Online Diagnosis of Systems with Rectangular Hybrid Automata Models

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Abstract: We propose an online diagnosis approach for a class of hybrid systems. The normal and the faulty behaviors of the system are modeled with rectangular hybrid automata. Our approach is based on the use of a diagnosis procedure which performs, online, an estimation of the system states, within a given time window, and based on the current record of observable timed events. Each new estimation can be triggered either, by a new event observation, or simply by the elapse of time. We give examples to illustrate the use of our hybrid systems diagnosis approach.

Keywords: Diagnosis, Online, Rectangular Hybrid Automata, Symbolic Analysis.

1. INTRODUCTION

Fault diagnosis is an important task for the design and development of man-made systems such as embedded systems, industrial process control systems, ... etc. This importance is due to the crucial role diagnosis plays in protecting human life, and increasing the efficiency and productivity of these systems. The diagnosis task consists of the detection of anomalous system behaviors, followed by the isolation and the identification of the causes behind these faults.

The diagnosis problem has been extensively studied in the automatic control and discrete event systems (DES) literatures, during the last two decades. Several model-based diagnosis techniques has been proposed within the context of continuous, discrete, and hybrid systems (Lin and Wonham, 1994; Sampath et al., 1996; McIlraith et al., 2000; Derbel et al., 2006; Tripakis, 2002; Bhowal et al., 2007). In the most widely used DES diagnosis approach (Sampath et al., 1995, 1996), a finite state machine called diagnoser is compiled offline, based on another finite state model, capturing the normal and the faulty behaviors of the system. In (Zad et al., 1999), a timed model of the system is used, capturing time with a tick event. In (Derbel et al., 2006; Tripakis, 2002) dense time extensions have been proposed, using *timed automata* (Alur, 1994) as a system model. In (Tripakis, 2002), the author proposes an online state estimator, and a good/faulty partition of the state space, to perform the system diagnosis.

Case studies of hybrid systems diagnosis have been developed, using system models capturing both the discrete events and the continuous variables. Most recently, in (Bhowal et al., 2007), a discrete time hybrid system model is used for constructing, offline, a diagnoser.

This paper is inspired from our contribution to the diagnosis of dense time systems (Derbel et al., 2006). It generalizes the online timed systems diagnosis approach in (Tripakis, 2002), to provide an effective online diagnosis for hybrid systems. The fact that our diagnosis approach works online helps us overcome many of the issues related to the undecidability and intractability of hybrid systems (Henzinger et al., 1998).

Our approach is based on Rectangular Hybrid Automata (RHA) (Henzinger et al., 1998) models, under the nonzeno condition. RHA represent an interesting class of hybrid systems, that can very closely capture the dynamics of real systems. They allow the description of arbitrary closed approximation of continuous behaviors, using lower and upper bounds on derivatives. Furthermore, the symbolic analysis of these automata can be easily performed using polyhedron structures (Alur et al., 1995).

The underlying idea of our approach consists of using a diagnosis procedure, which performs an online state estimation. The used system model captures both normal and faulty system behaviors. Failures are modeled as discrete transitions on unobservable events. The diagnosis procedure uses a symbolic analysis approach to perform online estimation of the system states, based on the current record of observable timed events, and within a lookahead time window (Ben.Hadj-Alouane et al., 1994). Our online diagnosis procedure is time/event driven. In fact, performing a new state estimation is triggered either by the arrival of an observable event, or the expiration of a timer. The expiration deadline of this timer, is dynamically computed, after performing each estimation.

This paper is organized as follows. The next section provides the necessary background on rectangular hybrid

automata. Section 3 formally defines our diagnosis model for hybrid systems. Section 4 describes, in detail, our online diagnosis procedure. We conclude in section 5. Throughout this paper, we use a simple example to illustrate the various features of our online diagnosis approach for hybrid systems.

2. AN OVERVIEW OF RECTANGULAR HYBRID AUTOMATA

Let $X = \{x_1, \ldots, x_n\}$ be a finite set of real-valued variables. We denote by $\dot{X} = \{\dot{x} | x \in X\}$ the set of first derivatives of the variables of X. A variable x is called *clock*, if $\dot{x} = 1$. We denote by $\sim$ an element of the operators $\{<, \leq, =, \geq, >\}$. A *rectangular inequality over X, is an inequality of the form $x \sim c$, where $x \in X$, and $c \in \mathbb{Z}$* [1]. *A* rectangular predicate over X is a conjunction of rectangular inequalities over X. We denote by $Rect(X)$ the set of rectangular predicates over X. A *polyhedral inequality over X is an inequality of the form $c_1x_1 + \cdots + c_kx_k \sim c$, where $x_1, \ldots, x_k \in X$, and $c, c_1, \ldots, c_k \in \mathbb{Z}$. A* polyhedral predicate over X is boolean combination of polyhedral inequalities over X. We denote by $\Psi(X)$ the set of polyhedral predicates over X.

The vector $\mathbf{v} = (\ v_1, \ldots, v_n\ )$, is an element of $\mathbb{R}^n$, that represents a value $v_i \in \mathbb{R}$, for every variable $x_i \in X$. [2] A subset of $\mathbb{R}^n$ is called a *region. For a region z and a variable $x_i \in X$, $z(x_i) = \{v_i | \mathbf{v} \in z\}$. We denote by $[\![\psi]\!]$, the region composed of the set of vectors $\mathbf{v} \in \mathbb{R}^n$, for which the predicate ψ is true, when each variable x_i is replaced by v_i for each $i \in \{1, \ldots, n\}$. For a rectangular predicate ψ, and a variable x_i, we write $[\![\psi]\!](x_i)$ to denote the interval of values described by v_i, for all $\mathbf{v} \in [\![\psi]\!]$.*

A Rectangular Hybrid Automaton *(RHA) (Henzinger et al., 1998), H, is a tuple $(L, X, \Sigma, E, inv, flow, init)$, where:*

- *L is a finite set of locations.*
- *X is a finite set of real valued variables.*
- *Σ is a set of events.*
- *$E \subseteq L \times \Sigma \times Rect(X) \times Rect(X) \times 2^X \times L$, is a finite set of edges. An edge (l, σ, g, r, R, l') corresponds to a switch from location l to location l', on the event σ, under the condition that $\mathbf{v} \in [\![g]\!]$, where the vector $\mathbf{v}$ corresponds to current values of the variables. Upon the location switch, each variable $x_i \in R$ is reset, nondeterministically, to a value in the interval $[\![r]\!](x)$, and the variables in $X - R$ remain unchanged.*
- *$inv : L \rightarrow Rect(X)$ captures the invariant conditions for the locations. The automaton H can remain in the same location as long as the value of each variable $x_i \in X$, belongs to the interval $[\![inv(l)]\!](x_i)$.*
- *The function $flow : L \rightarrow Rect(\dot{X})$, assigns a flow condition to each location $l \in L$. While the automaton H involves in the location l, the first time derivative of each variable $x_i \in X$ remains within the interval $[\![flow(l)]\!](x_i)$.*
- *$init \subseteq L \times Rect(X)$, specifies the initial condition of the automaton.*

[1] $\mathbb{Z}$ denotes the set of integers.

[2] $\mathbb{R}$ denotes the set of real numbers, and $\mathbb{R}^n$ denotes the n-dimensional euclidean space over $\mathbb{R}$.

A state $(l, \mathbf{v})$ *of the automaton H, consists of a discrete part $l \in L$ together with a continuous part $\mathbf{v} \in \mathbb{R}^n$ such that $\mathbf{v} \in [\![inv(l)]\!]$. The trajectories of the states of H progress, in the system state space, by performing one of the following transitions:*

- *Flow transition: The discrete part of the state remains in the same location l, and the continuous part involves from valuation $\mathbf{v}$ to valuation $\mathbf{v}'$, via any smooth trajectory satisfying the constraints imposed by $inv(l)$ and with derivatives of the variables remaining within the flow intervals specified by $flow(l)$. We denote this transition by $(l, \mathbf{v}) \xrightarrow{\delta} (l, \mathbf{v}')$, where $\delta \in \mathbb{R}_+$ is the time elapsed during the transition.* [3]
- *Discrete transition: corresponds to a discrete and instantaneous location switch, defined using the tuple $e = (l, \sigma, g, r, R, l') \in E$. We denote this transition by $(l, \mathbf{v}) \xrightarrow{e} (l', \mathbf{v}')$. The discrete transition is enabled only if the guard predicate is satisfied by the continuous part of the state; i.e., $\mathbf{v} \in [\![g]\!]$. The discrete part of the state changes from l to l', and the continuous part is updated according to the reset assignments.*

A run *is a finite or infinite sequence of transitions $(l_0, \mathbf{v_0}) \rightarrow (l_1, \mathbf{v_1}) \rightarrow (l_2, \mathbf{v_2}) \rightarrow \ldots$, where $(l_0, \mathbf{v_0}) \in init$. A state $(l_k, \mathbf{v_k})$ is said to be* reachable, *if there exist a finite run $(l_0, \mathbf{v_0}) \rightarrow (l_1, \mathbf{v_1}) \ldots \rightarrow (l_k, \mathbf{v_k})$. A* timed trace, *denoted as $\omega = (\sigma_1, \delta_1)(\sigma_2, \delta_2) \ldots (\sigma_n, \delta_n) \ldots$, is a finite or an infinite sequence of pairs (σ_i, δ_i), where σ_i is an event of Σ, and $\delta_i \in \mathbb{R}_+$ denotes the delay between the occurrences of σ_i and σ_{i+1}. We denote by $time(\omega) = \sum_{k=1}^{n} \delta_i$ the (limit of the) sum of all the delays in the timed trace ω. A trace ω is accepted by H, if there exists a run of H over the elements of ω. The run over ω are said to be* divergent, if ω is infinite and $time(\omega) = \infty$. The hybrid system H is said to be *nonzeno if every finite run of H is a prefix of some divergent run of H.*

Since the state space of a RHA is uncountably infinite, we use the symbolic representation to perform the analysis over this space. The symbolic state of the automaton H is a pair (l, z), where l corresponds to a location of L, and z is a region over X. The pair (l, z) represents the set of states $\{(l, \mathbf{v}) | \mathbf{v} \in z\}$. The continuous successor operator of a symbolic state (l, z) is defined as: $Post_c((l, z)) = \{(l, \mathbf{v}') | (l, \mathbf{v}) \xrightarrow{\delta} (l, \mathbf{v}'), \mathbf{v} \in z, \delta \in \mathbb{R}_+\}$. Similarly, we define the discrete successor operator of a symbolic state (l, z), over a transition $e \in E$, defined as: $Post_d((l, z), e) = \{(l', \mathbf{v}') | (l, \mathbf{v}) \xrightarrow{e} (l', \mathbf{v}'), \mathbf{v} \in z\}$. We define the successor operator $Post$, over an edge $e \in E$, as the composition of the discrete and continuous successor operators; i.e., $Post((l, z), e) = Post_c \circ Post_d((l, z), e)$.

Computing discrete and continuous successors is equivalent to performing some geometrical operations on n-dimensional regions (Alur et al., 1995). We note that some tools like HyTech (Henzinger et al., 1997), and PHAVer (Frehse, 2005), implement such operations on regions, using polyhedral libraries, to perform the symbolic

[3] $\mathbb{R}_+$ denotes the set of positive real numbers.

analysis of a more general class of automata called *linear hybrid automata*.

3. A MODEL FOR THE DIAGNOSIS OF HYBRID SYSTEMS

To perform a reliable model-based diagnosis, we must have a complete system model, describing both normal and faulty behaviors of the system. The normal behavior of the system corresponds to the expected operating modes and trajectories, whereas the faulty behaviors correspond to deviations from the normal behaviors, due to malfunctions called *failures*. These malfunctions typically affects the discrete and continuous dynamics of the system. Generally, due to limited instrumentations, these malfunctions are not directly measurable, and so, their detection cannot be straightforwardly performed. In a hybrid systems framework, a failure can affect the continuous dynamics; e.g., a stuck of a valve modifying the corresponding feeding rate, or the discrete behavior; e.g., a fluid level sensor generating an alarm when it detects an overflow.

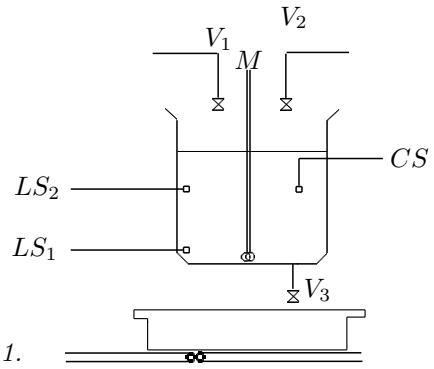
In our work, we use the RHA model as our basis for the diagnosis of hybrid systems. We assume that all continuous variables are not measurable. Furthermore, the event set Σ is partitioned into observable and unobservable sets of events $\Sigma = \Sigma_o \cup \Sigma_{uo}$ (Lin and Wonham, 1988). Hence, the diagnosis system can only observes the events in Σ_o, which correspond, typically, to discrete control commands and sensor feedbacks.

The diagnosis model, we use, must fulfill the following requirements:

- The system is modeled by an RHA, $H = (L, X, \Sigma, E, inv, flow, init)$.
- Failures are modeled using discrete transitions on unobservable events. We denote by $\Sigma_f \subseteq \Sigma_{uo}$, the set of failure events.
- The failure set Σ_f is partitioned into m disjoint failure subsets (or modes) $\Sigma_f = \{F_1, \ldots, F_m\}$.
- Failures are permanent; i.e., return to the normal behavior is not possible.
- Only single failure scenarios are considered. However, extensions of our work to deal with multiple failures scenarios can be easily developed.
- The system starts in normal mode; i.e., no failure has already happened.

To illustrate our approach, we consider the following manufacturing system example. This example is used for illustration purposes throughout the paper.

Consider the simple fluid mixing system, shown in Figure 1. Two fluids of different types are mixed into an empty container. The first fluid is introduced by opening the valve V_1. The second fluid, is introduced through the valve V_2 (the first fluid serves to dilute the second). As soon as the level of the first fluid reaches the maximal level, detected by the sensor LS_2, the valve V_1 is closed, and the second fluid is introduced until reaching the desired concentration of the mix, notified by the concentration sensor CS. In the next step, the mix is evacuated through the valve V_3, and



Example 1.

Fig. 1. *A Simple Fluid Mixing System.*

filled in a final product box. When the container becomes empty (notified by the sensor LS_1), the system waits for 5 t.u., then, starts the next mixing cycle.

The RHA in Fig. 2, captures the normal and the faulty behaviors of the system. The model has three continuous variables, x_1 measuring the mix volume in the container, x_2 measuring the concentration of the mix, and a clock x_3 used as a timer.

For simplicity purposes, we only capture the case of two failures, affecting the dynamics of the variables x_1 and x_2, of the mixing system. The first failure, modeled by the event sc_1, corresponds to a stuck of the valve V_1 in the close position. The second failure, modeled by the event so_3, corresponds to a stuck of the valve V_3 in the open position. We note that the transition edges on unobservable events (sc_1, so_3, and σ_u) are represented using dashed arrows. The observable events ls_1, ls_2, and cs, correspond to notifications of the sensors LS_1, LS_2, and CS, respectively.

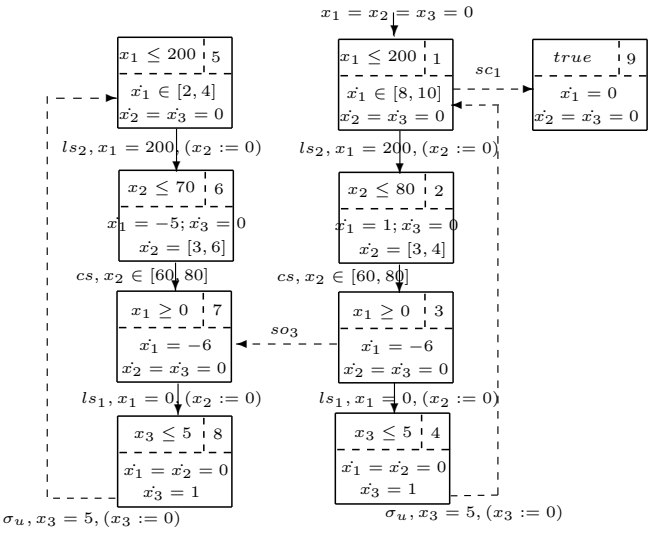


Fig. 2. *An RHA Model of the Simple Fluid Mixing System.*

4. DESCRIPTION OF THE DIAGNOSIS PROCEDURE

We describe in this section our failure diagnosis procedure which constitutes the heart of our diagnosis approach. We assume that the system is modeled as an RHA, $H =$

$(L, X, \Sigma, E, inv, flow, init)$, designed under the conditions discussed in Section 3. We assume that the system model H is nonzeno. This requirement assures the time progress of the system runs, which constitutes a necessary condition to apply our diagnosis procedure.

Before describing the diagnosis procedure, let define the needed formal background to perform that. We define the projection operator P, which erases all unobservable events from a given timed sequence, and summing the delays of the erased events. We associate to each failure mode $F_i,\ i \in \{1, \ldots, m\}$, a failure label $\mathcal{F}_i$. We denote by $\mathcal{N}$ the normal label, associated to normal states of the system. Let Φ denotes the set of $m+1$ diagnosis labels $\{\mathcal{N}, \mathcal{F}_1, \ldots, \mathcal{F}_m\}$. We define the label propagation operator $\otimes : \Phi \times \Sigma \rightarrow \Phi$ as follows:

$$\phi \otimes \sigma = \begin{cases} \mathcal{F}_i, \text{ if } & \phi = \mathcal{F}_i, \\ \mathcal{F}_i, \text{ if } & \phi = \mathcal{N} \text{ and } \sigma \in F_i, \\ \mathcal{N}, \text{ if } & \phi = \mathcal{N} \text{ and } \sigma \notin \Sigma_f. \end{cases}$$

A *diagnosis state* q is a set of triplets, $\{(l_i, z_i, \phi_i) \in L \times \Psi(X) \times \Phi, i \in \{1, \ldots, k\}\}$. Let Q denotes the set of all diagnosis states. We define the function $\mathcal{A} : Q \rightarrow \Phi \cup \{\perp\}$, which evaluates a given diagnosis state $q \in Q$. The function $\mathcal{A}$ returns a failure label $\mathcal{F}_i,\ i \in \{1, \ldots, m\}$ (respect. the normal label $\mathcal{N}$), whether all the elements in q contain the label $\mathcal{F}_i$ (respect. $\mathcal{N}$), and returns the symbol $\perp$, otherwise. The function $\mathcal{A}$ is formally defined as follows:

For a given state $q = \{(l_1, z_1, \phi_1), \ldots, (l_k, z_k, \phi_k)\}$,

$$\mathcal{A}(q) = \begin{cases} \phi, \text{ if } q \neq \emptyset \text{ and } \phi_1 = \cdots = \phi_k = \phi, \\ \perp, \text{ otherwise.} \end{cases}$$

A diagnosis state q is said to be F_i-certain, for $i \in \{1, \ldots, m\}$, if $\mathcal{A}(q) = \mathcal{F}_i$.

Before applying our diagnosis procedure, we add to the system model H, an extra clock y, taking the value 0, in all the initial states, and resets to 0 after any location switches, on observable events [4]. The clock y allows the measurement of the elapsed time, since the occurrence of the last observable event.

In the following, we give a formal definition of the *time bounded unobservable reach function*, denoted UR.

The function UR, illustrated in Algorithm 1, computes the set of reachable states from q_{in}, over executing any possible sequences of unobservable events, and the elapse of exactly y_{max} t.u., since the last occurrence of an observable event . In fact, the clock y will be equal to y_{max}, for all reached states. The failure labels are propagated during this computation using the operator $\otimes$.

We note that, due to the time divergence property provided by the nonzenoness assumption, the halting condition of this algorithm is guaranteed. Hence, the clock y will necessarily exceed the bound y_{max}, after a finite number of successor computations (especially, when considering the case of a cycle of transitions on unobservable events).

[4] We add the assignment $y := 0$ to the reset condition of any transition edges, labeled with observable events.

Algorithm 1 Function $UR(q_{in} \in Q, y_{max} \in \mathbb{N}) : Q$

1: Initially, $Waiting := \{(l, Post_c(z), \phi) | (l, z, \phi) \in q_{in}\}$, $Visited := Waiting$.
2: **repeat**
3: Get and remove an element (l, z, ϕ) from $Waiting$.
4: **for all** transitions $l \xrightarrow{\sigma_{uo}} l'$ *in* E, such that $\sigma_{uo} \in \Sigma_{uo}$ **do**
5: Compute (l', z', ϕ'), such that $(l', z') := Post((l, z), l \xrightarrow{\sigma_{uo}} l')$, and $\phi' := \phi \otimes \sigma_{uo}$.
6: Add (l', z', ϕ') to $Visited$, if $z' \neq \emptyset$.
7: Add $(l', z' \wedge (y < y_{max}), \phi')$ to $Waiting$, if $z' \wedge (y < y_{max}) \neq \emptyset$.
8: **end for**
9: **until** $Waiting = \emptyset$.
10: $q_{out} := \{(l, \tilde{z}, \phi)$, such that $(l, z, \phi) \in Visited$, $\tilde{z} := z \wedge (y = y_{max})$, and $\tilde{z} \neq \emptyset\}$.
11: return q_{out}.

The diagnosis procedure is a passive [5], state estimator algorithm, run on-line with the system. The procedure takes as input a sequence of observable events, and estimates the possible states of the system. Each estimated state can be reached over a run, on a timed trace, having an identical observable projection with the timed sequence of events, generated by the system. Using the estimated states, the procedure performs the diagnosis of the system.

Algorithm 2, illustrates our diagnosis procedure.

Our diagnosis procedure operates as a state machine. The current state, denoted $q_{cur} \in Q$, captures an estimation of the current system states, reached after observing a sequence of events generated by the system. The set of estimated states are symbolically captured, using symbolic states $\{(l_i, z_i), i \in \{1, \ldots, k\}\}$. We associate to each symbolic state (l_i, z_i) in q_{cur}, a diagnosis label ϕ_i. The label ϕ_i tracks the occurred failures in runs, reaching each state of (l_i, z_i). The label ϕ_i is equal to $\mathcal{F}_j, j \in \{1, \ldots, m\}$, whether a failure from the set F_j occurred. If the system operates in the normal mode, the label ϕ_i is equal to $\mathcal{N}$.

The initial diagnosis state is denoted $(l_0, z_0, \mathcal{N})$. A switch of the procedure, to the next diagnosis state, is triggered by (1) the observation of an event, or (2) the expiration of a timer t, without observing any events.

The diagram in Fig. 3, informally describes, the different operations performed while executing the main loop of the diagnosis procedure.

As illustrated in Fig. 3, we start the main loop of the procedure, by computing an integer waiting delay τ. The value τ corresponds to the smallest integer delay, after which, all the estimated states are F_i-faulty; i.e., a failure from the set F_i has affected the system before reaching any estimated states. So, the estimated diagnosis state q_{cur}, after this delay, will be F_i-certain. A finite value

[5] The procedure does not influence the behavior of the system to be diagnosed.

Algorithm 2 Diagnosis Procedure:

Require: Parameter T.

$q_{cur} := \{(l_0, z_0, \mathcal{N})\}$.
$y_{ref} := 0$.
loop
  if $\mathcal{A}(UR(q_{cur}, y_{ref} + T)) \neq \mathcal{F}_i$, for any $i \in \{1, \ldots, m\}$ **then**
    $\tau := T$.
  else
    $\tau := inf\{\theta \in \{1, \ldots, T\}$, such that $\mathcal{A}(UR(q, y_{ref} + \theta)) = \mathcal{F}_i, i \in \{1, \ldots, m\}\}$.
  end if
  Initialize the timer t to 0.
  Arm the expiration of the timer t after τ t.u.
  Await for an event $\sigma \in \Sigma_o$, or for the expiration of the timer t.
  Compute $q_{succ} := UR(q_{cur}, y_{ref} + t)$.
  if an event $\sigma \in \Sigma_o$ is observed **then**
    $q_{cur} := \{(l'_i, z'_i, \phi_i)$, such that $(l'_i, z'_i) = Post_d((l_i, z_i), \xrightarrow{\sigma})$, for all $(l_i, z_i, \phi_i) \in q_{succ}\}$.
    $y_{ref} := 0$.
  else
    $q_{cur} := q_{succ}$.
    $y_{ref} := y_{ref} + t$
  end if
  if $\mathcal{A}(q_{cur}) = \mathcal{F}_i, i \in \{1, \ldots, m\}$ **then**
    Announce the occurrence of a 'Failure F_i'.
    Halt the algorithm.
  else if $\mathcal{A}(q_{cur}) = \mathcal{N}$ **then**
    Announce that the system has 'No failures'.
  else
    Announce that current state of system is an 'Uncertain state.'
  end if
end loop

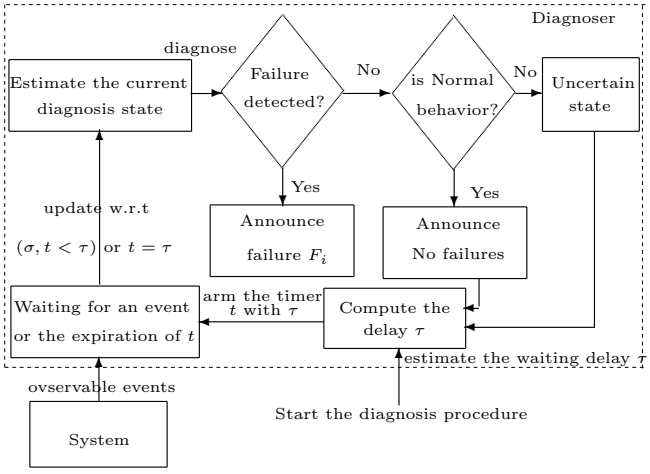


Fig. 3. *Diagnosis Procedure Flow-Chart.*

for τ may not exist in some cases (e.g., when the system operates permanently in the normal behavior). Therefore, we check the existence of the value τ, into a bounded set of discrete integer values $\{1, \ldots, T\}$, called the *lookahead time window* (Ben.Hadj-Alouane et al., 1994), given the parameter T. If the estimated diagnosis state q_{cur}, after the elapse of T t.u., is not F_i-certain, τ will be equal to the value T. In this case, the diagnosis procedure will only update the diagnosis state q_{cur}, without announcing any failures.

In the next step, we arm the expiration of the timer t after the elapse τ t.u., and we await for the occurrence of an observable event, or the expiration of the timer t. If one of these conditions occurs, the algorithm updates the diagnosis state q_{cur}, w.r.t. the elapsed time t, and the eventual observed event σ, as described in the following:

We compute, in the first step, the time successor of the diagnosis state q_{cur}, denoted q_{succ}, using the function UR.

In the second step, we compute the discrete successors of the elements of q_{succ}, if an event $\sigma \in \Sigma_o$ have been observed. The obtained result corresponds to the current diagnosis state q_{cur}. If no events have been observed, and the timer t expires, the current diagnosis state q_{cur} corresponds to the time successor q_{succ}.

In the last step, we evaluate the diagnosis state q_{cur}, using the function $\mathcal{A}$. As illustrated in Diagram 3, there are three possible diagnostics, regarding the result of $\mathcal{A}(q_{cur})$:

(1) $\mathcal{A}(q_{cur})$ is F_i-certain, for $i \in \{1, \ldots, m\}$: we announce the occurrence of a 'Failure F_i' In this case, all the estimated system states, elements of q_{cur}, are reached over runs, containing a failure from the set F_i.
(2) $\mathcal{A}(q_{cur}) = \mathcal{N}$: we announce that the system operates in the 'Normal' mode. So, no failures has been occurred, before reaching any estimated states in q_{cur}.
(3) $\mathcal{A}(q_{cur}) = \perp$: we announce that the system has an 'uncertain state'. In this case, the estimated states of the system contain at least a pair of states, where the first is F_i-faulty, $i \in \{1, \ldots, m\}$, and the second is Normal, or F_j-faulty, $j \in \{1, \ldots, m\}$ and $j \neq i$.

We note that our procedure may output, indefinitely, the diagnostic 'uncertain state'. In this case, our procedure cannot distinguish between two runs of the system, having identical observable behavior, the first contains a failure from F_i, and the second not, for a given $i \in \{1, \ldots, m\}$. In this case, the system model is said to be *not diagnosable* (Sampath et al., 1995). Diagnosability issues constitute the interest of our future works.

Example 2. To illustrate the execution of our diagnosis algorithm, let consider the Example 1. We suppose that the system generates the following traces: $\omega_1 = (ls_2, 22)(cs, 22)(so_3, 14)(ls_1, 23)(\sigma_u, 5)(ls_2, 50)$ and $\omega_2 = (ls_2, 22)(cs, 22)(ls_1, 37)(\sigma_u, 5)(sc_1, 10)(\epsilon, 100)$. Both traces ω_1 and ω_2 correspond to failure behaviors, the former contains a failure so_3, and the latter contains a failure sc_1. We note the use of the silent action ϵ, to represent the time progress of the run over ω_2, without observing any more events. We denote by $P(\omega_1) = (ls_2, 22)(cs, 22)(ls_1, 37)$ $(ls_2, 55)$ and $P(\omega_2) = (ls_2, 22)(cs, 22)$ $(ls_1, 37)(\epsilon, 115)$, the observable projections of the traces ω_1 and ω_2, respectively. We assume that the failure set $\Sigma_f = \{so_3, sc_1\}$ is partitioned as: $F_1 = \{so_3\}, F_2 = \{sc_1\}$, and T equals to 30. We give in the following, some execution statements, of our diagnosis procedure, given in the Algorithm 2. We

suppose that the system generates the timed trace $P(\omega_1)$ in our first execution example:

(1) Initially $q_{cur} := \{(1, (x_1 = x_2 = x_3 = y = 0), \mathcal{N})\}$.
(2) $\tau := 26$. The event ls_2 is observed at $t = 22$, $q_{cur} := \{(2, x_1 = 200 \wedge x_2 = x_3 = y = 0, \mathcal{N})\}$, the diagnostic 'Normal' is output.
(3) $\tau := 30$. The event cs is observed at $t = 22$, $q_{cur} := \{(3, x_1 = 222 \wedge x_2 \in [66, 80] \wedge x_3 = y = 0, \mathcal{N})\}$, the diagnostic 'Normal' is output.
(4) $\tau := 30$. The timer expires at $t = 30$, $q_{cur} := \{(3, x_1 = 42 \wedge x_2 \in [66, 80] \wedge y = 30 \wedge x_3 = 0, \mathcal{N}), (7, x_1 = 42 \wedge x_2 \in [66, 80] \wedge y = 30 \wedge x_3 = 0, \mathcal{F}_1)\}$, the diagnostic 'Uncertain state' is output.
(5) $\tau := 30$. The event ls_1 is observed at $t = 7$, $q_{cur} := \{(4, x_1 = x_2 = x_3 = y = 0, \mathcal{N}), (8, x_1 = x_2 = x_3 = y = 0, \mathcal{F}_1)\}$, the diagnostic 'Uncertain state' is output.
(6) $\tau := 30$. The timer expires at $t = 30$, $q_{cur} := \{(1, x_1 = 200 \wedge y = 30 \wedge x_2 = x_3 = 0, \mathcal{N}), (5, x_1 \in [50, 100] \wedge y = 30 \wedge x_2 = x_3 = 0, \mathcal{F}_1), (9, x_1 \in [0, 200] \wedge y = 30 \wedge x_2 = x_3 = 0, \mathcal{F}_2)\}$, the diagnostic 'Uncertain state' is output.
(7) $\tau := 30$. The event ls_2 is observed at $t = 25$, $q_{cur} := \{(6, x_1 = 200 \wedge x_2 = x_3 = y = 0, \mathcal{F}_1)\}$. The procedure announces a 'Failure F_1', and the algorithm halts.

We remark that, the observation of the event ls_2, later than the expected date in the normal behavior, allows the detection of the failure stuck open, affecting the valve V_3. In other cases, the system may become silent after the occurrence of a failure; i.e., the system stop generating events, as illustrated in the following example.

Let us consider now the example of the trace ω_2. We suppose that the system generates the timed sequence $P(\omega_2)$. When the timed sequence $(ls_2, 22)(cs, 22)(ls_1, 37)$, is observed, the diagnosis procedure repeats the statements 1 to 6, performed in the previous example. After that, the system will stop generating events, while the time continue to elapse for 115 t.u.. We give in the following, the continuation of the diagnosis procedure execution.

- $\tau := 30$. The timer expires at $t = 30$, $q_{cur} := \{(5, x_1 \in [110, 200] \wedge y = 60 \wedge x_2 = x_3 = 0, \mathcal{F}_1), (9, x_1 \in [0, 200] \wedge y = 60 \wedge x_2 = x_3 = 0, \mathcal{F}_2)\}$, the diagnostic 'Uncertain state' is output.
- $\tau := 30$. The timer expires at $t = 30$, $q_{cur} := \{(5, x_1 \in [170, 200] \wedge y = 60 \wedge x_2 = 0 \wedge x_3 = 0, \mathcal{F}_1), (9, x_1 \in [0, 200] \wedge y = 60 \wedge x_2 = 0 \wedge x_3 = 0, \mathcal{F}_2)\}$, the diagnostic 'Uncertain state' is output.
- $\tau := 16$. The timer expires at $t = 16$, $q_{cur} := \{(9, x_1 \in [0, 200] \wedge y = 76 \wedge x_2 = x_3 = 0, \mathcal{F}_2)\}$. The procedure announces the occurrence of a 'Failure F_2', and the algorithm halts.

5. CONCLUSION AND FUTURE WORKS

In this paper, we presented an online approach for the diagnosis of systems modeled with Rectangular Hybrid Automata. This approach is based on the use of a diagnosis procedure, performing an online estimation of the system states, within a lookahead time window, given a record of observable timed events, generated by the system. The diagnosis procedure is driven by event observations, or time elapsing. Our approach is applicable under the nonzenoness condition on the used system model.

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