass term  $\mathcal{H}_{\text{eff}}$  and some additional terms coming from spin-orbit coupling, spin and gradient coupling and coulomb interaction. The additional terms (2) modify the density dependent part,  $\mathcal{H}_3$  to be  $\mathcal{H}_3 + \mathcal{H}_3^s + \mathcal{H}_3^{st}$ , where the additional density dependent terms reads

$$\mathcal{H}_3^s = \frac{t_3^s}{12} \rho_s^{\gamma_s} \left[ \left(1 + \frac{x_3^s}{2}\right) \rho^2 + \frac{x_3^s}{2} \rho_s^2 - \left(x_3^s + \frac{1}{2}\right) (\rho_n^2 + \rho_p^2) - \frac{1}{2} (\rho_{sn}^2 + \rho_{sp}^2) \right], \quad (3)$$

$$\mathcal{H}_3^{st} = \frac{t_3^{st}}{12} \rho_{st}^{\gamma_{st}} \left[ \left(1 + \frac{x_3^{st}}{2}\right) \rho^2 + \frac{x_3^{st}}{2} \rho_s^2 - \left(x_3^{st} + \frac{1}{2}\right) (\rho_n^2 + \rho_p^2) - \frac{1}{2} (\rho_{sn}^2 + \rho_{sp}^2) \right], \quad (4)$$

where  $\rho_{sn} = \rho_{n\uparrow} - \rho_{n\downarrow}$  and  $\rho_{sp} = \rho_{p\uparrow} - \rho_{p\downarrow}$ . The mean field  $U_q$ , where  $q = n, p$ , is then corrected to be

$$U_q^{\text{add.}} = \frac{t_3^s}{12} \rho_s^{\gamma_s} \{ (2 + x_3^s) \rho - (1 + 2x_3^s) \rho_q \} + \frac{t_3^{st}}{12} \rho_{st}^{\gamma_{st}} \{ (2 + x_3^{st}) \rho - (1 + 2x_3^{st}) \rho_q \}. \quad (5)$$

In symmetric nuclear matter the Landau parameters  $G_0$  and  $G_0'$  [24] are also modified by the following additional terms

$$\frac{G_0^{\text{add.}}}{N_0} = \frac{t_3^s}{48} \gamma_s (\gamma_s - 1) [3\rho^2 \rho_s^{\gamma_s - 2} - (2x_3^s + 1) \rho_t^2 - \rho_{st}^2] + \frac{t_3^{st}}{12} (x_3^{st} - \frac{1}{2}) \rho_{st}^{\gamma_{st}} + \frac{t_3^s}{24} (x_3^s - \frac{1}{2}) (\gamma_s + 1) (\gamma_s + 2) \rho_s^{\gamma_s}, \quad (6)$$

$$\frac{G_0'^{\text{add.}}}{N_0} = \frac{t_3^{st}}{48} \gamma_{st} (\gamma_{st} - 1) [3\rho^2 + (2x_3^s - 1) \rho_s^2 - (2x_3^{st} + 1) \rho_t^2] \rho_{st}^{\gamma_{st} - 2} - \frac{t_3^s}{24} \rho_s^{\gamma_s} - \frac{t_3^{st}}{48} (\gamma_{st} + 2) (\gamma_{st} + 1) \rho_{st}^{\gamma_{st}}. \quad (7)$$

In pure neutron matter, the additional terms for  $G_0$  are

$$\frac{G_0^{\text{add.}}}{N_0} = \frac{t_3^s}{24} (1 - x_3^s) \gamma_s (\gamma_s - 1) \rho^2 \rho_s^{\gamma_s - 2} + \frac{t_3^{st}}{12} (x_3^{st} - 1) \rho_{st}^{\gamma_{st}} + \frac{t_3^s}{24} (x_3^s - 1) (\gamma_s + 2) (\gamma_s + 1) \rho_s^{\gamma_s}. \quad (8)$$

Non-trivial result for the Landau parameters  $G_0^{\text{add.}}$  and  $G_0'^{\text{add.}}$  imposes a condition on the powers of the density

dependence in Eq. (2) to be uniquely fixed as  $\gamma_s = 2$  and  $\gamma_{st} = 2$ . Notice that spin symmetry is also satisfied by these selections. It turns out that the additional contributions (5) to the mean field are simply null in spin-saturated systems, as the densities  $\rho_s = 0$  and  $\rho_{st} = 0$ . It is thus possible to add the new terms (2) to the existing Skyrme forces without destroying good properties of the original Skyrme interactions. We obtain the contri-

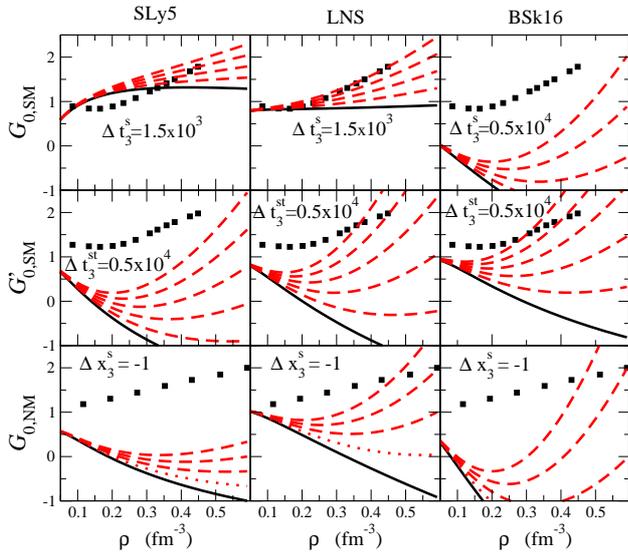


FIG. 3: (color online) Landau parameters for spin and spin-isospin channels  $G_0$  and  $G'_0$  in symmetric nuclear matter and neutron matter. The solid curves are the original ones, while the filled squares are obtained by the Brueckner HF calculations as shown in Fig. 2. The dashed curves correspond to different values of the parameters  $t_3^s$ ,  $t_3^{st}$  and  $x_3^s$ . The parameters are changed from bottom to top multiplying integers (1,2,3,-) by the mesh size given in each window. The Landau parameter  $G_{0,NM}$  is calculated with the optimal value for  $t_3^s$  in Tab. I, which reproduces best the Brueckner HF  $G_{0,SM}$  values. The thin dotted lines in the bottom panels correspond to the results with  $x_3^s=0$ .

contributions to the Landau parameters;

$$G_0^{\text{add.}} = \frac{N_0}{8} t_3^s \rho^2, \quad (9)$$

$$G_0'^{\text{add.}} = \frac{N_0}{8} t_3^{st} \rho^2, \quad (10)$$

in spin-saturated symmetric matter and

$$G_0^{\text{add.}} = \frac{N_0}{12} t_3^s (1 - x_3^s) \rho^2. \quad (11)$$

in spin-saturated neutron matter. The contributions to the Landau parameters  $G_0$  and  $G'_0$  in symmetric matter in Eq. (9) and (10) depend only on the parameters  $t_3^s$  and  $t_3^{st}$ , while that for the Landau parameter  $G_0$  in neutron matter in Eq. (11) depends on  $t_3^s$  and  $x_3^s$ . We shall then first fix  $t_3^s$  and  $t_3^{st}$  so as to reproduce the Landau parameters in symmetric matter, and then we fix  $x_3^s$  in neutron matter. Since the Landau parameters  $G_0$  and  $G'_0$  are independent of  $x_3^{st}$ , this parameter can be set as  $x_3^{st}=0$ .

We show in Fig. 3 the contributions of the new terms for the Landau parameters  $G_0$  and  $G'_0$  in symmetric matter and for  $G_0$  in neutron matter added to the three effective interactions SLy5, LNS and BSk16. We compare the original Landau parameters in Fig. 2 (solid lines) with

	$t_3^s$	$t_3^{st}$	$x_3^s$	$G_0$	$G_0^{new}$	$G'_0$	$G_0'^{new}$
SLy5st	$0.6 \times 10^4$	$2 \times 10^4$	-3	1.11	1.19	-0.14	0.15
LNSst	$0.6 \times 10^4$	$1.5 \times 10^4$	-1	0.83	0.95	0.14	0.45
BSk16st	$2 \times 10^4$	$1.5 \times 10^4$	-2	-0.65	-0.32	0.51	0.75

TABLE II: Parameters of the additional terms  $t_3^s$  (in  $\text{MeV} \cdot \text{fm}^{3\gamma_s-2}$ ),  $t_3^{st}$  (in  $\text{MeV} \cdot \text{fm}^{3\gamma_{st}-2}$ ) and  $x_3^s$  for the interactions SLy5st, LNSst and BSk16st in order to reproduce the realistic Landau parameters by Brueckner HF calculations. The powers of the density dependence in Eq. (2) are set to be  $\gamma_s = \gamma_{st} = 2$ , and  $x_3^{st} = 0$ . Are also shown the dimensionless Landau parameters  $G_0$ ,  $G_0^{new}$ ,  $G'_0$  and  $G_0'^{new}$  deduced from the original Skyrme interactions and the new interactions including the additional terms, respectively.

the ones which include the contributions of the new terms (6), (7) and (8). The dashed lines correspond to different values of the new parameters  $t_3^s$ ,  $t_3^{st}$  and  $x_3^s$  with the step indicated in each graph. The values for the parameters  $t_3^s$ ,  $t_3^{st}$  and  $x_3^s$  resulting from the best adjustment for asymptotic behavior at very high density to the Brueckner HF calculations are given in Tab. I and are labelled SLy5st, LNSst and BSk16st respectively. With the parameters given in Tab. I, it is confirmed that the correction terms to the Landau parameters (9)-(11) are repulsive enough to remove the spin instabilities. There are however still differences between the corrected Landau parameters and the ones given by the G-matrix which could be originated from the tensor interaction. We can see that the LNS interaction, fitted originally to the equation of state deduced from the G-matrix, gives the best fitted Landau parameters among the three interactions. Finally, it could be noticed that the corrected Landau parameters  $G_0^{new}$  are increased by about +0.3 which makes it closer to the empirical value. Notice that the empirical value is estimated to be  $1.3 \pm 0.2$  from Wood-Saxon single-particle states plus one-pion and rho meson models (see for instance Tab.I of Ref. [25] and references therein and also Ref. [26]) and  $0.7 \pm 0.1$  from self-consistent Skyrme mean-field models [27].

## II. RPA RESPONSE FUNCTIONS AND NEUTRINO PROPAGATION

As a consequence of the new terms the spin channels are more repulsive than the original Skyrme functional. The effect of the new terms could then modify the response function of collective spin modes already at  $\rho_0$ . For instance, we show in Tab. I the Landau parameters  $G_0$  and  $G'_0$  deduced from the original Skyrme interactions and the one including the new terms  $G_0^{new}$  and  $G_0'^{new}$ . The increased of the Landau parameters may have significant influence on spin and spin-isospin excitations such as Gamow-Teller states.

In the following, we calculate the RPA response func-

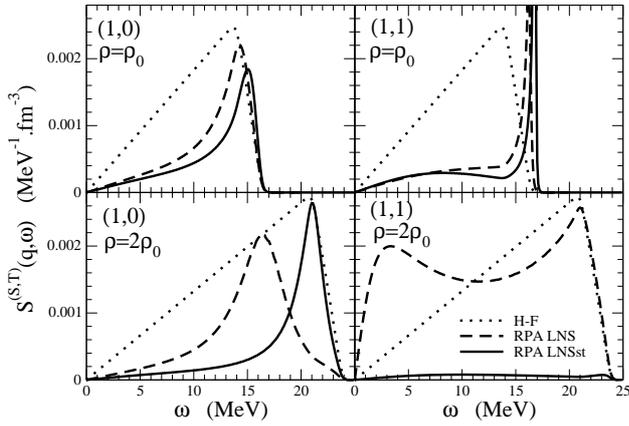


FIG. 4: RPA response functions  $S^{(S,T)}(q, \omega)$  for  $q=0.22 \text{ fm}^{-1}$  with the temperature  $T=0 \text{ MeV}$  calculated with LNS and LNSst interactions. Top panels are calculated for  $(S, T) = (1, 0)$  and  $(1, 1)$  with  $\rho = \rho_0$  and bottom panels are for the same channels with  $\rho = 2\rho_0$ .

tion in nuclear matter at finite temperature,

$$S^{(S,T)}(q, \omega, T) = -\frac{1}{\pi} \frac{1}{1 - e^{-\omega/T}} \text{Im} \chi^{(S,T)}(q, \omega, T) \quad (12)$$

where  $\chi^{(S,T)}(q, \omega, T)$  is the susceptibility [28] obtained as the solution of the Bethe-Salpeter equation [29]. In Fig. (4), we show the response functions in the spin channels ( $S = 1$ ) at  $T = 0 \text{ MeV}$  and at densities  $\rho = \rho_0$  and  $\rho = 2\rho_0$  calculated by the LNS and LNSst interactions. The HF solution (dotted line) is compared with the RPA using the original LNS Skyrme interaction (dashed line) and also with the RPA including the new terms in LNSst (solid line). According to the semi-classical Steinwedel-Jensen model [30] the optimal transferred momentum to compare with nuclei shall be  $q = \pi/2R$  where  $R$  is the radius of the nuclei. For  $^{208}\text{Pb}$ , one thus obtain  $q=0.22 \text{ fm}^{-1}$ . For  $\rho = \rho_0$ , the effect of the new terms is to move the collective mode to slightly higher energies by 0.5-1 MeV. For  $\rho = 2\rho_0$ , the  $(S = 1, T = 1)$  Gamow-Teller channel is not far from being unstable from the original Skyrme interaction and the low energy collective mode is being formed. By the new term  $t_3^{st}$ , the low energy mode is suppressed and the strength is reduced substantially at  $\omega > 0$ . A reduction of the strength at low energy is also observed for the  $(S = 1, T = 0)$  channel. This effect shall be seen in the calculation of microscopic processes such as the neutrino cross section.

Neutrinos play a crucial role in physics of supernova explosions [31] and in the early evolution of their compact stellar remnants [32, 33, 34]. To clarify the effect coming from the additional interaction (2), we consider the case of pure neutron matter. The scattering of neutrinos on neutrons is then mediated by the neutral current of the electroweak interaction. In the non-relativistic limit, the mean free path of neutrino with initial energy  $E_\nu$  is given

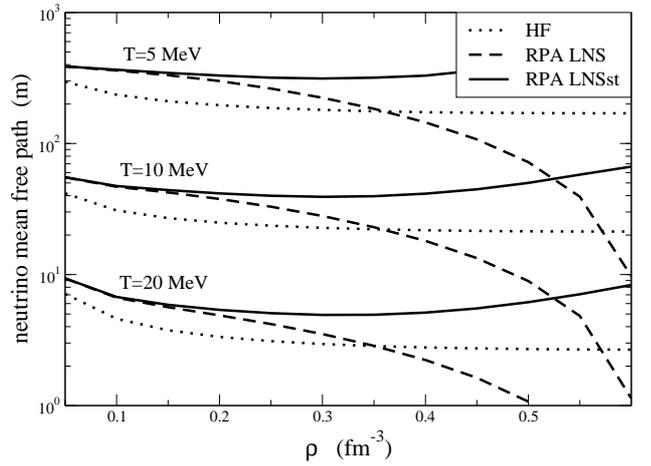


FIG. 5: Neutrino mean free path in neutron matter for different temperatures  $T=1, 10$  and  $20 \text{ MeV}$ . The neutrino energy is set to be  $E_\nu = 3T$ . The mean free path is calculated with the LNS Skyrme interaction by the HF mean field approximation (dotted line), including the RPA correlations with the original LNS interaction (dashed line) and RPA with the modified spin channel in LNSst (thick line).

by [35]

$$\lambda^{-1}(E_\nu, T) = \frac{G_F^2}{16\pi^2} \int d\mathbf{k}_3 \left( c_V^2 (1 + \cos \theta) S^{(S=0)}(q, \omega, T) + c_A^2 (3 - \cos \theta) S^{(S=1)}(q, \omega, T) \right), \quad (13)$$

where  $G_F$  is the Fermi constant,  $c_V$  ( $c_A$ ) is the vector (axial) coupling constant,  $k_1 = (E_\nu, \mathbf{k}_1)$  and  $k_3$  are the initial and final neutrino four-momenta, while  $q = k_1 - k_3$  is the transferred four-momentum, and  $\cos \theta = \hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_3$ . In the following, we impose the average energy  $E_\nu = 3T$  in MeV [34]. In Eq. (13), the contribution of the response function  $S^{(S)}(q, \omega, T)$  is clearly identified. It describes the response of neutron matter to excitations induced by neutrinos, and contain the relevant information on the medium. The vector (axial) part of the neutral current gives rise to density (spin-density) fluctuations, corresponding to the spin  $S = 0$  ( $S = 1$ ) channel.

The neutrino mean free path is shown in Fig. 5 as a function of the baryonic density and for different temperatures ( $T=1, 10, 20 \text{ MeV}$ ). The neutrino mean free path calculated with the original LNS Skyrme interaction (dashed line) is strongly reduced at high density as a consequence of the onset of the spin instabilities [36, 37]. The inclusion of the new density dependent interaction (2) in LNSst removes the spin instability and reduces the response function as shown in Fig. 4. As a consequence, the mean free path is increased and the matter is more transparent to neutrino as the density increases. This result is similar to a previous calculation of neutrino mean free path deduced directly from a microscopic G-matrix [38].

It illustrates the important contribution of the spin channel to the neutrino mean free path at high density. It is also clear from Fig. 5 that the effects of the correlations are not washed out by the temperature in the range going up to  $T = 20$  MeV.

### III. CONCLUSIONS

We have proposed the extension of the Skyrme interaction (2) which conserves its simplicity and extends its domain of stability to large densities and large isospin asymmetries. The parameters of the new terms have been adjusted based on microscopic G-matrix calculations. For the three interactions SLy5st, LNSst and BSk16st, the spin channels become more repulsive compared to the original interactions SLy5, LNS and BSk16; and the dimensionless Landau parameter  $G'_0$  is increased by about +0.3. The simplicity of the extended Skyrme interaction makes possible extensive calculations not only in asymmetric and dense matter but also in asymmetric finite nu-

clei. We have calculated RPA response functions of spin channels and also neutrino mean free path in dense matter. The collapse of the neutrino mean free path at high density is suppressed by the newly added terms in the Skyrme interaction and the overall effect of the RPA correlations makes dense matter more transparent for neutrino propagation by a factor of 2 to 10 depending on the density.

We would like to extend further the applications of our new spin-density dependent interactions for the systematic study of odd mass nuclei, the spin-dependent excitations in finite nuclei and also infinite nuclear matter. Eventually our ultimate goal is to construct the global energy density functional for the spin and spin-isospin dependent interactions including tensor and two-body spin-orbit forces.

### Acknowledgments

This work was supported by the Japanese Ministry of Education, Culture, Sports, Science and Technology by Grant-in-Aid for Scientific Research under the Program number C(2) 20540277.

- 
- [1] D. Vautherin and D.M. Brink, Phys. Rev. C **5**, 626 (1972).
  - [2] J.R. Stone, J.C. Miller, R. Koncewicz, P.D. Stevenson, M.R. Strayer, Phys. Rev. C **68** 034324 (2003).
  - [3] J. Margueron, J. Navarro, and N.V. Giai, Phys. Rev. C **66**,014303 (2002).
  - [4] P. Haensel and S. Bonazzola, Astron. Astrophys. 314, 1017 (1996).
  - [5] D. Bhattacharya and V. Soni, arXiv:0705.0592(astro-ph).
  - [6] C. Thompson and R. C. Duncan, Mon. Not. R. Astron. Soc. 275, 255 (1995).
  - [7] D. Lazzati, Nature 434, 1075 (2005).
  - [8] N. Chamel, S. Goriely, J. M. Pearson, Nucl. Phys. **A 812**, 72 (2008).
  - [9] M. Rayet, M. Arnould, F. Tondeur and G. Paulus, Astron. Astrophys. 116, 183 (1982).
  - [10] J.Bartel et al., Nucl. Phys. **A 386**, 79 (1982).
  - [11] E. Chabanat et al., Nucl. Phys. **A 627** 710 (1997); *ibid.*, **A 635** 231 (1998), *ibid.*, **A 643** 441 (1998).
  - [12] N. Van Giai and H. Sagawa, Nucl. Phys. **A 371**,1 (1981).
  - [13] L. G. Cao, U. Lombardo, C. W. Shen, N.V. Giai, Phys.Rev. C **73**, 014313 (2006).
  - [14] S. Fantoni, A. Sarsa, and K.E. Schmidt, Phys. Rev. Lett. 87, 181101 (2001)
  - [15] I. Vidaña, A. Polls and A. Ramos, Phys. Rev. C **65**, 035804 (2002).
  - [16] I. Vidaña and I. Bombaci, Phys. Rev. C 66, 045801 (2002).
  - [17] I. Bombaci, A. Polls, A. Ramos, A. Rios, and I. Vidaña, Phys. Lett. **B 632**, 638 (2006).
  - [18] W. Zuo, C. Shen and U. Lombardo, Phys. Rev. C **67**, 037301 (2003); C. Shen, U. Lombardo, N. Van Giai, and W. Zuo, Phys. Rev. C **68**, 055802 (2003).
  - [19] J. W. Negele and D. Vautherin, Phys. Rev. C **5**, 1472 (1972); *ibid.* **C 11**, 1031 (1975).
  - [20] M. Waroquier, K. Heyde and W. Wenes, Nucl. Phys. **A 404** (1983) 269.
  - [21] K.-F. Liu, H. Luo, Z. Ma, Q. Shen and S. A. Moszkowski, Nucl. Phys. **A 534** (1991) 1.
  - [22] M. Farine, J. M. Pearson and F. Tondeur, Nucl. Phys. **A696** (2001) 396.
  - [23] M. Waroquier, J. Sau, K. Heyde, P. Van Isacker and H. Vincx, Phys. Rev. C. **19** (1979) 1983.
  - [24] S. Hernández, J. Navarro, A. Polls, Nuclear Physics **A 658**, 327 (1999); *ibid.* Nuclear Physics **A 627**, 460 (1997).
  - [25] F. Osterfeld, Rev. Mod. Phys. **64** (1992) 491.
  - [26] E. T. Suzuki and H. Sakai, Phys. Lett. B455 (1999) 25-29; M. Ichimura, H. Sakai and T. Wakasa, Prog. Part. Nucl. Phys., 56 (2006) 446-531; T. Wakasa, M. Ichimura and H. Sakai, Phys. Rev. **C 72** (2005) 067303.
  - [27] S. Fracasso and G. Colò, Phys. Rev. **C 76**, 044307 (2007).
  - [28] A. L. Fetter and J. D. Walecka, quantum Theory of Many-Particle Systems, McGraw- Hill, New-York (1971).
  - [29] C. Garcia-Recio, J. Navarro, N. Van Giai and L. L. Salcedo, Ann. Phys. (N.Y.) **214**, 293 (1992).
  - [30] P. Ring and P. Schuck, The nuclear many body problem, Springer, Berlin, 1982, page 558.
  - [31] H.A. Bethe, Rev. Mod. Phys. 62 (1990) 801.
  - [32] A. Burrows and J. M. Lattimer, ApJ **307** (1986) 178.
  - [33] H.-Th. Janka and E. Müller, ApJ **448** (1995) L109.
  - [34] S. Reddy, M. Prakash, J. M. Lattimer, and J. A. Pons, Phys. Rev. **C 59**, 2888 (1999).
  - [35] N. Iwamoto and C.J. Pethick, Phys. Rev. **D 25**, 313 (1982).
  - [36] J. Margueron, J. Navarro, N. Van Giai and W. Jiang, "The Nuclear Many-Body Problem 2001", NATO Science Series II (Kluwer Academic Publishers).
  - [37] J. Navarro, E.S. Hernández and D. Vautherin, Phys. Rev. **C 60**, 045801 (1999).
  - [38] J. Margueron, I. Vidaña and I. Bombaci, Phys.Rev. **C 68**, 055806 (2003).