On the improved use of Chebyshev expansion for mechanical system identification

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Abstract

Differential equations, and particularly those governing mechanical system behavior, can be transformed into algebraic equations by using the well known properties of polynomial functions. In this paper, by using Chebyshev polynomials, improvements of classical methods are proposed for the identification and inverse formulation of mechanical systems. The drawbacks of these polynomials compel us to propose alternative formulations by dissociating signal expansions and parameter estimation.

Before any identification, excitation and response signals have to be correctly expanded in the polynomial basis. With Chebyshev polynomials, this requires the signals to be in a the same narrow frequency bandwidth. In the presence of noisy signals, correct parameter identification can be performed with a new formulation by taking only central points within the time interval. This methodology can be generalized by defining any other point selection criterion.

This new formulation is first tested on a linear 3-DOF mechanical system and then extended to the parameter identification of a non linear mecahnical component by using the product property of Chebyshev polynomials. The identification carried out on the hysteretic behavior measured and exhibited by a tape-spring actuator gives relevant parameter estimation.

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The proposed methodologies and formulations can be easily extended to other orthogonal functions in association with differential or integral transformations of differential equations.

Key words: polynomial, identification, Chebyshev, mechanical system, non linear

1 Introduction

Predicting the dynamic behavior of complex mechanisms and/or structures requires reliable models. But boundary conditions, uncertain parameters, especially damping parameters make them difficult to formulate. Therefore, experimental models could be used in particular when operating conditions and or mechanical properties change during the life-cycle of a mechanism. Generally speaking, experimental models are mainly used for:

- modeling behavior by identifying model parameters for given measured excitations and responses. This identification problem is also related to the updating of theoretical or numerical models.
- predicting responses due to a specific excitation. Models are solved with differential equation resolution, by using classical integration methods.
- predicting excitations from the responses with a well defined model. These inverse problems are used when the direct measurement of excitations is ineffective or impossible.

In all these problems, the main difficulty lies in the transformation of differential equations into a set of easy to process algebraic equations.
Traditionally, a discrete form of these equations is obtained by applying the Z transform on signals when linear assumptions are available. This leads to

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an equivalent recurrent equation when the signals are sampled at a constant rate and when the Zero Order Holder assumption is assumed. However an over-high sampling rate makes it difficult to obtain the physical parameter [11]. Moreover, this method requires specific excitations (e.g. Pseudo-Random Binary Signal, white noise, etc) which are not compatible with operating conditions.

Over the last few decades, alternative methods have been proposed and gathered under the banner of Continuous Time Identification methods (see [1; 5; 9; 11]). In these approaches, orthogonal functions are frequently used in an integral formulation of differential equations. Their main advantage is that they transform the integration of signals into a simpler integration of these functions by making use of a square matrix that depends on the orthogonal functions. Therefore, the differential equations governing the behaviour of the mechanical system can be transformed into algebraic equations.

In [12], the authors describe several applications that have been developped since the 1990's for identifying controlled systems and MDOF systems. They compare different kinds of orthogonal bases such as Jacobi, Legendre or Chebyshev polynomials, Block-Pulse or Walsh functions and, of course, Fourier series. They also mention the easiness of integral formulation in the field of inverse problems and the simplification of the calculation in sensitivity analysis problems. In [13], they also use this integral formulation in the presence of nonlinearities such as the Duffing oscillator or dry friction damper. In the field of identification of nonlinear behavior, other contributors have proposed similar approaches using Chebyshev polynomials early in the 1980's [4; 8; 10]. Other contributors (see [6; 7]) have proposed a similar approach based on a derivative formulation with wavelet bases. The authors reach the same general conclusions on the relevancy of the using of orthogonal functions for solving problems based on differential equations, with linear, time variant or invari-

ant, or non linear behavior. However, the derivative formulation is much more convenient for identification, where the estimation of initial states is not required, and for inverse problems, where no matrix inversion is necessary.
Moreover, Pacheco and Steffen in [12] showed that the results obtained with Chebyshev, Jacobi or Legendre polynomials are not satisfactory when random excitations are applied, although they provided no explanation for this. This difficulty is mainly due to the particular shape of Chebyshev polynomials which leads to very poor expansions for wide frequency bandwidth signals or edges effects for smooth signals (Gibbs phenomenon). Therefore, attention must be paid to the quality of the signal expansion before any parameter estimation. This drawback of Chebyshev polynomials compels us to propose a more general identification methodology which can be easily extended to other classical polynomials.

In Section 2 the properties of Chebyshev polynomials and a derivative formulation based on these polynomials are recalled. In our approach, the general identification process is clearly split into signal expansion and parameter estimation. Two general indicators are introduced in order to qualify these two steps.
In Section 3, expansion quality is analysed through different excitations. An improved general identification methodology is proposed for cases of poor signal expansion. An illustration is given for the case of noisy signal expansion. Then, nonlinear behavior is identified by introducing the product property of Chebyshev polynomials in this formulation.
A particular case of nonlinear identification is treated in Section 4. The hysteretic behavior of a tape-spring coiling device is taken as an experimental application of the proposed method.

2 Chebyshev polynomials properties and derivative formulation

2.1 Basic properties of the Chebyshev polynomials

The n order Chebyshev polynomials defined in the interval [-1,+1] by the following equation :

$$\mathrm{T}_n(\tau) = cos(n \cdot arccos(\tau)) \tag{1}$$

compose an orthogonal basis $\{T^n(\tau)\} = \langle T_0(\tau)\ T_1(\tau)\ T_2(\tau) \cdots T_n(\tau) \rangle^T$. Any function $x(\tau)$ can be expanded as follows :

$$x(\tau) = \left\langle x_0\ x_1\ x_2 \cdots\ x_n \right\rangle . \{T^n(\tau)\} \tag{2}$$

where x_i are the coordinates of $x(\tau)$ in the basis $\{T^n(\tau)\}$.
The derivative of each polynomial can be expressed as a combination of polynomials with a lower order, that leads to :

$$\frac{dT_n}{d\tau} = \begin{cases} T_0 & \text{for } n = 1 \\ 2n \sum\limits_{m=0}^{\frac{n}{2}-1} T_{2m+1} & \text{for } n \geq 2 \text{ and } n \text{ even} \\ nT_0 + 2n \sum\limits_{m=1}^{\frac{n-1}{2}} T_{2m} & \text{for } n \geq 3 \text{ and } n \text{ odd} \end{cases} \tag{3}$$

Consequently, the derivation in the time domain of a function has a very simple expression:

$$\tfrac{dx}{dt} = \langle x_0\ x_1\ x_2 \cdots x_n \rangle . \left\{\dot{T^n}\right\} = \langle x_0\ x_1\ x_2 \cdots x_n \rangle . [D] . \{T^n\} \tag{4}$$

For instance, in the case of even n, the square matrix $[D]_{n\times n}$ is :

$$[D]_{n\times n} = \frac{2}{T}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 3 & 0 & 6 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 8 & 0 & 8 & 0 & 0 & \cdots & 0 \\ 5 & 0 & 10 & 0 & 10 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 2n & 0 & 2n & 0 & 2n & \cdots & 0 \end{bmatrix} \tag{5}$$

Let $P(\tau)$ and $Q(\tau)$ be two polynomials :

$$\begin{aligned} P(\tau) &= \sum_{k=0}^{n} p_k \cdot T_k(\tau) \\ Q(\tau) &= \sum_{k=0}^{n} q_k \cdot T_k(\tau) \end{aligned} \tag{6}$$

their product can be expressed as a linear combination of polynomials

$$P(\tau)\cdot Q(\tau) = \sum_{i=0}^{2n} c_i \cdot T_i(\tau) \tag{7}$$

where :

$$c_i = \begin{cases} p_0 q_0 + \frac{1}{2}\sum_{j=1}^{n} p_j q_j & \text{if } i = 0 \\ \frac{1}{2}\sum_{j=0}^{i} p_j q_{i-j} + \frac{1}{2}\sum_{j=0}^{n-i}\left(p_j q_{j+i} + p_{j+i} q_j\right) & \text{if } 1 \le i \le n \\ \frac{1}{2}\sum_{j=i-n}^{n} p_j q_{i-j} & \text{if } i > n \end{cases} \tag{8}$$

This property is particularly interesting for non linear mechanical systems.

Let the dynamic behavior of a N-DOF mechanical system be governed by the following set of equations :

$$[M] \cdot \{\ddot{x}(t)\} + [C] \cdot \{\dot{x}(t)\} + [K] \cdot \{x(t)\} = \{f(t)\} \tag{9}$$

where $[M]$, $[C]$, $[K] \in \Re^{N,N}$ are the mass, viscous damping and stiffness matrices respectively and $\{x(t)\} \in \Re^{N,1}$ is the displacement vector. Expanding the time history responses and excitations on the orthogonal basis and applying the derivative operator yields :

$$\left[[M]\ [C]\ [K] \right] \cdot \begin{bmatrix} [X] \cdot [D]^2 \\ [X] \cdot [D] \\ [X] \end{bmatrix} = [F] \tag{10}$$

where $[X]$ and $[F] \in \Re^{N,n+1}$ are the matrices of the expansion coefficients of the time responses and excitations, respectively. This general formulation can be expressed in the following compact form :

$$[H]_{N,3N} \cdot [J]_{3N,n+1} = [F]_{N,n+1} \tag{11}$$

In the case of inverse problems with a known model and responses, the derivative formulation permits the direct expression of the excitations without determining initial conditions and without using a pseudo-inverse calculation. In the case of identification, Eq. (11) can be solved with a least square approximation leading to the solution :

$$[H] = [F] \cdot [J]^T \cdot \left([J] \cdot [J]^T \right)^{-1} \tag{12}$$

which can be achieved with a computationally stable method like the Singular Value Decomposition Technique. It should be mentioned that the expansions must contain $n+1 > 3N$ terms in the basis. This formulation is exactly equivalent to the integration formulation proposed in [12].

2.3 Illustration of the derivative formulation

The 3-DOF system sketched in Fig.1 permits illustrating the use of the derivative formulation with no loss of generality. Let the displacement of each DOF be known and only the first DOF be subjected to an excitation force. The equations governing the dynamic behavior can be written in the following state-space form :

$$\begin{Bmatrix} \dot{x}^{(1)} \\ \dot{x}^{(2)} \\ \dot{x}^{(3)} \\ \dot{x}^{(4)} \\ \dot{x}^{(5)} \\ \dot{x}^{(6)} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & 0 & -\frac{c_1+c_2}{m_1} & \frac{c_2}{m_1} & 0 \\ \frac{k_2}{m_2} & -\frac{k_2+k_3}{m_2} & \frac{k_3}{m_2} & \frac{c_2}{m_2} & -\frac{c_2+c_3}{m_2} & \frac{c_3}{m_2} \\ 0 & \frac{k_3}{m_3} & -\frac{k_3+k_4}{m_3} & 0 & \frac{c_3}{m_3} & -\frac{c_3+c_4}{m_3} \end{bmatrix} \cdot \begin{Bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \\ x^{(4)} \\ x^{(5)} \\ x^{(6)} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ F^{(1)}/m_1 \\ 0 \\ 0 \end{Bmatrix} \tag{13}$$

Expanding the vectors on the Chebyshev basis leads to two algebraic equations

:

$$\begin{bmatrix} \left\langle x^{(1)} \right\rangle \\ \left\langle x^{(2)} \right\rangle \\ \left\langle x^{(3)} \right\rangle \end{bmatrix} \cdot [D] = \begin{bmatrix} \left\langle x^{(4)} \right\rangle \\ \left\langle x^{(5)} \right\rangle \\ \left\langle x^{(6)} \right\rangle \end{bmatrix} \tag{14}$$

and

$$\begin{bmatrix} \left\langle x^{(4)} \right\rangle [D] \\ \left\langle x^{(5)} \right\rangle [D] \\ \left\langle x^{(6)} \right\rangle [D] \end{bmatrix} = \begin{bmatrix} -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & 0 & -\frac{c_1+c_2}{m_1} & \frac{c_2}{m_1} & 0 \\ \frac{k_2}{m_2} & -\frac{k_2+k_3}{m_2} & \frac{k_3}{m_2} & \frac{c_2}{m_2} & -\frac{c_2+c_3}{m_2} & \frac{c_3}{m_2} \\ 0 & \frac{k_3}{m_3} & -\frac{k_3+k_4}{m_3} & 0 & \frac{c_3}{m_3} & -\frac{c_3+c_4}{m_3} \end{bmatrix} \cdot [X_{SS}] + \begin{bmatrix} \frac{1}{m_1} \left\langle F^{(1)} \right\rangle \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

Transposing Eq. 15 and rearranging all the terms yields:

$$\begin{bmatrix} [X_{SS}]^T & [0] & [0] & \left\{F^{(1)}\right\} \\ [0] & [X_{SS}]^T & [0] & \{0\} \\ [0] & [0] & [X_{SS}]^T & \{0\} \end{bmatrix} \cdot \begin{Bmatrix} -\frac{k_1+k_2}{m_1} & \vdots & \vdots \\ \frac{k_2}{m_1} & \frac{k_2}{m_2} & 0 \\ 0 & -\frac{k_2+k_3}{m_2} & \frac{k_3}{m_3} \\ -\frac{c_1+c_2}{m_1} & \frac{k_3}{m_2} & -\frac{k_3+k_4}{m_3} \\ \frac{c_2}{m_1} & \frac{c_2}{m_2} & 0 \\ 0 & -\frac{c_2+c_3}{m_1} & \frac{c_3}{m_3} \\ \vdots & \frac{c_3}{m_2} & -\frac{c_3+c_4}{m_3} \\ & \vdots & \frac{1}{m_1} \end{Bmatrix} = \begin{Bmatrix} [D]^T \left\{x^{(4)}\right\} \\ [D]^T \left\{x^{(5)}\right\} \\ [D]^T \left\{x^{(6)}\right\} \end{Bmatrix} \tag{16}$$

which makes the identification of the mass, damping and stiffness parameters possible and where $[X_{SS}]^T = \left[\left\{x^{(1)}\right\} \left\{x^{(2)}\right\} \left\{x^{(3)}\right\} \left\{x^{(4)}\right\} \left\{x^{(5)}\right\} \left\{x^{(6)}\right\} \right]$. It should be mentioned that the procedure presented can be performed with velocity or acceleration states.

3 How to improve polynomial identification ?

In [12], the authors reported that Chebyshev, Legendre and Jacobi polynomials estimate parameters badly. This fact is mainly due to the poor expansion of the excitation signals (impulse and random) on these polynomials. In order to underline this point regarding the Chebyshev polynomials, let an indicator be used to estimate the quality of the excitation expansions and response signals given by the 3DOF mechanical system presented in Fig.1. The coefficients of the expansion (2) are obtained by the measure of the signal at N_e points, not necessarily equally distributed within the time interval. Then, let the Average Deviation be an indicator of the expansion quality and defined as follows:

$$AD = \frac{1}{N_e} \sum_{i=1}^{N_e} \left| x\left(t_i\right) - \langle x_j \rangle \cdot \left\{T^n\left(t_i\right)\right\} \right| \tag{17}$$

where t_i are the instants of the signal measurements. A Total Average Deviation gives the same information by taking into account all the signals. For the harmonically excited 3-DOF system, Fig.2 shows that the size of the basis must be large enough but not too large in order to describe excitation and response signals accurately.

A similar indicator can be formulated for describing the quality of the parameter estimation for the identification problem. To estimate N_p parameters, let an Average Absolute Deviation be defined by:

$$AAD = \frac{1}{N_p} \sum_{i=1}^{N_p} \left| \hat{p}_i - p_i \right| \tag{18}$$

where p_i are the parameters and $\hat{p}_i$ their estimates. Table 1 associates the AAD indicator with the quality of the parameter estimation and shows that an AAD less than the unit can be considered as a very good identification. The largest deviation on the worst estimated parameter is about 5%.

Fig. 3 gives the evolution of the error on the identified parameters versus the number of polynomials in the basis. Estimation performance decreases with the size of the basis.

From Figs. 2 and 3, a necessary condition to obtain a good identification first resides in the correct expansion of the excitation and response signals. For Chebyshev polynomials, this correct description can be obtained by increasing the size of the basis when the signals are smooth enough on the interval. However, increasing basis size increases the number of equations to be satisfied by the parameters, thus explaining the loss of quality for large sizes in Fig. 3. This indicates clearly that Chebyshev expansion must be dedicated to identification in the presence of smooth excitations.

3.1 Utilisation of adapted excitation signals

Traditionally, excitations with a large bandwidth are preferred for identification, such as random noise or impulse signals. Appropriate signals for classical identification must also be persistent such as random noise and Pseudo-Random Binary Signals. When using orthogonal decomposition methods, these signals are not suitable for identification due to the difficulty of obtaining a good expansion in the polynomial basis, see [12]. The orthogonal polynomials have to be chosen for their ability to describe both excitation and response signals with the same accuracy. Then, excitation has to be chosen in the same frequency bandwidth of natural responses of the mechanical system to be identified. Thus, it is more relevant to choose a pure harmonic excitation in order

to keep both excitation and response signals in the same frequency range.
When the mechanical system is harmonically excited, the quality of the identification is better in the bandwidth of its natural frequencies, as shown in Fig.4. The natural frequencies of the 3-DOF system are 9.7, 22.2 and 25.8 Hz for the parameter values of Table 1. Applying a pure harmonic excitation generates both transient and steady-state components and makes it possible to estimate the whole set of parameters (see Table 1).

3.2 By identifying adapted mechanical systems

In order to test the robustness of the proposed identification method, the previous investigation is performed on the stiffness and damping parameters varying within [100, 20000 N/m] and [1,200 Ns/m] intervals respectively and assuming $m_i = 1kg$, $k_i = k$ and $c_i = c$. Then, the AAD is calculated on damping coefficient $-\frac{c_1+c_2}{m_1}$ and on stiffness coefficient $-\frac{k_1+k_2}{m_1}$ and plotted in Fig. 5. The two graphs show that stiffness (damping) parameter identification becomes unreliable for a stiffness parameter in the presence of high damping and low stiffness, and vice versa for a damping parameter. In comparison, the system chosen in [12] is defined by low stiffness and damping parameters that corresponds to an advantageous configuration.
This result clearly indicates that the proposed method is well adapted to the identification of reasonably damped systems.

3.3 Selection of only a few points within the interval

In the case of noisy signals, the natural property of low-pass filtering of the polynomials should constitute an advantage although the edge effects represent an important drawback. In fact, signal expansion requires a minimum

number of polynomials which involves an oscillating shape on the edges of the time interval. Therefore a part of the noise is captured on the edges through the expansion process (see Fig. 6). Moreover the use of the derivative form is prejudicial since it increases these oscillations on the estimate of velocity and acceleration.

Consequently, astute use of Chebyshev polynomials is required for obtaining a satisfactory parameter estimation. The proposed alternative formulation resides in the possibility to satisfy the algebraic equations [11] only at some particular points selected to avoiding the Gibbs phenomenon. According to equation [13], the relation remains valid at any point t_i where the signals are measured and can be written as:

$$[X]\,.\,[D]\,.\,\{T^n\,(t_i)\} = [A]\,.\,[X]\,.\,\{T^n\,(t_i)\} + [U]\,.\,\{T^n\,(t_i)\} \tag{19}$$

The parameter estimation is then processed only on points where the expansion is satisfactory. Table 2 gives the parameters estimated by taking only the points located in the time interval [0.2 s; 1.8 s] with simulated noise on the displacements (Signal to Noise Ratio of 20 dB). The values identified are acceptable for the set of parameters. In comparison, the non-improved polynomial identification gives a value of 8412.6($\frac{1}{s^2}$ for the $\frac{k_2}{m_1}$ parameter.

Moreover, this alternative method is an efficient way to dissociate the number of polynomials, critical for good signal expansion, and the number of equations to be satisfied by the set of parameters to be identified. As shown in Fig.7, simulations are performed on the 3-DOF system with the approach proposed in Section 2.2 and by retaining only a few points on the interval (20, 50, 100 or 200 points).

For all these simulations, the signals without noise are known in the time interval [0 s, 2 s] with a sampling frequency of 2000 Hz. In the case of solving the equations for only a low number of points, they are regularly sampled within

the time interval [0.2 s, 1.8 s]. Quality is improved initially by eliminating edge effects. Then, reducing the number of equations degrades identification quality as the AAD indicator increases but remains at a value below the AAD indicator that is obtained by keeping the whole time interval (2000 points, without noise). Moreover, the number of polynomials can be chosen in a larger range since the AAD indicator remains constant when the size of the basis increases. This method can be applied and generalized to other polynomial basis (see for example [7]) and to any kind of point selection used for identification. Retaining only the points where signals are close to their expansion should improve identification quality and reduce calculation time. The demonstration described for the derivative form can evidently be extended to the integral form proposed by [12].

4 Non linear identification

The product property of polynomial approach (see Eqs. 7 and 8) has a great advantage in the field of non linear system identification. Two original approaches are described in a SDOF system in order to aid comprehension though without any loss of generality. Particularly, extension to non linearity with higher order like Duffing oscillator lies on the iteration of the product property (equation 8) of the following formulations. A comparison between the two methods is here the main objective and focuses on identification quality. The first method is then applied to an actual mechanical system with hysteretic behavior.

4.1 Identification by using the truncated formulation

Parameter nonlinearity can be easily described by using Expression (7). Let the following equation governs the non-linear behavior of an SDOF system:

$$\ddot{x}(t) + \frac{c}{m}\dot{x}(t) + \frac{k}{m}x^2(t) = \frac{1}{m}f(t) \tag{20}$$

The square of a function expanded on the Chebyshev basis is expressed by the truncated form:

$$x^2 = \langle \xi \rangle . \{T^n\} = \left\langle \xi_0 \ \xi_1 \ \xi_2 \ \cdots \ \xi_n \right\rangle . \{T^n\} \tag{21}$$

with

$$\xi_i = \begin{cases} x_0 x_0 + \frac{1}{2}\sum_{j=1}^{n} x_j x_j & \text{if } i = 0 \\ \frac{1}{2}\sum_{j=0}^{i} x_j x_{i-j} + \frac{1}{2}\sum_{j=0}^{n-i}(x_j x_{j+i} + x_{j+i}x_j) & \text{if } 1 \leq i \leq n \end{cases} \tag{22}$$

After rewriting the governing equation (20) in a state-space form, the formulation for the parameter estimation is given by:

$$\left[\{\xi\} \ \left\{x^{(2)}\right\} \right] . \begin{Bmatrix} \frac{k}{m} \\ \\ \frac{c}{m} \end{Bmatrix} = \frac{1}{m} . \{f\} - [D]^t \left\{x^{(2)}\right\} \tag{23}$$

with $\left\{x^{(2)}\right\} = [D]^t \{x\}$. A Least Square solution leads to the estimation of the unknown stiffness and damping parameters for a given mass (which can be estimated by another formulation). In what follows, this formulation is called the truncated form.

4.2 Identification by using a non truncated formulation

Another way to take into account the nonlinear term is to express this term naturally as the product of the signal expansion by:

$$x^2(\tau) = \langle x \rangle . \{T^n(\tau)\} . \langle x \rangle . \{T^n(\tau)\} \tag{24}$$

Therefore it is no longer possible to take the problem as a whole and it must be solved on a set of sampled points:

$$\left[\langle x \rangle . \{T^n(t_i)\} . \langle x \rangle . \{T^n(t_i)\} \left\langle x^{(2)} \right\rangle . \{T^n(t_i)\} \right] . \left\{ \begin{array}{c} \frac{k}{m} \\ \\ \frac{c}{m} \end{array} \right\} =$$

$$\frac{1}{m} . \langle T^n(t_i) \rangle . \{f\} - \langle T^n(t_i) \rangle . [D]^t \left\{ x^{(2)} \right\} \tag{25}$$

4.3 Comparison between the truncated and non-truncated formulations

Evaluation of the loss of accuracy induced by the truncation is based on the AAD plot versus the basis order, Fig. 8. The truncation leads to a significant difference, but for a basis size greater than 40, the quality of the parameter estimation remains acceptable because the AAD is below the accuracy threshold. Therefore the truncated form can be used without any significant loss of estimation quality or any significant increase of basis size. For more complex systems, the truncated form requires particular attention as presented in the appendix.

4.4 Application on an real nonlinear system

The truncated formulation is applied to a tape-spring coiling device with non linear behavior. This component is implemented as a one-dimensional actuator in a self-deployable hexapod, previously described in [3]. This actuator contains a rotating roll module with a spiral groove in order to guide the tape-spring. The coiling of the strip leads to the flattening of the natural curved section. The tape-spring tends to naturally recover its curved section due to the stress generated by its elastic deformation. Fig. 9 shows the coiling device and the experimental set-up used for its characterization. The hub of the spool is fixed on a frame, which alloxs changes in the length of the unrolled tape-spring. Moreover, the spool is free to rotate. The unrolled end of the tape-spring is connected to a load sensor and sine excited in displacement by an electro-dynamic shaker.

The global behavior of the coiling device can be described by force-deflection loops that exhibit hysteretic behavior. Different models are available for the description of such hysteretic loops. In 1976, Dahl proposed a dynamic model to avoid discontinuity which he introduced in a classical Coulomb friction model. The generalized Dahl model, developed by Al Majid [2], is a more general restoring force model. For the coiling device, this model can be simplified and described by the following first order differential equation:

$$\dot{R} = \beta \cdot \dot{u} \cdot (h(u) - R \cdot sgn\,(\dot{u})) \tag{26}$$

where R is the restoring force, $\dot{u}$ is the elongation velocity of the tape-spring length, β a parameter that adjusts the shape of the friction slope function. The hysteretic behavior is captured by defining the function $h(u)$ as:

$$h(u) = \frac{1}{2}\left[(h_u + h_l)\, sign(\dot{u}) + (h_l - h_u)\right] \tag{27}$$

with the two limit curves h_u and h_l defined by:

$$\begin{cases} h_u = a_1 \cdot u + b_1 \\ \\ h_l = a_2 \cdot u + b_2 \end{cases} \tag{28}$$

Fig. 10 shows an example of a hysteretic loop and the two limit curves.
By expanding the restoring force and the displacement in the Chebyshev basis, using the truncated formulation and combining Eq. (26),(27) and (28), we obtain:

$$\begin{bmatrix} A & B & C & D & E \end{bmatrix} \begin{Bmatrix} \beta \cdot a_1 \\ \beta \cdot b_1 \\ \beta \cdot a_2 \\ \beta \cdot b_2 \\ \beta \end{Bmatrix} = \langle F \rangle \cdot \begin{Bmatrix} \beta \cdot a_1 \\ \beta \cdot b_1 \\ \beta \cdot a_2 \\ \beta \cdot b_2 \\ \beta \end{Bmatrix} = 2 \langle T^n \rangle [D]^t \{R\} \tag{29}$$

with $R = \langle R \rangle . \{T^n\}$ $\qquad u = \langle u \rangle . \{T^n\}$

$A = (sgn(\dot{u}) - 1) \langle T^n \rangle \{\dot{u}\} \langle T^n \rangle \{u\}$ $\quad B = (sign(\dot{u}) - 1) \langle T^n \rangle \{\dot{u}\}$

$C = (sgn(\dot{u}) + 1) \langle T^n \rangle \{\dot{u}\} \langle T^n \rangle \{u\}$ $\quad D = (sign(\dot{u}) + 1) \langle T^n \rangle \{\dot{u}\}$

$E = -2sgn(\dot{u}) \langle T^n \rangle \{R\} \langle T^n \rangle \{\dot{u}\}$

Therefore, the terms which depend on the displacement and the velocity can

be evaluated for any sampling point, leading to the formulation:

$$\begin{bmatrix} \langle F(\tau_i)\rangle \\ \langle F(\tau_{i+1})\rangle \\ \langle F(\tau_{i+2})\rangle \\ \langle F(\tau_{i+3})\rangle \\ \vdots \\ \langle F(\tau_j)\rangle \end{bmatrix} \begin{Bmatrix} \beta a_1 \\ \beta b_1 \\ \beta a_2 \\ \beta b_2 \\ \beta \end{Bmatrix} = 2 \begin{bmatrix} \langle T^n(\tau_i)\rangle \\ \langle T^n(\tau_{i+1})\rangle \\ \langle T^n(\tau_{i+2})\rangle \\ \langle T^n(\tau_{i+3})\rangle \\ \vdots \\ \langle T^n(\tau_j)\rangle \end{bmatrix} [D]^t \{R\} \tag{30}$$

solved by the Least Square Method.

The measurements of the restoring force and the displacement are acquired during 2 seconds. Fig. 11 shows the force simulated with the identified model compared to the measured restoring force. The estimation of the generalized Dahl model obtained by a regression method exposed in [3] is also reported. Furthermore, it can be noticed that the restoring force oscillations are not correctly described. In order to illustrate the compliance of the method, a simple modification is proposed to take these oscillations into account, leading to the introduction of an additional parameter. This is done by implementing a sinusoid in the definition of the upper limit curve as follows:

$$h_u = a_1 u + b_1 + A\sin(\omega u - \phi) \tag{31}$$

Then, it is possible to introduce the sine magnitude A as a new parameter to estimate, the angular frequency ω and phase ϕ being imposed. The equation

to be solved is re-written in the following way:

$$\left[A\ B\ C\ D\ E\ G \right] \left\{ \begin{array}{l} \beta \cdot a_1 \\ \beta \cdot b_1 \\ \beta \cdot a_2 \\ \beta \cdot b_2 \\ \beta \\ \beta \cdot A \end{array} \right\} = 2 \left\langle T^n \right\rangle [D]^t \{R\} \tag{32}$$

with $G = \left\langle T^n \right\rangle \{\dot{u}\} \left(sgn(\dot{u}). \sin(\omega u - \phi) - \sin(\omega u - \phi) \right)$. This equation is solved for all the data acquired and the results are grouped in Table 3, proving the good agreement of the estimated values. It should be mentionned that the β value given by [3] is not completely reliable.

Fig. 12 shows the improvement obtained on the restoring force in the time domain. In conclusion, this example highlights the simplicity of implementing of new specific nonlinearities with the proposed formulation.

5 Conclusions

This paper presents improved methods for identifying the parameters of mechanical systems through expansion using a Chebyshev polynomial basis. Its chief contribution is to propose formulations and methodology in which the drawbacks of Chebyshev polynomial are avoided. Thus their interesting properties can be explored to achieve an efficient description of linear and non linear behaviors.

When noisy signals are recorded, it is possible to identify parameters correctly

by taking only central points within the time interval. This solution leads to a new formulation by using only a few points from the acquired data, improving the quality of parameter estimation. The general identification procedure is then divided into two main parts: first by expanding signals in the basis and then by solving parameter estimation using a Least Square Approach.

In the case of nonlinear behavior, advantage can be taken of the product property of Chebyshev polynomials, leading to a truncated formulation. The loss of accuracy caused is shown to be insignificant and nonlinear systems can be identified. Applying this approach to a real system whith hysteretic behavior permits identifying a generalized Dahl model. Moreover, this model can be improved by introducing a more complex formulation without any difficulty and any loss of efficiency.

All the developments are proposed in a differential form, for which inverse problems can also be solved directly without any inverse matrix calculation. Extensions to a classical integral formulation can obviously be obtained and adapted. All the approaches and results presented in this paper can also be extended to other polynomials or orthogonal functions without any difficulty.

Acknowledgment

The authors are indebted to Pierre Argoul and Eric Walter for useful and interesting discussions about this work.

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Appendix : remark on the truncated form for a non linear system

The truncated form of a non linear system requires a particular arrangement of equations. Considering a 3-DOF system similar to the one shown in Fig. 1 but with nonlinear stiffnesses ($K_1 = k_1.x_1$, $K_2 = k_2.(x_2 - x_1)$, $K_3 = k_3.(x_3 - x_2)$

and $K_4 = k_4.x_3$), the second part of the state-space form (Eq. 15) is given by :

$$\begin{Bmatrix} \dot{x^4} \\ \dot{x^5} \\ \dot{x^6} \end{Bmatrix} = \begin{bmatrix} -\frac{k_1.x_1+k_2.(x_2-x_1)}{m_1} & \frac{k_2.(x_2-x_1)}{m1} & 0 \\ \frac{k_2.(x_2-x_1)}{m_2} & -\frac{k_2.(x_2-x_1)+k_3.(x_3-x_2)}{m_2} & \frac{k_3.(x_3-x_2)}{m_2} \\ 0 & \frac{k_3.(x_3-x_2)}{m_3} & -\frac{k_3.(x_3-x_2)+k_4.x_3}{m_3} \end{bmatrix} [C] \cdot \begin{Bmatrix} x^1 \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \end{Bmatrix} + \begin{Bmatrix} \frac{u^1}{m_1} \\ 0 \\ 0 \end{Bmatrix} \tag{33}$$

where $[C]$ is the damping matrix. The latter equation must be rewritten to estimate the parameters only once, taking into account the nil valued parameters. This specific expression is given by:

$$\begin{Bmatrix} \dot{x^4} \\ \dot{x^5} \\ \dot{x^6} \end{Bmatrix} = \begin{bmatrix} -\frac{k_1}{m_1} & \frac{k_2}{m_1} & 0 & 0 & 0 & 0 \\ 0 & -\frac{k_2}{m_2} & 0 & \frac{k_3}{m_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{k_3}{m_3} & 0 & -\frac{k_4}{m_3} \end{bmatrix} [C] \cdot \begin{Bmatrix} x_1^2 \\ (x_2-x_1)^2 \\ x_3 \\ (x_3-x_2)^2 \\ x_1 \\ x_3^2 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} + \begin{Bmatrix} \frac{u^1}{m_1} \\ 0 \\ 0 \end{Bmatrix} \tag{34}$$

Applying a Chebyshev expansion on all the signals and expressing the unknown parameters as a column vector results in:

$$\begin{bmatrix} [F_1]^T & [0] & [0] \\ [0] & [F_2]^T & [0] \\ [0] & [0] & [F_3]^T \end{bmatrix} \cdot \begin{Bmatrix} -\frac{k_1}{m_1} & \vdots & \vdots \\ \frac{k_2}{m_1} & -\frac{k_2}{m_2} & \frac{k_3}{m_3} \\ 0 & \frac{k_3}{m_2} & 0 \\ -\frac{c_1+c_2}{m_1} & \frac{c_2}{m_2} & -\frac{k_4}{m_3} \\ \frac{c_2}{m_1} & -\frac{c_2+c_3}{m_2} & 0 \\ 0 & \frac{c_3}{m_2} & \frac{c_3}{m_3} \\ \vdots & \vdots & -\frac{c_3+c_4}{m_3} \end{Bmatrix} = \begin{Bmatrix} [D]^T \{x^4\} - \left\{\frac{u^1}{m_1}\right\} \\ [D]^T \{x^5\} \\ [D]^T \{x^6\} \end{Bmatrix} \tag{35}$$

with $[F_1] = \begin{bmatrix} \langle x_1^2 \rangle \\ \langle (x_2 - x_1)^2 \rangle \\ \langle x_3 \rangle \\ \langle x_4 \rangle \\ \langle x_5 \rangle \\ \langle x_6 \rangle \end{bmatrix}$, $[F_2] = \begin{bmatrix} \langle (x_2 - x_1)^2 \rangle \\ \langle (x_3 - x_2)^2 \rangle \\ \langle x_4 \rangle \\ \langle x_5 \rangle \\ \langle x_6 \rangle \end{bmatrix}$ and $[F_3] = \begin{bmatrix} \langle (x_3 - x_2)^2 \rangle \\ \langle x_1 \rangle \\ \langle x_3^2 \rangle \\ \langle x_4 \rangle \\ \langle x_5 \rangle \\ \langle x_6 \rangle \end{bmatrix}$

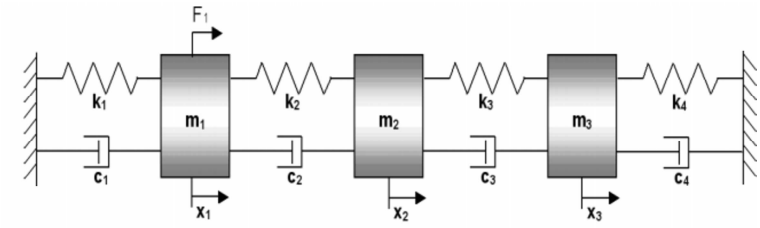


Figure 1. Scheme of the 3-DOF system used for performance characterisation.

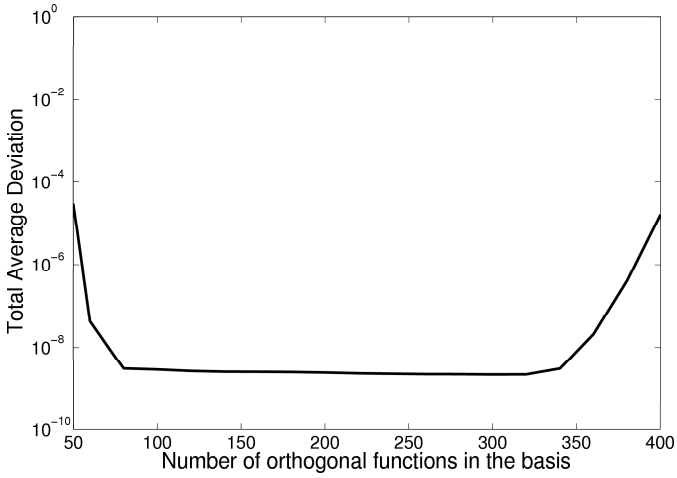


Figure 2. Total Average Deviation for a harmonic excitation ($N_e = 2000$ in Eq.(17)).

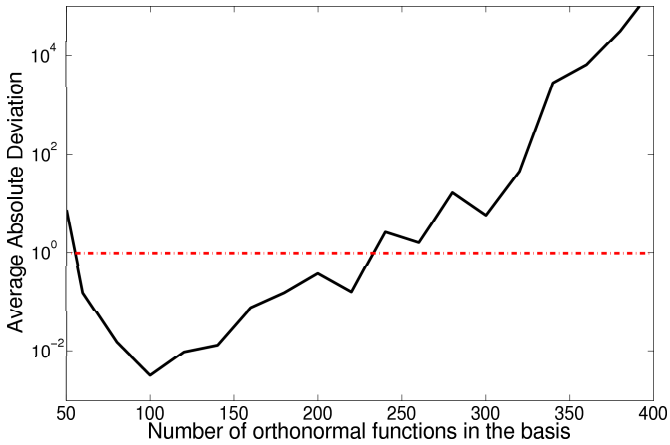


Figure 3. Average Absolute Deviation of all parameters in the case of a harmonic excitation ($N_e = 2000$). The red line represents the accuracy threshold.

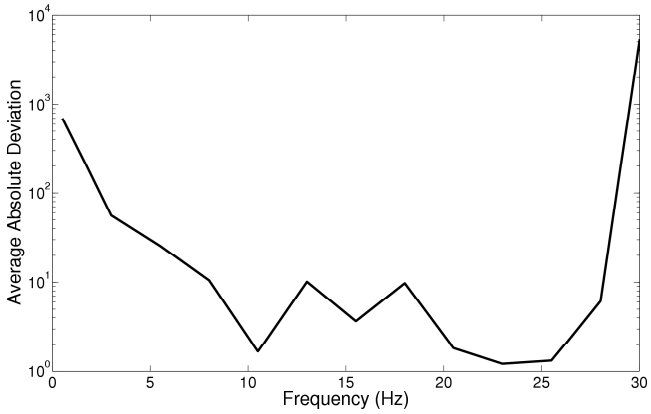
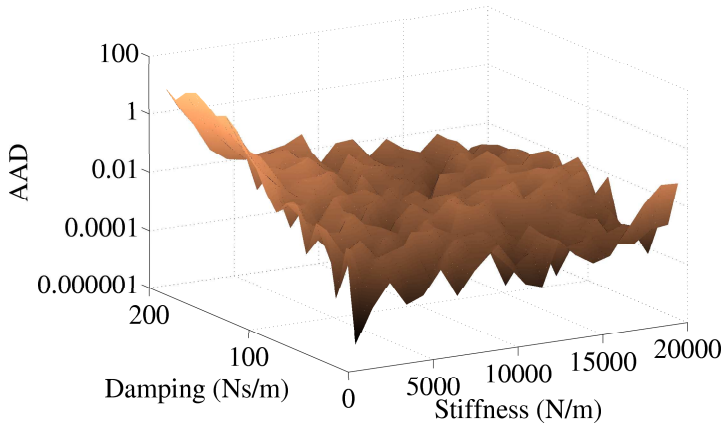
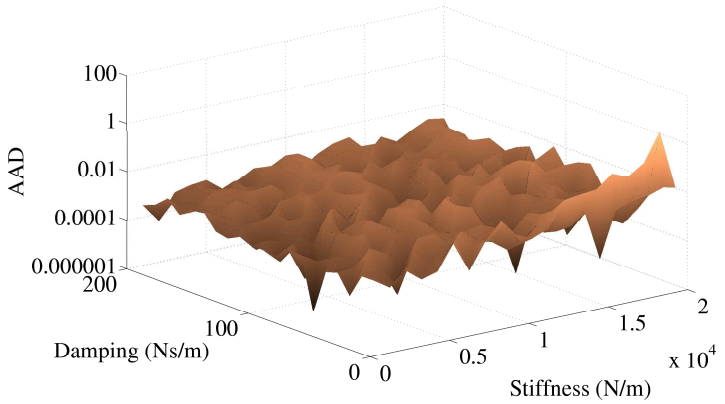


Figure 4. Evolution of Average Absolute Deviation with frequency of a pure harmonic excitation.

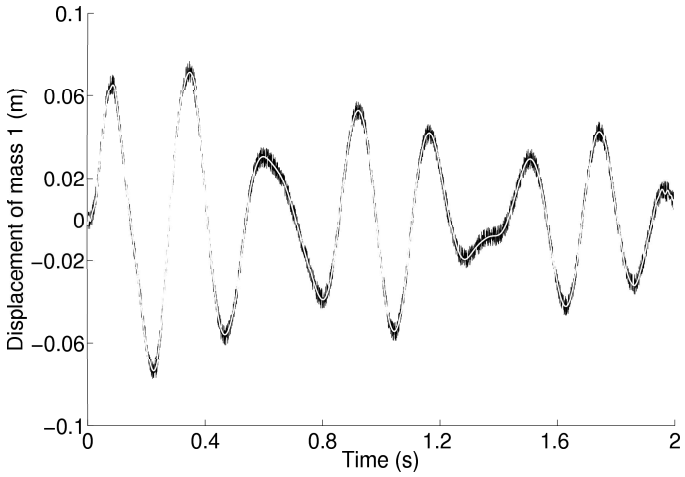


(a)

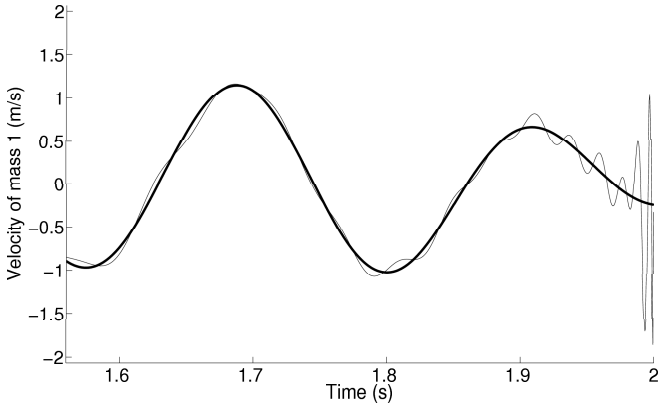


(b)

Figure 5. Evolution of Average Absolute Deviation on the stiffness coefficient $\frac{k_1+k_2}{m_1}$ (a) and on the damping coefficient $\frac{c_1+c_2}{m_1}$ (b).



(a)



(b)

Figure 6. (a) Displacement with noise (SNR = 20 dB) and its expansion on a Chebyshev basis (order 100) (b) Speed signal after derivation on the right edge of the time interval.

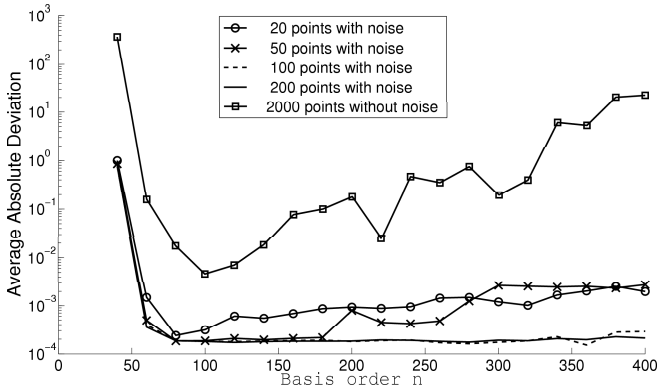


Figure 7. Improvement of identification capability by retaining only a few points for identification.

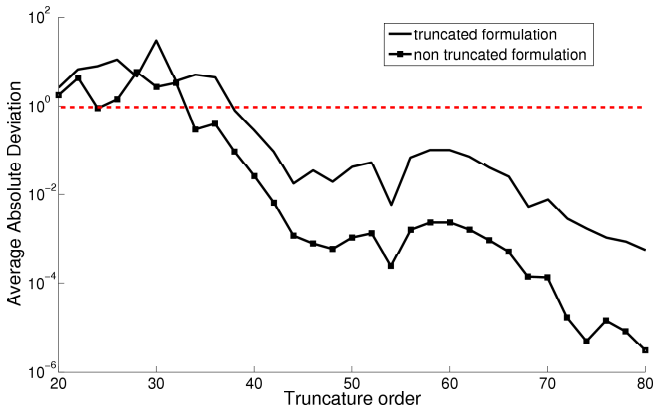
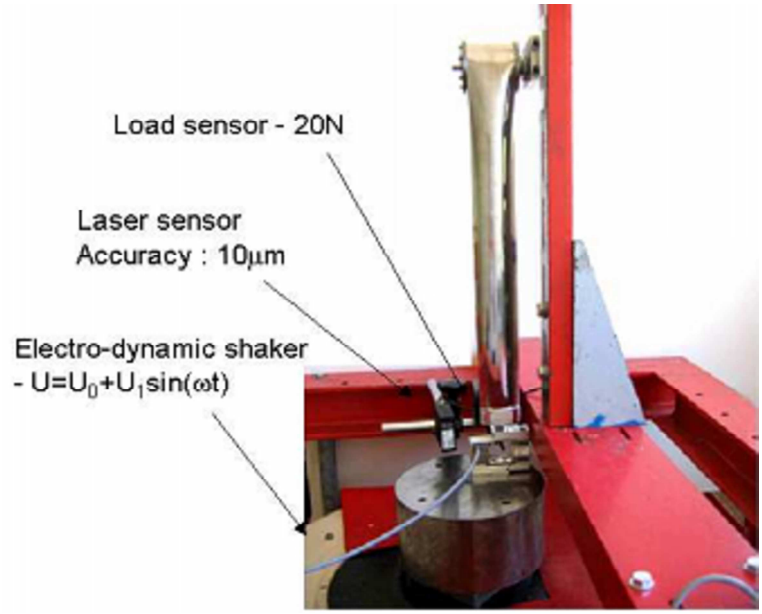


Figure 8. Influence of truncation on identification quality for an SDOF system. The red line represents the accuracy threshold.

(a)



(b)

Figure 9. (a) Tape-spring coiling device and (b) test apparatus for nonlinear behavior characterization.

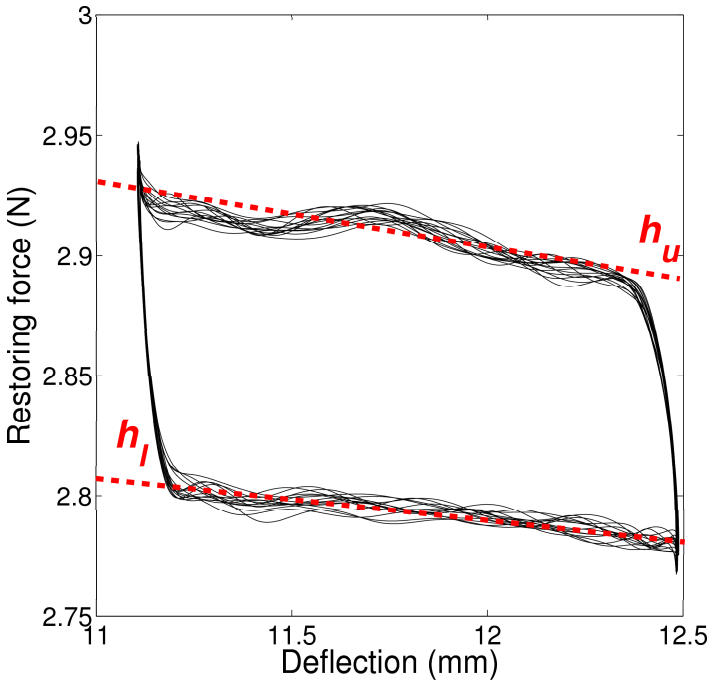


Figure 10. Hyteretic loop of a tape-spring coiling device under forcing deflection.

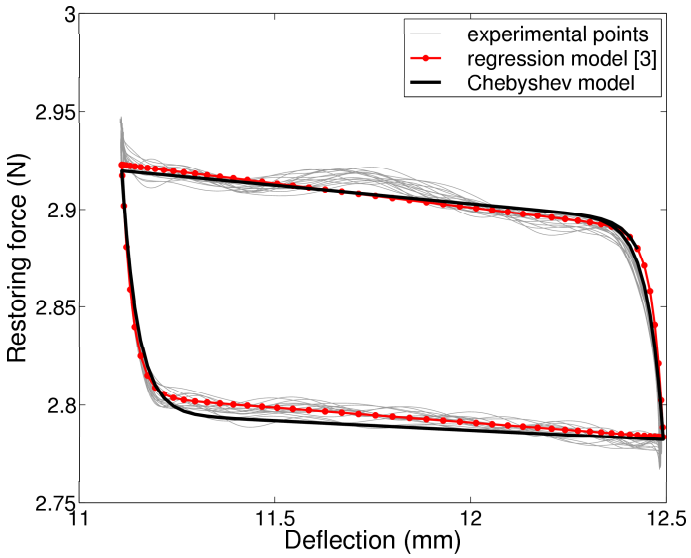
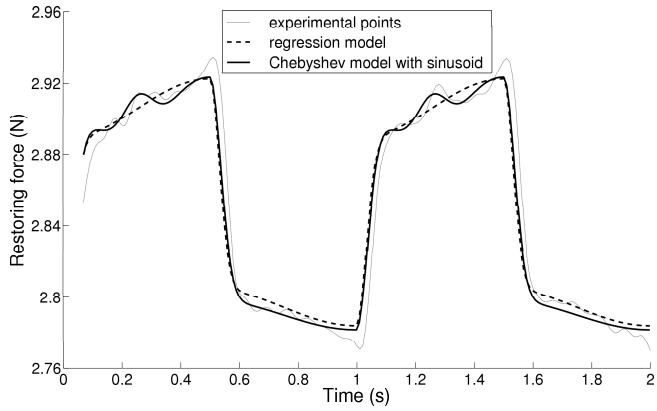
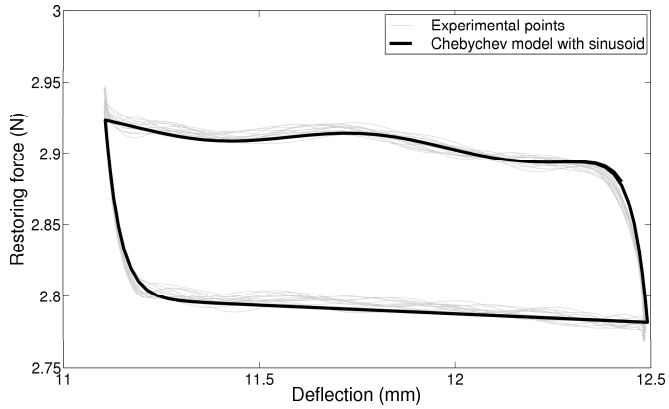


Figure 11. Hysteretic loops: measured and simulated with the proposed method or with [3].



(a)



(b)

Figure 12. (a) Measured and simulated time evolution of restoring force and (b) corresponding hysteretic loops.

parameter	actual	estimated	parameter	actual	estimated
$-\frac{k_1+k_2}{m_1}$	-3000	-3000.6	$-\frac{c_2+c_3}{m_2}$	-4	-4
$\frac{k_2}{m_1}$	2000	1998.8	$\frac{c_3}{m_2}$	2	2
0	0	0.0044	0	0	0.0001
$-\frac{c_1+c_2}{m_1}$	-6	-6.1	$\frac{k_2}{m_2}$	1000	1000.5
$\frac{c_2}{m_1}$	4	4.1	$-\frac{k_3+k_4}{m_2}$	-3000	-3001.1
0	0	0.0001	0	0	0.0001
$\frac{k_2}{m_2}$	1000	999.7	$\frac{c_3}{m_3}$	4	4
$-\frac{k_2+k_3}{m_2}$	-1500	-1498.6	$-\frac{c_3+c_4}{m_3}$	-6	-6
$\frac{k_3}{m_2}$	500	495.9	$\frac{1}{m_1}$	1	1
$\frac{c_2}{m_2}$	2	2.1			
Average Absolute Deviation			0.752		

Table 1

Parameters values and AAD indicator level

coefficient	actual	estimated	coefficient	actual	estimated
$-\frac{k_1+k_2}{m_1}$	-3000	-2991.42	$-\frac{c_2+c_3}{m_2}$	-4	-3.7
$\frac{k_2}{m_1}$	2000	1961.4	$\frac{c_3}{m_2}$	2	1.34
0	0	68.15	0	0	0.01
$-\frac{c_1+c_2}{m_1}$	-6	-5.94	$\frac{k_2}{m_2}$	1000	999.97
$\frac{c_2}{m_1}$	4	3.91	$-\frac{k_3+k_4}{m_2}$	-3000	-2999.95
0	0	0.2	0	0	0.000007
$\frac{k_2}{m_2}$	1000	1003.1	$\frac{c_3}{m_3}$	4	4
$-\frac{k_2+k_3}{m_2}$	-1500	-1504.4	$-\frac{c_3+c_4}{m_3}$	-6	-6
$\frac{k_3}{m_2}$	500	509.13			
$\frac{c_2}{m_2}$	2	1.93			
Average Absolute Deviation			7.4		

Table 2

Parameter values for noisy signal and level of AAD indicator

parameter	Chebyshev estimation	Limit curves estimation [3]
a_1	-0.0190	-0.0244
b_1	3.1324	3.1943
a_2	-0.0106	-0.0150
b_2	2.9140	2.9710
β	26.7025	35.4010
A	0.0063	–

Table 3

Comparison of estimated parameter values.