HAL
open science

Stable chaos in the 55Cnc exoplanetary system?

J. Gayon, F. Marzari, H. Scholl

To cite this version:

J. Gayon, F. Marzari, H. Scholl. Stable chaos in the 55Cnc exoplanetary system?. Monthly Notices of the Royal Astronomical Society, 2008, 389, pp.L1-L3. 10.1111/j.1745-3933.2008.00505.x . hal-00382479

HAL Id: hal-00382479

https://hal.science/hal-00382479v1

Submitted on 15 Dec 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Mon. Not. R. Astron. Soc. **389,** L1–L3 (2008) doi:10.1111/j.1745-3933.2008.00505.x

Stable chaos in the 55Cnc exoplanetary system?

J. Gayon,[1★] F. Marzari[2] and H. Scholl[1]

[1]*Laboratoire Cassiopée, Université de Nice Sophia Antipolis, CNRS, Observatoire de la Côte d'Azur, B.P. 4229, F-06304 Nice, France*
[2]*Department of Physics, University of Padova, Via Marzolo 8, 35131 Padova, Italy*

Accepted 2008 May 23. Received 2008 May 21; in original form 2008 April 25

ABSTRACT

The five planets discovered around the main-sequence star 55Cnc may represent a case of stable chaos. By using both the Frequency Map Analysis and MEGNO, we find that about 15 per cent of the systems that can be build from the nominal orbital elements of the system are highly chaotic. However, in spite of the fast diffusion rate in the phase space, the planetary system is not destabilized over 400 Myr and close encounters between the planets are avoided.

Key words: planets and satellites: formation – stars: individual: 55Cnc – planetary systems: formation.

Downloaded from https://academic.oup.com/mnrasl/article/389/1/L1/995790 by guest on 15 December 2020

1 INTRODUCTION

The most crowded extrasolar planetary system discovered so far is that around the main-sequence star 55Cnc ($=\rho^1$ Cancri). Doppler shift measurements strongly suggest the existence of five planets orbiting the star with semimajor axes ranging from 0.038 to 5.901 au (Fischer et al. 2008). The innermost and smallest planet, *e*, is a Neptune-mass object while the outer and most massive one, planet *d*, has a minimum mass of about four Jupiter masses. A self-consistent dynamical fit to the stellar wobble data performed by Fischer et al. (2008) gives a set of orbital elements for the planets reported in Table 1. It is claimed (Fischer et al. 2008) that this nominal system of five planets described in Table 1 is dynamically stable at least over a time-scale of 1 Myr. We would like to point out that planets *b* and *c* are no longer in a 3:1 mean motion resonance as in the previous solution given by McArthur et al. (2004). In addition, the eccentricity of planet *c* in Fischer et al. (2008) is significantly smaller compared to that derived by McArthur et al. (2004).

We have performed a detailed exploration of the stability of the nominal system by applying the Frequency Map Analysis (FMA) method (Laskar, Froeschlé & Celletti 1992; Laskar 1993a,b; Marzari, Scholl & Tricarico 2006) on 400 varied systems. These systems are obtained by varying orbital elements of the nominal system. The varied systems are analysed by adding randomly some inclination (lower than 5°) to the Keplerian orbits to make the system more realistic. The masses of the planets are scaled accordingly. Among all the systems analysed with FMA, we obtain cases with large diffusion speed in the phase space suggesting a chaotic evolution and possible instability. However, integrating a few of these chaotic systems over time-scales of 10^8 yr, we never obtain destabilization. We, therefore, think that these chaotic systems are examples for stable chaos. This term was introduced by Milani & Nobili (1992) to indicate the peculiar behaviour of asteroids with short Lyapunov times (of the order of some 10^3 yr) which, however, show a remarkable stability over the age of the Solar system (Milani & Nobili 1992). For the 55Cnc planetary system, we find that in about 15 per cent of all cases the orbital elements of the four inner planets exhibit a chaotic evolution on a time-scale of a few million of years. This stable chaos may be due to the topology of the phase space: quasi-periodic solutions in the proximity of a chaotic island can act like 'quasi-barriers' for the diffusion (Tsiganis, Varvoglis & Hadjidemetriou 2000). The chaotic evolution appears to be confined and the system does not destabilize at least on a time-scale of 400 Myr. This peculiar behaviour may be due to the closeness of the system to the planetary 3:1 mean motion resonance between planets *b* and *c* and secular resonances.

To confirm that stable chaos is not limited to the nominal case but can be retrieved also in a larger region of the phase space, we randomly varied the orbital elements of the nominal case and found a similar behaviour. For a few selected cases, we also applied the Mean Exponential Growth factor of Nearby Orbits (MEGNO) method proposed by Cincotta & Simò (2000). The results obtained by MEGNO confirm the FMA results.

2 FMA ANALYSIS

We have performed the FMA on 400 different systems derived from Table 1. Random mean anomalies, node longitudes and inclinations lower than 5° are assigned to each planet. All other orbital elements were taken from Table 1. The frequency analysis is performed over a time-scale of 2×10^4 yr. The orbital elements computed with the numerical integrator SYMBA (Duncan, Levison & Lee 1998) are Fourier analysed and the values of intrinsic frequencies are obtained over running windows. The relative changes of these frequencies are estimated and used to compute the diffusion rate in the phase space. We have applied FMA to the signal $s_{\Delta\varpi_{b,c}}$, the difference between the periapse longitude of planets *b* and *c*, and to the more conventional signal $s = h_e + \mathrm{i}k_e$. The usual non-singular variables $h_e = \mathrm{ecc} * \cos(\varpi)$ and $k_e = \mathrm{ecc} * \sin(\varpi)$

★E-mail: Julie.Gayon@oca.eu

© 2008 The Authors. Journal compilation © 2008 RAS

Table 1. Orbital parameters for the self-consistent dynamical fit of the 55Cancri planetary systems. These data are taken from table 4 of Fischer et al. (2008).

Planet	Period (d)	T_p (JD 2440000)	e	ω (°)	$M \sin i$ ($M_{\rm Jup}$)	a (au)
b	14.651 262	7572.0307	0.0159	164.001	0.8358	0.115
c	44.378 710	7547.5250	0.0530	57.405	0.1691	0.241
d	5371.8207	6862.3081	0.0633	162.658	3.9231	5.901
e	2.796 744	7578.2159	0.2637	156.500	0.0241	0.038
f	260.6694	7488.0149	0.0002	205.566	0.1444	0.785

refer to the innermost planet *e* where ecc refers to its eccentricity. The diffusion rate is computed by using the standard deviation σ of the main frequency of the signal computed over the running windows. Slow diffusion rates, characterized by small values of σ, mean quasi-periodic systems, while large values of σ imply chaotic evolution. As in Marzari et al. (2006), we measure the diffusion speed by the logarithmic number $v_{\rm d} = -\log_{10}(\sigma) + \log_{10}(\sigma_0)$ where σ_0 is the smallest value of σ we observed in our sample of systems. A small value of $v_{\rm d}$ means a low dispersion of the frequencies in the considered time interval and, hence, a slow diffusion speed in the phase space. Large values of $v_{\rm d}$ indicate fast changes of the system frequencies and, therefore, chaos.

In Fig. 1, we show the distribution of the diffusion speed measured by $v_{\rm d}$ in our sample of 400 55Cnc planetary systems. The values smaller than four suggest long-term stability according to our previous experience with FMA, while larger values indicate chaos. We concentrate on systems with a diffusion speed of about five or larger which represent about 15 per cent of the whole sample we analysed. These systems have major frequencies which change on short time-scales. If the fast diffusion of major frequencies also induces drastic changes of amplitudes, in particular of planetary eccentricities, close encounters between planets may occur resulting in the ejection of one or more planets. We will show in the next section that planetary eccentricities do not increase in longer time-scales which suggest a 'stable chaos' state for the Cnc55 system.

An additional set of FMA simulations has been performed for 400 systems where we have randomly varied the last significant digit of all the orbital elements given in Table 1. We obtain a histogram that is very similar to that shown in Fig. 1 confirming that the behaviour of stable chaos we have found for the nominal case of Table 1 is extended in phase space.

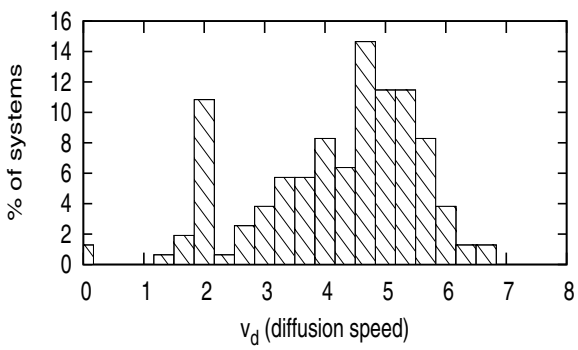

Figure 1. Histogram showing the distribution of the diffusion rate within our sample of 400 55Cnc planetary systems.

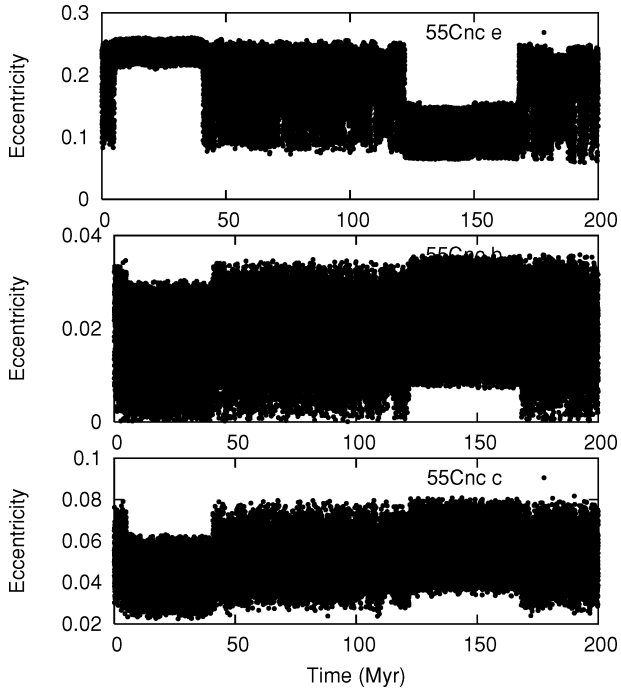

Figure 2. Evolution with time of the eccentricity of the three inner planets of the system. The top plot shows the eccentricity of 55Cnc *e*, the medium plot that of 55Cnc *b* and the bottom plot that of 55Cnc *c*.

3 LONG-TERM EVOLUTION OF THE CHAOTIC SYSTEMS

For those systems with a large diffusion speed ($v_{\rm d} > 4$), we performed numerical integrations over a time-scale of 400 Myr with SYMBA (Duncan et al. 1998). In Fig. 2, we show for a typical run the evolution of the eccentricity of the three inner planets *e*, *b*, *c*. The eccentricity jumps reveal the chaotic nature of the system. The jumps are obviously correlated among the three planets. The major jumps occur simultaneously. Also the eccentricity of planet *f* shows simultaneous but more moderate jumps. All jumps, although they appear to be significant, do not result in close approaches among the planets. The planetary system is not destabilized on this time-scale while it is chaotic.

Fig. 2 suggests that we have the case of stable chaos. The eccentricity evolution is characterized by jumps. It does not have a random walk growth but remains limited. It appears that the system is bouncing between the limited regions.

A different case with similar high diffusion speed $v_{\rm d}$ is shown in Fig. 3. The semimajor axes of the three inner planets are shown on a shorter time-scale to highlight the chaotic evolution. Also in this case, the system remains confined in a stable configuration over 400 Myr.

4 MEGNO ANALYSIS

Besides the FMA method which uses the diffusion rate of intrinsic frequencies to measure chaos, there is a large class of methods which use the divergence of nearby orbits in phase space as a measure. Exponential divergence means chaos while linear divergence means non-chaotic which guarantees stability of the dynamical system. The stability means here that, in particular, no planet is ejected out of the system. A nearly exponential divergence does not necessarily mean instability in our sense over the lifetime of the system. As

Downloaded from https://academic.oup.com/mnrasl/article/389/1/L1/995790 by guest on 15 December 2020

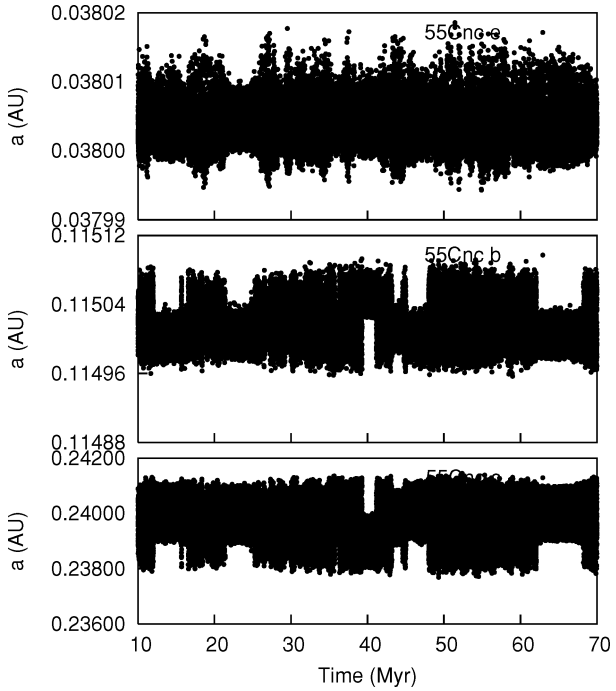

Figure 3. Semimajor axis of the three inner planets of the system over a short time-scale.

outlined above, we would qualify such a system as stable chaotic. The widely used Lyapunov characteristic numbers (LCNs) yield the necessary information about the divergence and are used to measure the chaocity of the system. Since the computation of the LCNs is very time consuming, more rapid methods were introduced which were shown to yield a very good estimation for the LCNs. One of these fast LCN estimators is the MEGNO (Cincotta & Simò 2000) method. It was applied to several exoplanetary systems (Bois et al. 2003). We computed the MEGNO indicator for about 50 varied systems. They were obtained by varying only the eccentricities of planets e, b, c and f while taking the other orbital elements from the nominal system of Table 1. Eccentricities of planet e are varied between 0.22 and 0.26, of planets b and c between 0.0 and 0.10 and of planet f between 0.0 and 0.04. We integrated the systems over 100 000 yr. For eccentricities of planets b and c ranging from 0.0 to 0.02, we obtain weak chaos. The estimator for the maximal Lyapunov exponent is not linear but far from exponential. For larger eccentricities, the indicator has a behaviour between linear and exponential. MEGNO shows, like FMA, that the four planetary orbits close to the nominal system are chaotic.

5 CONCLUSIONS

Among the possible dynamical configurations of the 55Cnc planetary system, there is also a stable chaos. About 15 per cent of the systems built from the nominal dynamical fit given in Fischer et al. (2008) have a high diffusion speed in the phase space. However, the chaotic island seems to be limited in extent and surrounded by quasi-periodic systems. In this way, the chaotic systems do not increase their eccentricity to planet crossing values and they are stable over a long time-scale. Systems with intermediate values of diffusion speed $v_{\rm d}$ as measured by FMA ($\sim$4) do not show this behaviour. They possibly populate the outer border of the chaotic region and their chaotic evolution is much slower. Systems with smaller values of $v_{\rm d}$ are possibly stable over time-scale of 10^9 yr. Additional FMA and MEGNO computations show that this behaviour is not only peculiar for the nominal system given by Table 1 but it is also extended over a wider range of orbital elements for the system. The weak resonances responsible for the stable chaos are present also for different combinations of the initial orbital elements.

ACKNOWLEDGMENTS

We thank J. Hadjidemetriou for his useful comments and suggestions that helped to improve the manuscript. The MEGNO computations were performed on the 'Mesocentre SIGAMM' machine, hosted by the Observatoire de la Côte d'Azur.

REFERENCES

Bois E., Kiseleva-Eggleton L., Rambaux N., Pilat-Lohinger E., 2003, ApJ, 598, 1312
Cincotta P., Simó C., 2000, A&AS, 147, 205
Duncan M. J., Levison H. F., Lee M. H., 1998, AJ, 116, 2067
Fischer D. A. et al., 2008, ApJ, 675, 790
Laskar J., 1993a, Physica D, 67, 257
Laskar J., 1993b, Celest. Mech. Dyn. Astron., 56, 191
Laskar J., Froeschlé C., Celletti A., 1992, Physica D, 56, 253
McArthur B. E. et al., 2004, ApJ, 614, L81
Marzari F., Scholl H., Tricarico P., 2006, A&A, 453, 341
Milani A., Nobili A., 1992, Nat, 357, 569
Tsiganis K., Varvoglis H., Hadjidemetriou D., 2000, Icarus, 146, 240

This paper has been typeset from a TEX/LATEX file prepared by the author.

Downloaded from https://academic.oup.com/mnrasl/article/389/1/L1/995790 by guest on 15 December 2020